## 5.4 Indefinite Integrals, Net Change Theorum

Goals:  Understand

1. The process of finding an antiderivative or indefinite integral requires the reverse process of finding a derivative.
2. The result of a Definite Integral is a numerical value.  The result of an Indefinite Integral is a function.
3. The Net Change Theorum is an interpretation of the Fundamental Theorum of Calculus (FTC).

Homework:  Study 5.4  # 1;  5 - 11, 15, 19, ...31; 37, 41, 45, 49, 51, 59, 61

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## 5.4 Indefinite Integrals, Net Change Theorum

Find $\frac{dy}{dx}$:   $y = x^5 + 3$

Find $\frac{dy}{dx}$:   $y = x^5 + 2\pi$

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## 5.4 Indefinite Integrals, Net Change Theorum

*Suppose you are given:*

$$f'(x) = 5x^4$$

How would you find *f(x)*?

Inverse of finding the derivative:
1. multiply coefficient by exponent
2. subtract 1 from the exponent

Therefore:
1. Add 1 to exponent
2. Divide by new exponent.

Is there only one *f(x)*?

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$$\frac{5x^{4+1}}{4+1} = x^5 + C$$

## 5.4 Indefinite Integrals, Net Change Theorum

Solve the differential equation: $\frac{dy}{dx} = 5x^4$

$$\int 5x^4\,dx$$

$$= \frac{5x^5}{5} + c$$

$$= x^5 + c$$

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## 5.4 Indefinite Integrals, Net Change Theorum

**Antidifferentiation** or **Indefinite Integration**

The **operation of finding all the solutions** to the differential equation, $dy = f(x)\,dx$, is called antidifferentiation or indefinite integration and is denoted by the integral sign $\int$. The solution when $F'(x) = f(x)$ is:

$$y = \int f(x)dx = F(x) + c$$

integrand — variable of integration — constant of integration

$$y = \int F'(x)dx = F(x) + c$$

[ looking for the function, F(x), for which the integrand is the derivative, F'(x) = f(x) ]

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## 5.4 Indefinite Integrals, Net Change Theorum

$$y = \int f(x)dx = F(x) + c$$

integrand — variable of integration — constant of integration

$$\int 5x^4\,dx = \frac{5x^5}{5} + c = x^5 + c$$

Note: **Operators $\int$ and $dx$ are no longer present after the operation of integration is performed.**

The integral sign $\int$ and $dx$ indicate the operation of integration the same way that a plus sign indicates the operation of addition.

For the division problem, $12 \div 2 = 6$, the result no longer has the operator, $\div$ Instead, it contains only the result, 6.

Likewise, for integration, the result no longer has either the integral sign $\int$ or the $dx$. Therefore, to continue **to write the $\int$ or the $dx$** after the operation of integration has been performed **is not correct**.

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## 5.4 Indefinite Integrals, Net Change Theorum

Rules of:

| Differentiation | Integration |
|---|---|
| $\frac{d(c)}{dx} = 0$ | $\int 0\,dx = c$ |
| $\frac{d(kx)}{dx} = k$ | $\int k\,dx = kx + c$ |
| $\frac{d(x^n)}{dx} = nx^{n-1}$ | $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$ |
| $\frac{d(\sin x)}{dx} = \cos x$ | $\int \cos x\,dx = \sin x + c$ |
| $\frac{d(\cos x)}{dx} = -\sin x$ | $\int \sin x\,dx = -\cos x + c$ |

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## 5.4 Indefinite Integrals, Net Change Theorum

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \sqrt[4]{x^5}\,dx$$

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## 5.4 Indefinite Integrals, Net Change Theorum

