5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

Goals:

1. Recognize and understand the Fundamental Theorem of Calculus.
2. Use the Fundamental Theorum of Calculus to evaluate Definite Integrals.
3. Recognize and understand the Mean Value Theorem for Integrals.
4. Find the average value of a function on [a,b].
5. Understand the significance of the Second Fundamental Theorem of Calculus.

Study 5.3 # 149-153, 157; 5.2 # 111, ~~113~~ 110

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ex: Indefinite Integrals

5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

Fundamental Theorem of Calculus (FTC)

If :

1. a function ***f*** **is continuous on [a, b]** and
2. ***F*** **is an antiderivative of f** on the interval,

then:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

The integral of *f* from *a* to *b* is the difference:
(antiderivative of *f* evaluated at *x*=*b*) - (antiderivative of *f* evaluated at *x*=*a*.)

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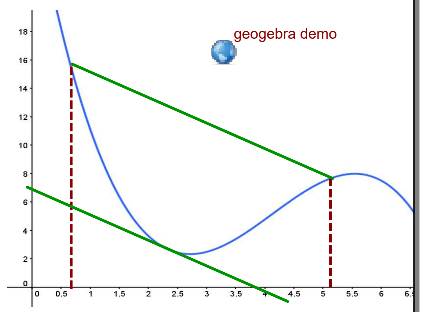
FTC

5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

Mean Value Theorum for Derivatives



Let f be:
1. continuous on closed interval [a,b] and
2. differentiable on open interval (a,b)

then $\exists$ at least one c $\in$ (a,b) $\ni$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Interpretation:

There exists at least one *c* on the interval from *a* to *b* such that the derivative at *c* equals the slope of the secant line joining the endpoints.

ALSO: There exists at least one c on the interval where the instantaneous rate of change equals the average value.

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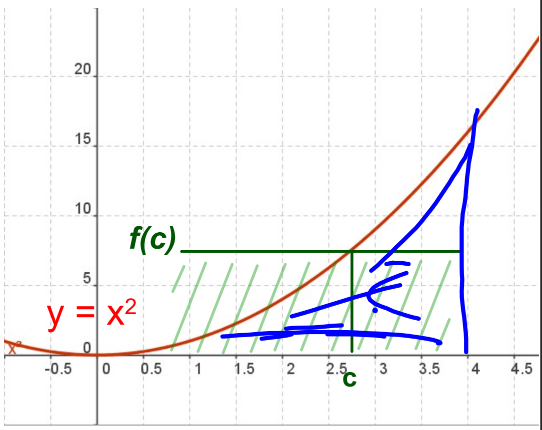
Mar 6-2:30 AM

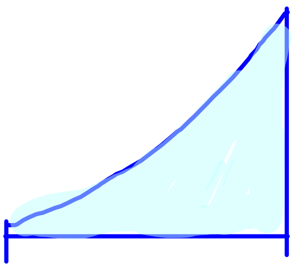
5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

Mean Value Theorum for Integrals

Let f be continuous on a closed interval [a,b]
then $\exists$ at least one c $\in$ [a,b]

$$\int_a^b f(x)dx = f(c)(b - a)$$





(equal areas: area of rectangle, f(c)*(b-a)= area under curve from a to b)

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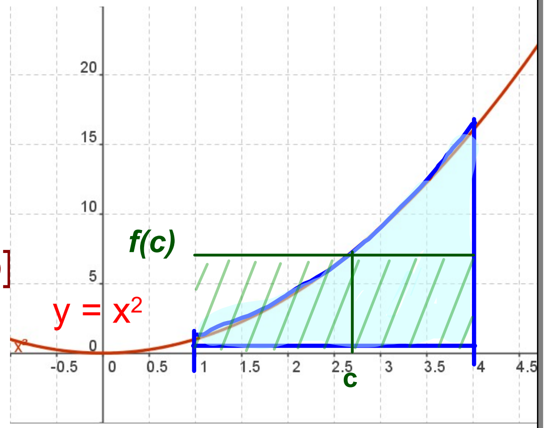
MVT

5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

Mean Value Theorum for $\boxed{\text{Integrals}}$

Let f be continuous on a closed interval [a,b]
then ∃ at least one c ∈ [a,b]

$$\int_a^b f(x)dx = f(c)(b - a)$$



$$\int_1^4 x^2\,dx = 21 = (2.64)^2\,(4 - 1)$$

(equal areas: area of rectangle, f(c)*(b-a)= area under curve from a to b)

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MVT

5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

Average Value of a Function on [a,b]

From MVT: $\int_a^b f(x)dx = f(c)(b-a)$

Multiply both members of the equation by 1/(b-a)

$$\frac{1}{b-a}\int_a^b f(x)dx = \frac{f(c)(b-a)}{(b-a)}$$

Results in an equation for **f(c), the average value of the function f(x)** on the interval [a,b]

