5.2 Reimann Sums and the Definite Integral

Goals:

1. Recognize that Riemann Sums use Δx_i that are not all equal.
2. Learn the definition of the Definite Integral.
3. Understand how the Definite Integral relates to Area under a curve.
4. Learn basic properties of definite integrals.

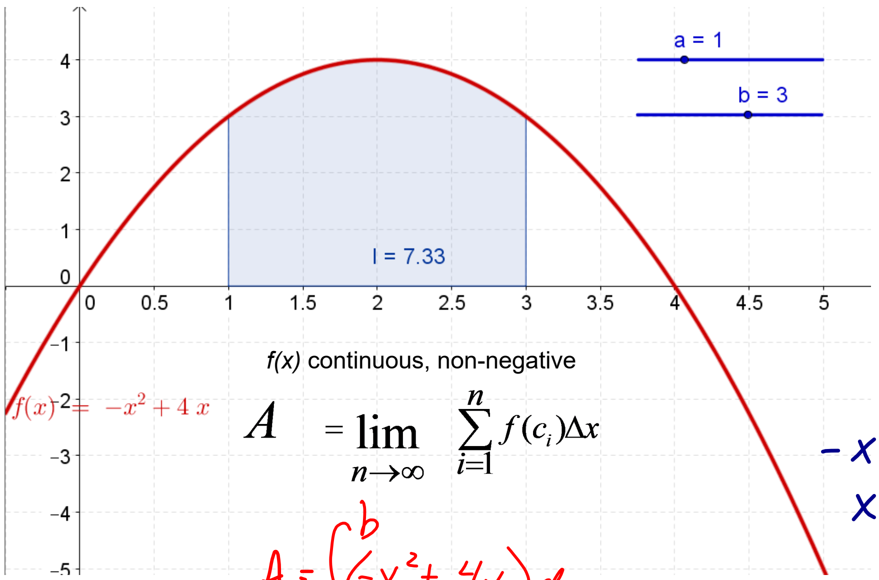
Study 5.2 #61, 63, 71, 73, 77, 79, 89, 91

video

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Homework

5.2 Reimann Sums and the Definite Integral



f(x) continuous, non-negative

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

$$A = \int_a^b (-x^2 + 4x)\,dx$$

$-x^2 + 4x = 0$

$x(-x+4) = 0$

$x = 0,\ x = 4$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Homework video

5.2 Reimann Sums and the Definite Integral

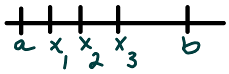
Definition of Riemann Sum

Let f be defined on [a,b], and let
Δ be a partition of [a,b], given by
$a = x_0 < x_1 < x_2 < \dots < x_{n-1} < x_n = b$
where Δx_i is the width of the ith sub-interval
If c_i is any point on the ith sub-interval, then
the sum

$$\sum_{i=1}^{n} f(c_i)\,\Delta x_i, \qquad x_{i-1} < c_i < x_i$$

is called a Riemann Sum of f for the partition Δ

Δx_i not all equal



norm of the partition $||\Delta||$ =
width of longest subinterval

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Homework

5.2 Reimann Sums and the Definite Integral

Definition of Definite Integral

If f is defined on [a,b], and the following limit exists

$$\lim_{||\Delta||\to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i$$

Then f is integrable on [a,b] and the limit is denoted as:

$$\lim_{||\Delta||\to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i = \int_a^b f(x)\,dx$$

This is called the Definite Integral of f from a to b

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Homework

5.2 Reimann Sums and the Definite Integral

$$\int x^3\,dx = \frac{x^4}{4} + c$$

Indefinite

$$\int_0^1 x^3\,dx = \frac{x^4}{4}\Big]_0^1$$

Definite

Class Notes: Prof. G. Battaly, Westchester Community College, NY

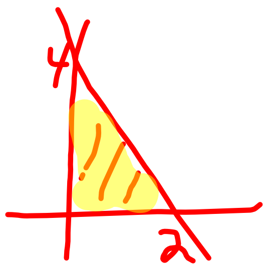
Calculus Home Homework

5.2 Reimann Sums and the Definite Integral

G: $f(x) = 4 - 2x$

F: Set up the definite integral that yields the area of the shaded region.

$$\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i = \int_a^b f(x)\,dx$$



Need to rewrite with *a*, *b*, and *f(x)* for this problem.

$$\int_a^b f(x)\,dx$$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Homework

5.2 Reimann Sums and the Definite Integral

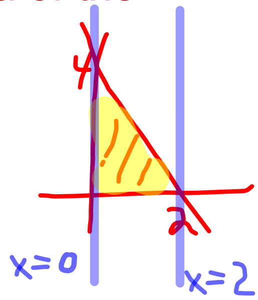
G: f(x) = 4 - 2x

$\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i = \int_a^b f(x)\,dx$

Need to rewrite with *a, b*, and *f(x)* for this problem. $\int_a^b f(x)\,dx$

F: Set up the definite integral that yields the area of the shaded region.

$$\int_0^2 (4-2x)\,dx$$



Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Homework

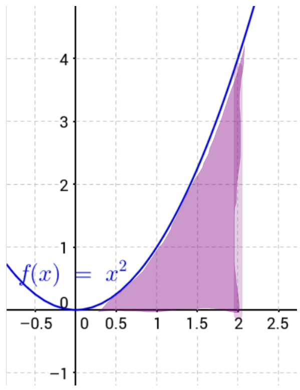
5.2 Reimann Sums and the Definite Integral

G: f(x) = x^2

Need to rewrite with *a, b*, and *f(x)* for this problem.

$$\int_a^b f(x)\,dx$$

F: Set up the definite integral that yields the area of the shaded region.



Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Homework

5.2 Reimann Sums and the Definite Integral

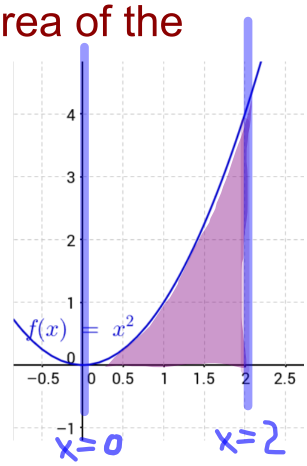
G: $f(x) = x^2$

Need to rewrite with *a, b*, and *f(x)* for this problem.

$$\int_a^b f(x)\,dx$$

F: Set up the definite integral that yields the area of the shaded region.

$$\int_0^2 x^2\,dx$$



Class Notes: Prof. G. Battaly, Westchester Community College, NY

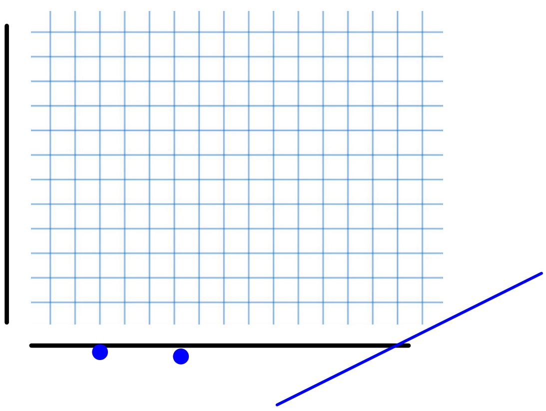
Calculus Home

Homework

5.2 Reimann Sums and the Definite Integral

G: $\int_0^4 \frac{x}{2}\,dx$

F: 1. Sketch region
2. Use geometry to find the area



Class Notes: Prof. G. Battaly, Westchester Community College, NY

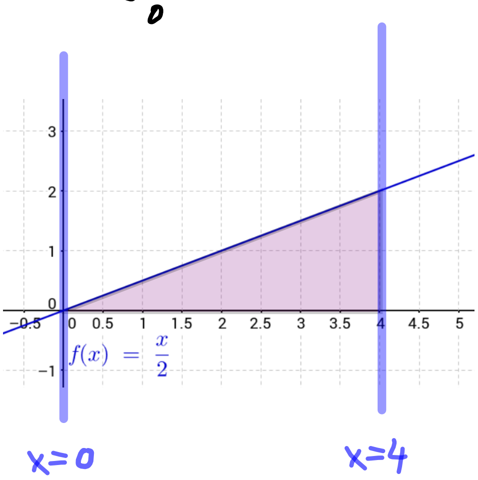
Calculus Home

Homework

5.2 Reimann Sums and the Definite Integral

G: $\int_0^4 \frac{x}{2}\,dx$

F: 1. Sketch region
2. Use geometry to find the area



$A = \frac{1}{2}bh$

$= \frac{1}{2}(4)(2) = 4$ sq. units

$\int_0^4 \frac{x}{2}dx = \frac{1}{2}\frac{x^2}{2}\Big]_0^4 = \frac{1}{4}x^2\Big]_0^4 = \frac{1}{4}[4^2-0^2] = 4$ sq units