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \overline{x^5}\,dx$$

$$= \int x^{5/4}\,dx$$

$$= \frac{x^{\frac{9}{4}}}{\frac{9}{4}} + C = \frac{4}{9}x^{\frac{9}{4}} + C$$

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## 5.4 Indefinite Integrals, Net Change Theorum

$$\int \left(8x^3 + \frac{1}{2x^2}\right) dx$$

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5.4 Indefinite Integrals, Net Change Theorum

$$\int \left(8x^3 + \frac{1}{2x^2}\right) dx = \int \left(8x^3 + \frac{1}{2}x^{-2}\right) dx$$

$$= \frac{8x^4}{4} + \frac{1}{2}\frac{x^{-1}}{-1} + C$$

$$= 2x^4 - \frac{1}{2x} + C$$

$$= 2x^4 - \frac{1}{2x} + C$$

check: $\dfrac{d\left(2x^4 - \frac{1}{2}x^{-1} + c\right)}{dx}$

$$= 8x^3 + \frac{1}{2}x^{-2} + 0$$

$$= 8x^3 + \frac{1}{2x^2} \checkmark$$

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5.4 Indefinite Integrals, Net Change Theorum

$$\int \sqrt{t}\,(t^2 + 3t + 2)\,dt$$

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5.4 Indefinite Integrals, Net Change Theorum

$$\int \sqrt{t}\,(t^2 + 3t + 2)\,dt$$

$$\int \left(t^{5/2} + 3t^{3/2} + 2t^{1/2}\right) dt$$

$$= \frac{t^{7/2}}{7/2} + \frac{3t^{5/2}}{5/2} + \frac{2t^{3/2}}{3/2} + C$$

$$= \frac{2}{7}t^{7/2} + \frac{6}{5}t^{5/2} + \frac{4}{3}t^{3/2} + C$$

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5.4 Indefinite Integrals, Net Change Theorum

$$\int \frac{\sin x}{1 - \sin^2 x}\,dx$$

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## 5.4 Indefinite Integrals, Net Change Theorum

$$\int \frac{\sin x}{1 - \sin^2 x}\,dx = \int \frac{\sin x}{\cos^2 x}\,dx$$

$$= \int \frac{1}{\cos x}\,\frac{\sin x}{\cos x}\,dx$$

$$= \int \sec x \tan x \,dx$$

$$= \sec x + c$$

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## 5.4 Indefinite Integrals, Net Change Theorum

Fundamental Theorem of Calculus (FTC)

If : 1. a function ***f* is continuous on [a, b]** and

2. ***F* is an antiderivative of f** on the interval,

then: $\int_a^b f(x)dx = F(b)-F(a)$

$$\int_a^b F'(x)dx = F(b)-F(a)$$

Interpret FTC as **NET CHANGE** in the antiderivative from a to b

$$\int_0^{10} v(t)dt = h(10)-h(0)$$

when v(t) = h'(t), this definite integral results in net change in distance from t=0 to t=10

graphs

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5.4 Indefinite Integrals, Net Change Theorum

G:  Honey bee population starts with 100 bees and increases at a rate of n'(t) bees per week.
F:  What does the following represent?

$$100 + \int_0^{15} n'(t)dt$$

graphs

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5.4 Indefinite Integrals, Net Change Theorum

G:  $a(t) = 2t + 3$ (in m/s²), $v(0)=-4$,  $0 \leq t \leq 3$

F:  a)  velocity at time t
    b)  distance traveled from t=0 to t=3 sec

graphs

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## 5.4 Indefinite Integrals, Net Change Theorum

G: $a(t) = 2t + 3$ (in m/s²), $v(0) = -4$, $0 \le t \le 3$

F: a) velocity at time t

b) distance traveled from t=0 to t=3 sec

a) velocity at time t

$v(t) = \int a(t)\,dt = \int (2t + 3)dt = \frac{2t^2}{2} + 3t + c$

$v(t) = t^2 + 3t + c$

$v(0) = 0^2 + 3(0) + c = -4 \quad \therefore \quad c = -4$ and

$\boxed{v(t) = t^2 + 3t - 4}$ velocity at time t in m/s

b) distance traveled from t=0 to t=3 sec

$h(t) = \int_0^3 v(t)dt = \int_0^3 (t^2 + 3t - 4)dt = \left[\frac{t^3}{3} + \frac{3t^2}{2} - 4t\right]_0^3$

graphs

$= \frac{27}{3} + \frac{27}{2} - 12 - 0$

$= (54+81-72)/6 = 63/6 = \boxed{10.5 \text{ m}}$ distance traveled from t=0 to t=3 sec