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

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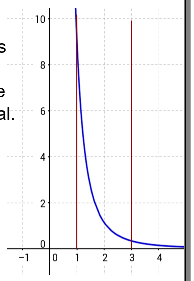
Ave Value of f

5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

G: $f(x) = \frac{9}{x^3}$

F: Ave value $[1,3]$

The values of f(x) on the interval from 1 to 3 varies from f(1)=9 to f(3)=1/3. Here we find the average value of f over the interval.



$$\text{Ave.} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

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MVT example

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G: $f(x) = \frac{9}{x^3}$

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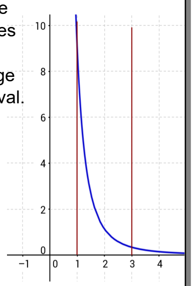
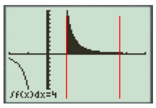


$$\text{Ave.} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{3-1}\int_1^3 9x^{-3}\,dx$$

$$= \frac{1}{2}\cdot\frac{9x^{-2}}{-2}\Bigg]_1^3 = -\frac{9}{4}\cdot\frac{1}{x^2}\Bigg]_1^3$$



$$-\frac{9}{4}\cdot\frac{1}{9} - \left[-\frac{9}{4}\cdot 1\right] =$$

$$-\frac{1}{4} + \frac{9}{4} = \frac{8}{4} = 2$$

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MVT example

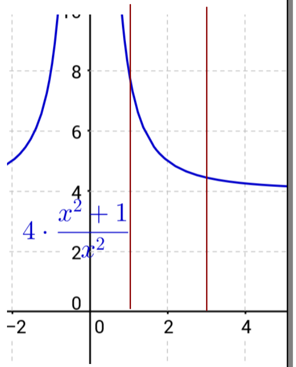
5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

G: $f(x) = \frac{4(x^2+1)}{x^2}$ F: Ave. val. on $[1,3]$

$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$

The values of f(x) on the interval from 1 to 3 varies from f(1)=8 to f(3)=40/9. We need to find the average value of f over the interval.



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Ave.Value ex 1

5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

G: $f(x) = \frac{4(x^2+1)}{x^2}$ F: Ave. val. on $[1,3]$

$\text{Ave. val} = \frac{1}{b-a}\int_a^b f(x)\,dx$

$\text{Ave} = \frac{1}{3-1}\int_1^3 \frac{4(x^2+1)}{x^2}\,dx$

$\frac{x^2+1}{x^2} = \frac{x^2}{x^2} + \frac{1}{x^2}$

$= 1 + x^{-2}$

$= 2\int_1^3 (1+x^{-2})\,dx$

$= 2\left[x + \frac{x^{-1}}{-1}\right]_1^3 = 2\left[x - \frac{1}{x}\right]_1^3 = 2\left[\left(3-\frac{1}{3}\right) - \underbrace{(1-1)}_{0}\right]$

$= 2\left[3 - \frac{1}{3}\right] = 2\left(\frac{8}{3}\right) = \frac{16}{3}$

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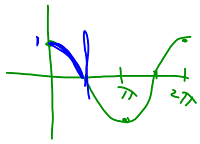
Ave.Value ex 1

5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

G: $f(x) = \cos x$ F: ave. value on $[0, \pi/2]$



$$\text{Ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

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Ave Value,ex 2

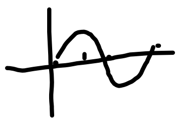
5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

G: $f(x) = \cos x$ F: ave. value on $[0, \pi/2]$

$$\text{Ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\text{Ave} = \frac{1}{\pi/2 - 0}\int_0^{\pi/2} \cos x\,dx$$

$$= \frac{2}{\pi}\sin x\Big]_0^{\pi/2} = \frac{2}{\pi}\Big[\overset{1}{\sin\tfrac{\pi}{2}} - \overset{0}{\sin 0}\Big] = \frac{2}{\pi}$$



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Ave Value,ex 2

5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

2nd Fundamental Theorem of Calculus - Concept

$$\int_1^2 t\,dt = \frac{t^2}{2}\Big]_1^2 =$$

$$\int_1^x t\,dt = \frac{t^2}{2}\Big]_1^x =$$

$$\frac{d}{dx}\left[\int_1^x t\,dt\right] = \frac{d(\qquad)}{dx} =$$

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2nd FTC, concept

5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

2nd Fundamental Theorem of Calculus - Concept

$$\int_1^2 t\,dt = \frac{t^2}{2}\Big]_1^2 = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$$

$$\int_1^x t\,dt = \frac{t^2}{2}\Big]_1^x = \frac{x^2}{2} - \frac{1}{2}$$

$$\frac{d}{dx}\left[\int_1^x t\,dt\right] = \frac{d\left(\frac{x^2}{2} - \frac{1}{2}\right)}{dx} = x$$

