5.3 Fundamental Theorem of Calculus

Goals:

1. Recognize and understand the Fundamental Theorem of Calculus.
2. Use the Fundamental Theorum of Calculus to evaluate Definite Integrals.
3. Recognize and understand the Mean Value Theorem for Integrals.
4. Find the average value of a function on [a,b].
5. Understand the significance of the a Second Fundamental Theorem of Calculus.

Study 5.3 #171, 175-197

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Calculus Home Homework

ex: Indefinite Integrals

5.3 Fundamental Theorem of Calculus

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

Start with Indefinite Integration: complete the following

1. $\int (x^{3/2} + 2x - 1)\,dx =$

2. $\int (x+1)(3x-2)\,dx = \int (3x^2 + x - 2)\,dx$

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Calculus Home Homework

ex: Indefinite Integrals

5.3 Fundamental Theorem of Calculus

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

Start with Indefinite Integration: complete the following

1. $\int (x^{3/2} + 2x - 1)dx = \frac{x^{5/2}}{\frac{5}{2}} + \frac{2x^2}{2} - x + C$

$= \frac{2}{5}x^{5/2} + x^2 - x + C$

2. $\int (x+1)(3x-2)dx = \int (3x^2 + x - 2)dx$

$x^3 + \frac{x^2}{2} - 2x + C$

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Calculus Home Homework

ex: Indefinite Integrals

5.2 Fundamental Theorem of Calculus

G: $\frac{dy}{dx} = 3x^2$ F: solve for y

$dy = \frac{dy}{dx} dx$

Start with the differential of y, *dy*

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Nov 28-6:27 PM

5.3 Fundamental Theorem of Calculus

$G: \dfrac{dy}{dx} = 3x^2$ $\qquad$ $F:$ solve for y

$dy = \boxed{\dfrac{dy}{dx}}\, dx$

Start with the differential of y, *dy*

$\int dy = \int 3x^2\, dx$

$y = \dfrac{3x^3}{3} + c = x^3 + c$

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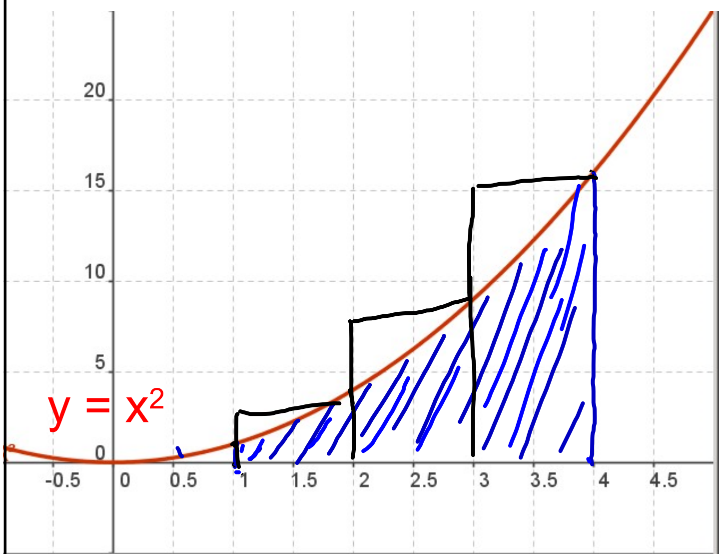
5.3 Fundamental Theorem of Calculus

Problem: Find the area under $y = x^2$ from x=1 to x=4



Estimates:

Sum of rectangles:
1(4) + 1(9) + 1(16)
= 29

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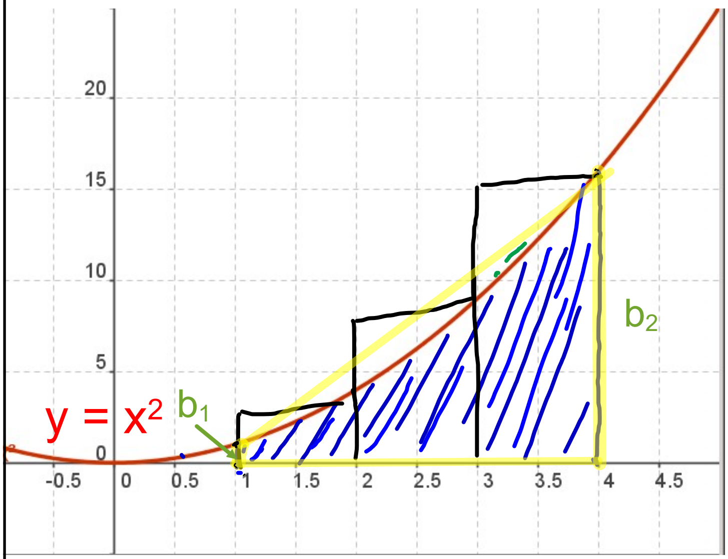
Calculus Home $\qquad$ Homework

Estimating Area, Definite Integral

5.3 Fundamental Theorem of Calculus

Problem: Find the area under $y = x^2$ from x=1 to x=4



Estimates:

Sum of rectangles:
$1(4) + 1(9) + 1(16)$
$= 29$

a single trapazoid:
$(1/2)(b_1+b_2)h =$
$(1/2)(1+16)(3) =$
25.5

Use multiple trapazoids for better estimate.

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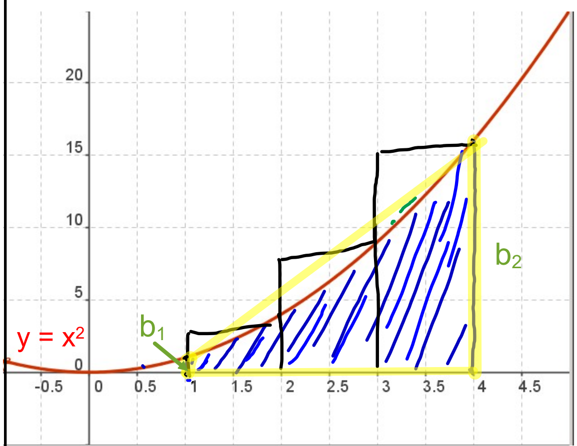
Estimating Area, Definite Integral

5.3 Fundamental Theorem of Calculus

Problem: Find the area under $y = x^2$ from x=1 to x=4



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25.5

Use multiple trapazoids for better estimate.

$$\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i = \int_a^b f(x)\,dx$$

Estimating Area under a Curve, since y is non-negative.
Also estimating the definite integral.
Therefore, use the definite integral: $\int_1^4 x^2\,dx$

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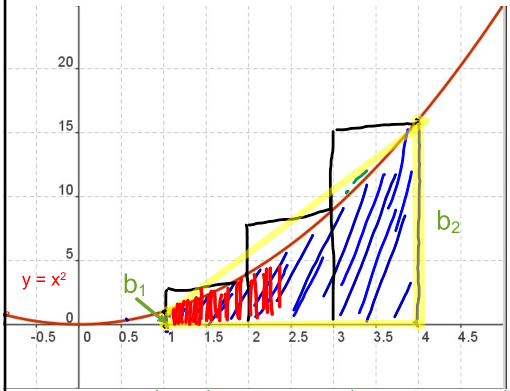
Estimating Area, Definite Integral

5.2 Fundamental Theorem of Calculus

Problem: Find the area under $y = x^2$ from x=1 to x=4



Estimates:

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Use multiple trapazoids for better estimate.

$$\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i = \int_a^b f(x)\,dx$$

Estimating Area under a Curve, since y is non-negative.
Also estimating the definite integral.
Therefore, use the definite integral: $\int_1^4 x^2\,dx$

How do we use the definite integral to actually compute the area?

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Estimating Area, Definite Integral

$$\int_1^4 x^2\,dx = \left.\frac{x^3}{3} + c\right]_1^4$$

$$= \frac{4^3}{3} + c - \left[\frac{1}{3} + c\right]$$

$$\frac{64}{3} + c - \frac{1}{3} - c = \frac{63}{3} = 21$$

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5.2 Fundamental Theorem of Calculus

Fundamental Theorem of Calculus (FTC)

If :

1. a function ***f* is continuous on [a, b]** and
2. ***F* is an antiderivative of f** on the interval,

then:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

The integral of *f* from *a* to *b* is the difference:
(antiderivative of *f* evaluated at *x*=*b*) - (antiderivative of *f* evaluated at *x*=*a*.)