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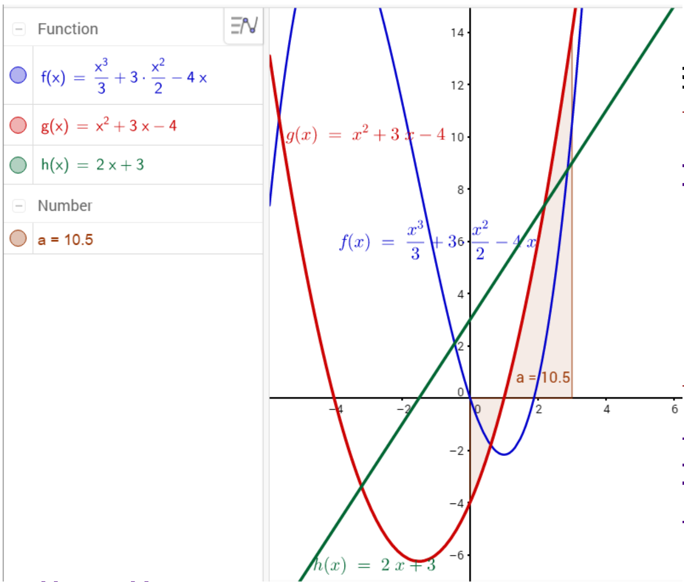
## 5.4 Indefinite Integrals, Net Change Theorum

G: $a(t) = 2t + 3$ (in m/s²), $v(0) = -4$, $0 \le t \le 3$

F ... sec

a) v




b) d

h(t

graphs

$= (54+81-72)/6 = 63/6 = \boxed{10.5 \text{ m}}$ distance traveled from t=0 to t=3 sec

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## 5.4 Indefinite Integrals, Net Change Theorum

Think Questions: True or False?

1. The antiderivative of *f(x)* is unique.

2. Each antiderivative of an *n*th-degree polynomial function is an *(n+1)*th degree polynomial function.

3. If *p(x)* is a polynomial function,then *p* has exactly one antiderivative whose graph contains the origin.

4. If *F(x)* and *G(x)* are antiderivatives of *f(x)*, then

   $F(x) = G(x) + c$

5. If $f'(x) = g(x)$, then $\int g(x)\,dx = f(x) + c$

6. $\int f(x)g(x)dx = \int f(x)dx \int g(x)dx$

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## 5.4 Indefinite Integrals, Net Change Theorum

Think Questions: True or False?

F 1. The antiderivative of *f(x)* is unique.

T 2. Each antiderivative of an *n*th-degree polnomial function is an *(n+1)*th degree polynoial function.

T 3. If *p(x)* is a polynomial function,then *p* has exactly one antiderivative whose graph contains the origin.

T 4. If *F(x)* and *G(x)* are antiderivatives of *f(x)*, then

   $F(x) = G(x) + c$

T 5. If $f'(x) = g(x)$, then $\int g(x)\,dx = f(x) + c$

F 6. $\int f(x)g(x)dx = \int f(x)dx \int g(x)dx$

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5.4 Indefinite Integrals, Net Change Theorum Extras

G: $f''(x) = x^2,\ f'(0) = 8,\ f(0) = 4$

F: Solve the diff. eq. F: $f(x)$

1. Start with 2nd Derivative. Integrate
2. Substitute given to find c.
3. State 1st Derivative
4. Integrate 1st Derivative
5. Substitute given to find $c_2$.
6. State function.