***Other Fundamental Theorum of Calculus

$$\boxed{\frac{d}{dx}\left[\int_1^x t\,dt\right] = x}$$

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2nd FTC, concept

5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

Try that, again.....

$$\int_1^2 (t^2+2)\,dt =$$

$$\int_1^x (t^2+2)\,dt =$$

$$\frac{d}{dx}\left[\int_1^x (t^2+2)\,dt\right] =$$

When upper limit is variable, fill place holder for t with the variable and the place holder for dt with the derivative of the variable.

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2nd FTC, more

5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

Try that, again.....

$$\int_1^2 (t^2+2)\,dt = \frac{t^3}{3}+2t\Big]_1^2 = \frac{8}{3}+4-\left(\frac{1}{3}+2\right)$$

$$= \frac{8}{3}-\frac{1}{3}+2 = \frac{7}{3}+2 = \frac{13}{3}$$

$$\int_1^x (t^2+2)\,dt = \frac{t^3}{3}+2t\Big]_1^x = \frac{x^3}{3}+2x-\left(\frac{1}{3}+2\right)$$

$$= \frac{x^3}{3}+2x-\frac{7}{3}$$

$$\frac{d}{dx}\left[\int_1^x (t^2+2)\,dt\right] = x^2+2$$

When upper limit is variable, fill place holder for t with the variable and the place holder for dt with the derivative of the variable.

2nd FTC, more

5.2 FTC, Part 2: MVT, Ave. Value, 2nd FTC

$$\frac{d}{dx}\left[\int_1^x (t^2+2)\,dt\right] = x^2+2$$

$$\frac{d}{dx}\left[\int_1^{x^3} (t^2+2)\,dt\right] =$$

$$\left[(x^3)^2+2\right](3x^2)$$

When upper limit is variable, fill place holder for t with the variable and the place holder for dt with the derivative of the variable.

if $t = x^3$

then $dt = 3x^2\,dx$

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May 1-7:17 PM

5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

$$\frac{d}{dx}\left[\int_1^x (t^2+2)\,dt\right] = x^2+2$$

$$\frac{d}{dx}\left[\int_1^{x^3} (t^2+2)\,dt\right] = \left.\frac{t^3}{3}+2t\right]_1^{x^3}$$

$$\frac{d}{dx}\left(\frac{x^9}{3}+2x^3-\frac{1}{3}-2\right) = 3x^8+6x^2$$

$$\left[(x^3)^2+2\right](3x^2)$$

$$(x^6+2)(3x^2) = 3x^8+6x^2$$

When upper limit is variable, fill place holder for t with the variable and the place holder for dt with the derivative of the variable.

if $t = x^3$

then $dt = 3x^2\,dx$

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5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

2nd FTC

If f is continuous on an open interval I containing a,

then, for every x on the interval:

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

***

$$\boxed{\frac{d}{dx}\left[\int_1^x t\,dt\right] = x}$$

2nd FTC

5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC

***

$$\boxed{\frac{d}{dx}\left[\int_1^x t\,dt\right] = x}$$

Advantage:

$$\frac{d}{dx}\left[\int_1^x \sqrt{t-1}\,dt\right]$$

$$= \sqrt{x-1}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

cannot integrate - no rule yet

$$\sqrt{x-1} = u^n \neq x^n$$

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2nd FTC

5.3 FTC, Part 2: MVT, Ave. Value, 2nd FTC ***MVT concept covered, but MVT problems not covered this semester.***

G: $f(x) = \frac{9}{x^3}$ F: $c \in [1,3] \ni$ MVT applies

f(x) cont on [1,3] $\therefore$ MVT applies $\int_a^b f(x)dx = f(c)(b-a)$

$$\int_1^3 9x^{-3}dx = 9\frac{x^{-2}}{-2}\Big]_1^3$$

$$\frac{-9}{2x^2}\Big]_1^3 = \frac{-9}{2\cdot 3^2} - \left[-\frac{9}{2\cdot 1^2}\right]$$

$$-\frac{1}{2} + \frac{9}{2} = \frac{8}{2} = 4$$

$$\int_a^b f(x)dx = f(c)(b-a)$$

$$\boxed{\int_1^3 9x^{-3}dx} = f(c)(3-1)$$

$$4 = f(c)(3-1)$$

$$f(c) = 2 = \frac{9}{c^3}$$

$$c^3 = \frac{9}{2} \quad c = \sqrt[3]{\frac{9}{2}} = 1.65$$

$\in [1,3]$

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MVT example