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Calculus Home Homework

FTC

5.2 Fundamental Theorem of Calculus

$$\int_1^4 x^2\,dx = ?$$

Fundamental Theorem of Calculus (FTC)

If :

1. a function ***f* is continuous on [a, b]** and
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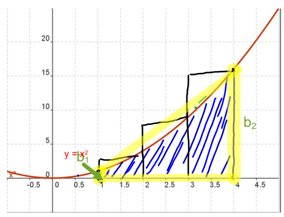

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FTC

5.3 Fundamental Theorem of Calculus

$$\int_1^4 x^2\,dx = ?$$

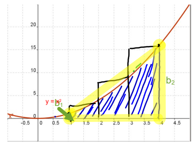
Fundamental Theorem of Calculus (FTC)

If :

1. a function f **is continuous on [a, b]** and
2. **F is an antiderivative of f** on the interval,

then:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$



$$\int_1^4 x^2\,dx = \left[\frac{x^3}{3} + \cancel{c}\right]_1^4$$

Do not need c, the constant of integration. It gets added and subtracted to add to 0.

$$= \left[\frac{4^3}{3} + c\right] - \left[\frac{1}{3} + c\right]$$

$$= \frac{4^3}{3} + c - \frac{1}{3} - c = \frac{63}{3} = 21$$

Since y is non-negative, the **area = 21 sq. units**

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Calculus Home Homework

FTC

5.3 Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

G: $\int_2^7 3\,dx$ F: Evaluate

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Calculus Home Homework

Homework, Ex.

5.2 Fundamental Theorem of Calculus

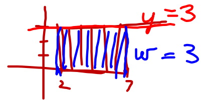
G: $\int_2^7 3\,dx$

F: Evaluate

$$= 3x\Big]_2^7$$

$$= 3(7) - 3(2)$$

$$= 21 - 6 = 15$$



$l = 7-2 = 5$

$A = 15$

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Homework

Homework, Ex.

5.3 Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\int_1^3 (3x^2 + 5x - 4)\,dx$$

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Calculus Home

Homework

Example 2

5.3 Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(b)-F(a)$$

10. $\int_1^3 (3x^2+5x-4)\,dx$

$$= \frac{3x^3}{3} + \frac{5x^2}{2} - 4x \Big]_1^3$$

$$= x^3 + \frac{5}{2}x^2 - 4x \Big]_1^3$$

$F(b)-F(a)$
$=F(3)-F(1)$

$$= 3^3 + \frac{5}{2}\cdot 3^2 - 4(3) - \left[1+\frac{5}{2}-4\right]$$

$$= 27 + \frac{45}{2} - 12 - 1 - \frac{5}{2} + 4 = 27+4-13+\frac{45-5}{2}$$

$$= 31-13+\frac{40}{2} = 18+20 = 38$$

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Example 2

5.3 Fundamental Theorem of Calculus

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}+c$$

$$\int_1^8 \sqrt{\frac{2}{x}}\,dx = \sqrt{2}\int_1^8 x^{-1/2}\,dx$$

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Examples 3, 4

5.3 Fundamental Theorem of Calculus

$$\int_1^8 \sqrt{\frac{2}{x}}\,dx = \int_1^8 \sqrt{2}\,x^{-1/2}\,dx$$

$$\sqrt{\frac{b}{a}} = \frac{\sqrt{b}}{\sqrt{a}}$$

$$\frac{\sqrt{2}}{\sqrt{x}} = \frac{\sqrt{2}}{x^{1/2}}$$

$$= \sqrt{2}\int_1^8 x^{-1/2}\,dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$= \sqrt{2}\left.\frac{x^{1/2}}{\frac{1}{2}}\right]_1^8 = 2\sqrt{2}\left.\sqrt{x}\right]_1^8$$

$$= 2\sqrt{2}\sqrt{8} - 2\sqrt{2}\sqrt{1}$$

$$F(b) - F(a)$$
$$= F(8) - F(1)$$

$$= 2\sqrt{2}\,2\sqrt{2} - 2\sqrt{2}$$
$$= 4\cdot 2 - 2\sqrt{2}$$
$$= 8 - 2\sqrt{2}$$