5.4 Indefinite Integrals, Net Change Theorum Extras

G: $f''(x) = x^2,\ f'(0) = 8,\ f(0) = 4$

F: Solve the diff. eq. F: $f(x)$

1. Start with f '' (x). Integrate

$$f'(x) = \int f''(x)\,dx = \int x^2\,dx = \frac{x^3}{3} + c$$

$$f'(x) = \frac{1}{3}x^3 + c$$

2. Substitute given to find c.

$$f'(0) = 0 + c = 8 \quad \therefore c = 8$$

3. State f ' (x).

$$f'(x) = \frac{1}{3}x^3 + 8$$

half way there

4. Integrate f ' (x).

$$f(x) = \int f'(x)\,dx = \int \left(\frac{1}{3}x^3 + 8\right) dx$$

$$= \frac{1}{3}\frac{x^4}{4} + 8x + c_2 = \frac{1}{12}x^4 + 8x + c_2$$

5. Substitute given to find $c_2$.

$$f(0) = \frac{1}{12}\cdot 0 + 8(0) + c_2 = 4 \quad \therefore c_2 = 4$$

6. State f (x).

$$f(x) = \frac{1}{12}x^4 + 8x + 4$$

## 5.4 Indefinite Integrals, Net Change Theorum

Extras

Find: Indefinite Integral & Check by Differentiation

$$\int \frac{x+6}{\sqrt{x}}\,dx$$

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## 5.4 Indefinite Integrals, Net Change Theorum

Extras

Find: Indefinite Integral & Check by Differentiation

$$\int \frac{x+6}{\sqrt{x}}\,dx = \int \frac{x+6}{x^{1/2}}\,dx = \int \left(\frac{x^1}{x^{1/2}} + \frac{6}{x^{1/2}}\right)dx$$

$$= \int \left(x^{1/2} + 6x^{-1/2}\right)dx$$

$$= \frac{x^{3/2}}{3/2} + \frac{6x^{1/2}}{1/2} + c = \frac{2}{3}x^{3/2} + 12x^{1/2} + c$$

check: $\frac{d(\quad)}{dx} = x^{1/2} + 6x^{-1/2}$

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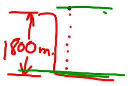
## 5.4 Indefinite Integrals, Net Change Theorum

Extras

$G: a(t) = -9.8\,m/s^2 \quad F: a)\ h(t)$




$v(t) = h'(t)$

$a(t) = v'(t) = h''(t)$

$a(t) = -9.8\,m/s^2$

b) How long to hit bottom?
ie: find t when h = 0

1. Start with 2nd Derivative. Integrate
2. Substitute given to find c.
3. State 1st Derivative
4. Integrate 1st Derivative
5. Substitute given to find $c_2$.
6. State function.

b) How long to hit bottom? At bottom when h(t) = 0

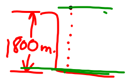


## 5.4 Indefinite Integrals, Net Change Theorum

Extras

$G: a(t) = -9.8\,m/s^2 \quad F: a)\ h(t)$

b) How long to hit bottom?
ie: find t when h = 0




$v(t) = h'(t)$

$a(t) = v'(t) = h''(t)$

$a(t) = -9.8\,m/s^2$

$v(t) = \int a(t)\,dt = \int -9.8\,dt$

$= -9.8t + c_1$

1. Start with h "(t) = a(t). Integrate

2. Substitute given to find c.

$t=0,\ v(0) = -9.8(0) + c_1 \quad \therefore c_1 = v_0 = 0$

$\therefore v(t) = -9.8t \ m/s$

3. State h ' (t) = v(t)

$h(t) = \int v(t)\,dt$

4. Integrate h '(t)=v(t)

$h(t) = \int -9.8t\,dt$

$h(t) = -\frac{9.8t^2}{2} + c_2 = -4.9t^2 + c_2$

5. Substitute given to find $c_2$.

$t=0:\ h(0) = -4.9(0) + c_2$

$c_2 = h_0 = 1800m.$

6. State h(t).

$$h(t) = -4.9t^2 + 1800 \ m.$$

b) How long to hit bottom? At bottom when h(t) = 0

$-4.9t^2 + 1800 = 0$

$+4.9t^2 = +1800$

$t^2 = \frac{1800}{4.9}$

$t = \pm 10\sqrt{\frac{18}{4.9}} = 19.17\,sec.$

negative meaningless

4.1  Antiderivatives:  Basic Concepts

Extras

Find a function f such that the graph of f has a horizontal tangent at (2,0) and $f''(x) = 2x$

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