Class Notes: Prof. G. Battaly, Westchester Community College, NY

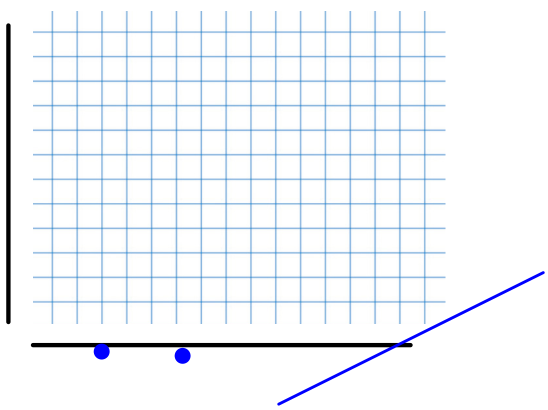
Calculus Home

Homework

5.2 Reimann Sums and the Definite Integral

G: $\int_0^8 (8-x)\,dx$

F: 1. Sketch region
2. Use geometry to find the area



Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home

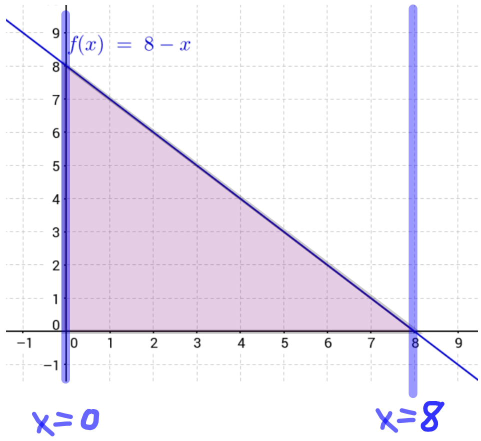
Homework

5.2 Reimann Sums and the Definite Integral

G: $\int_0^8 (8-x)\,dx$

F: 1. Sketch region
2. Use geometry to find the area

$A = \frac{1}{2} b h$
$= \frac{1}{2} \cdot 8 \cdot 8 = 32$ sq units

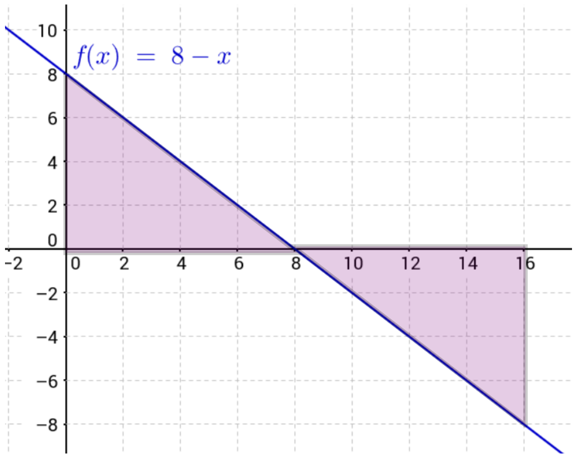


Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Homework

5.2 Reimann Sums and the Definite Integral

What about $\int_0^{16} (8-x)\,dx$?



video

NOT non-negative

$\int_0^{16} (8-x)\,dx = 0$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Homework

5.2 Reimann Sums and the Definite Integral

Definite Integral as the Area of a Region

If ***f*** is continuous and non-negative on [a,b], then the **area of the region** bounded by:

- the graph of ***f***,
- the **x-axis**, and
- the vertical lines **x = a and x = b**

is given by:

$$\text{Area} = \int_a^b f(x)\,dx$$

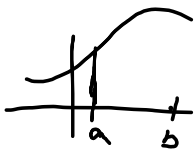
Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Homework

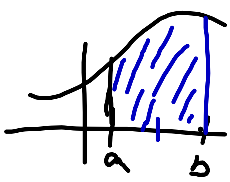
5.2 Reimann Sums and the Definite Integral

Special Integrals

① $\int_a^a f(x)\,dx = 0$



② $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$



③ $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Homework

5.2 Reimann Sums and the Definite Integral

Continuity Implies Integrability

If a function f is continuous on [a,b],

then f is integrable on [a,b].

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Homework

5.2 Reimann Sums and the Definite Integral

$$\int_2^2 x^3\,dx$$

$$\int_2^4 15\,dx$$

$$\int_2^4 (x^3+4)\,dx$$

Given:

$$\boxed{\begin{gathered}\int_2^4 x^3\,dx = 60 \\ \int_2^4 x\,dx = 6 \\ \int_2^4 dx = 2\end{gathered}}$$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Homework

5.2 Reimann Sums and the Definite Integral

Given:

$$\boxed{\begin{aligned}&\int_2^4 x^3\,dx = 60\\ &\int_2^4 x\,dx = 6\\ &\int_2^4 dx = 2\end{aligned}}$$

$$\int_2^2 x^3\,dx = 0$$

$$\int_2^4 15\,dx = 15\int_2^4 dx$$
$$= 15(2) = 30$$

$$\int_2^4 (x^3+4)\,dx$$
$$= \int_2^4 x^3\,dx + \int_2^4 4\,dx = 60 + 8 = 68$$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Homework

5.2 Reimann Sums and the Definite Integral

$$G: \int_0^3 f(x)\,dx = 4 \qquad \int_3^6 f(x)\,dx = -1$$

$$\int_0^6 f(x)\,dx$$

$$\int_6^3 f(x)\,dx$$

$$\int_3^3 f(x)\,dx$$

$$\int_3^6 -5f(x)\,dx$$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Homework

5.2 Reimann Sums and the Definite Integral

G: $\int_0^3 f(x)\,dx = 4 \qquad \int_3^6 f(x)\,dx = -1$

$\int_0^6 f(x)\,dx = \int_0^{\boxed{3}} f(x)\,dx + \int_{\boxed{3}}^6 f(x)\,dx = 4 - 1 = 3$

$\int_6^3 f(x)\,dx = -\int_3^6 f(x)\,dx = -(-1) = +1$

$\int_3^3 f(x)\,dx = 0$

$\int_3^6 -5 f(x)\,dx = (-5)(-1) = 5$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home

Homework