~ 5.17

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Examples 3, 4

5.3 Fundamental Theorem of Calculus

$$\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}}\,dx =$$

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Homework

Examples 3, 4

5.3 Fundamental Theorem of Calculus

$$\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}}\,dx = \frac{1}{2}\int_{-8}^{-1} \frac{x - x^2}{x^{1/3}}\,dx$$

$$= \frac{1}{2}\int_{-8}^{-1} \left(x^{2/3} - x^{5/3}\right) dx = \frac{1}{2}\left[\frac{x^{5/3}}{\frac{5}{3}} - \frac{x^{8/3}}{\frac{8}{3}}\right]_{-8}^{-1}$$

$$= \frac{1}{2}\left[\frac{3}{5}x^{5/3} - \frac{3}{8}x^{8/3}\right]_{-8}^{-1}$$

$$= \frac{1}{2}\left[\left[\frac{3}{5}(-1)^{5/3} - \frac{3}{8}(-1)^{8/3}\right] - \left[\frac{3}{5}(-8)^{5/3} - \frac{3}{8}(-8)^{8/3}\right]\right]$$

$$= \frac{1}{2}\left(\left[\frac{3}{5}(-1) - \frac{3}{8}(1)\right] - \left[\frac{3}{5}(-32) - \frac{3}{8}(+256)\right]\right)$$

Can use calculator after substitution.

$$= \frac{1}{2}\left[\left[-\frac{3}{5} - \frac{3}{8}\right] - \left[-\frac{96}{5} - 96\right]\right] = \frac{1}{2}\left[-\frac{3}{5} + \frac{96}{5} - \frac{3}{8} + 96\right]$$

$$= \frac{1}{2}\left(\frac{93}{5} - \frac{3}{8} + 96\right) = \frac{1}{2}\left(\frac{744 - 15}{40} + 96\right)$$

$$= \frac{1}{2} \cdot 114.225 = 57.1125$$

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Calculus Home

Homework $\frac{729}{40} = 18.225$

Examples 3, 4

5.3 Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\int_{-2}^{-1} \left(u - \frac{1}{u^2}\right) du$$

$$\frac{1}{2} - 1 - \left(2 - \frac{1}{2}\right) = \frac{1}{2} - 1 - 2 + \frac{1}{2} = -2$$

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Homework

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5.3 Fundamental Theorem of Calculus

$$\int_{-2}^{-1} \left(u - \frac{1}{u^2}\right) du$$

$$= \int_{-2}^{-1} (u - u^{-2})\, du = \left.\frac{u^2}{2} - \frac{u^{-1}}{-1}\right]_{-2}^{-1}$$

$$= \left.\frac{u^2}{2} + \frac{1}{u}\right]_{-2}^{-1}$$

$$= \frac{(-1)^2}{2} + \frac{1}{-1} - \left[\frac{(-2)^2}{2} + \frac{1}{-2}\right]$$

$$= \underline{\frac{1}{2} - 1} - \left[\frac{4}{2} - \frac{1}{2}\right] = -\frac{1}{2} - \left[\frac{3}{2}\right] = -\frac{1}{2} - \frac{3}{2} = \frac{-4}{2} = -2$$

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5.3 Fundamental Theorem of Calculus

$$\int_0^{\pi/4} (\sec^2\theta - \sin\theta)\, d\theta$$

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$
$\csc x$	$-\csc x \cot x$

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5.3 Fundamental Theorem of Calculus

$$\int_0^{\pi/4} (\sec^2\theta - \sin\theta)\, d\theta$$

$$= \tan\theta + \cos\theta \Big]_0^{\pi/4}$$

$$= \left[1 + \frac{1}{\sqrt{2}}\right] - \left[0 + 1\right]$$

$$= \frac{1}{\sqrt{2}}$$

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$
$\csc x$	$-\csc x \cot x$

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5.3 Fundamental Theorem of Calculus

$$\int_1^4 (3 - |x-3|)\, dx$$

Presents some problems.

1. Start with definiton of absolute value and
2. consider what this means regarding the interval from lower to upper limits.

Step #1: Absolute value:

Step #2: About the interval

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FTC with Absolute Value

5.3 Fundamental Theorem of Calculus

$$\int_1^4 (3-|x-3|)\,dx$$

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$$

$$|x-3| = \begin{cases} x-3, & x-3 \geq 0,\ x \geq 3 \\ -(x-3), & x-3<0,\ x<3 \end{cases}$$

$-x+3$

1 2 3 4

The integrand is defined differently on the interval.
Since definite integrals are defined as limits of sums, we replace the original integral with the sum of 2 integrals which have integrands and limits that correspond to the 2 part definition of the original:

a c b

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Rewrite the original integral as the sum of 2 integrals.

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FTC: sums of intervals

5.3 Fundamental Theorem of Calculus

$$\int_1^4 (3-|x-3|)\,dx$$

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$$

$$|x-3| = \begin{cases} x-3, & x-3 \geq 0,\ x \geq 3 \\ -(x-3), & x-3<0,\ x<3 \end{cases}$$

$-x+3$

1 2 3 4

$$\int_1^4 (3-|x-3|)\,dx = \int_1^3 [3-(3-x)]\,dx + \int_3^4 [3-(x-3)]\,dx$$

a c b

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

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FTC with Absolute Value

5.3 Fundamental Theorem of Calculus

$$\int_1^4 (3-|x-3|)\,dx$$

$$|a| = \begin{cases} a, & a\ge 0 \\ -a, & a<0\end{cases}$$

$$|x-3| = \begin{cases} x-3, & x-3\ge 0,\ x\ge 3 \\ -(x-3), & x-3<0,\ x<3\end{cases}$$

$-x+3$

$$\int_1^4 (3-|x-3|)\,dx = \int_1^3 [3-(3-x)]\,dx + \int_3^4 [3-(x-3)]\,dx$$

$$= \int_1^3 x\,dx + \int_3^4 (6-x)\,dx$$

$$= \left[\frac{x^2}{2}\right]_1^3 + \left[6x-\frac{x^2}{2}\right]_3^4$$

$$= \left[\frac{9}{2}-\frac{1}{2}\right] + \left[24-\frac{16}{2}-\left(18-\frac{9}{2}\right)\right]$$

$4 \quad + \quad 16-18+\frac{9}{2}$

$4-2+\frac{9}{2} = 2+\frac{9}{2} = 13/2$

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FTC with Absolute Value

5.3 Fundamental Theorem of Calculus

$$\int_{\pi/4}^{\pi/2} (2-\csc^2 x)\,dx = 2x + \cot x\Big]_{\pi/4}^{\pi/2}$$

$$2-\frac{1}{\sin^2 x}$$

$$2\cdot\frac{\pi}{2}+\cot\frac{\pi}{2} - \left[2\cdot\frac{\pi}{4}+\cot\frac{\pi}{4}\right]$$

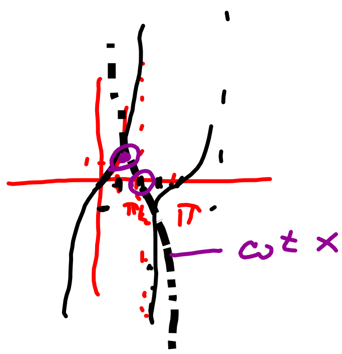
$$\pi + \cot\frac{\pi}{2} - \frac{\pi}{2} - \cot\pi/4$$

$$\frac{\pi}{2}+\cot\frac{\pi}{2} - \cot\frac{\pi}{4}$$

$$\frac{\pi}{2} + 0 - 1$$

$$\frac{\pi}{2} - 1 \sim 0.5$$

$$(\csc x)^2 = \frac{1}{(\sin x)^2}$$



$\tan\frac{\pi}{4} = 1$

$\tan\frac{3\pi}{4} = -1$

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