5.1 Area

Goals:
1. Understand the operation of summation and the symbol Σ
2. Understand the index of summation, usually indicated by an i or j
3. Learn the definition of Area and understand how it relates to limits of sums.

5.1 # 3, 22, 43, 46 (opt), definition of Area

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Homework

5.1 Area

Sigma Notation Σ

$\sum_{i=1}^{n}$ is a representation of **summation** from i = 1 to i = n

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \ldots + a_n$$

where ***i*** is the **index of summation**
and $\boldsymbol{a_i}$ is the **ith term** of the sum

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5.1 Area

$$\sum_{i=1}^{6} \frac{i}{2} =$$

$$\sum_{j=3}^{5} \frac{1}{j} =$$

$$\sum_{k=0}^{4} \frac{1}{k^2+1} =$$

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5.1 Area

$$\sum_{i=1}^{6} \frac{i}{2} = \frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \frac{4}{2} + \frac{5}{2} + \frac{6}{2} = 10.5$$

$$\sum_{j=3}^{5} \frac{1}{j} = \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = 0.783$$

$$\sum_{k=0}^{4} \frac{1}{k^2+1} = \frac{1}{0+1} + \frac{1}{1+1} + \frac{1}{4+1} + \frac{1}{9+1} + \frac{1}{16+1}$$

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17} = 1.859$$

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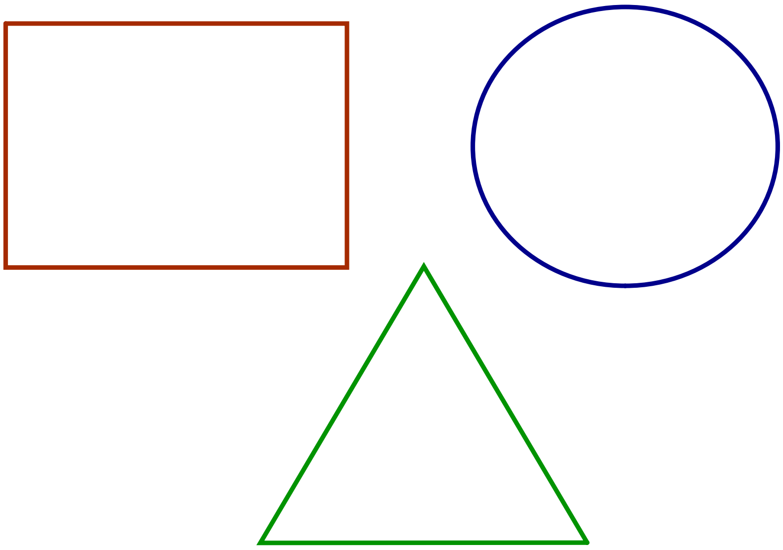
5.1 Area

How do we find the area of:



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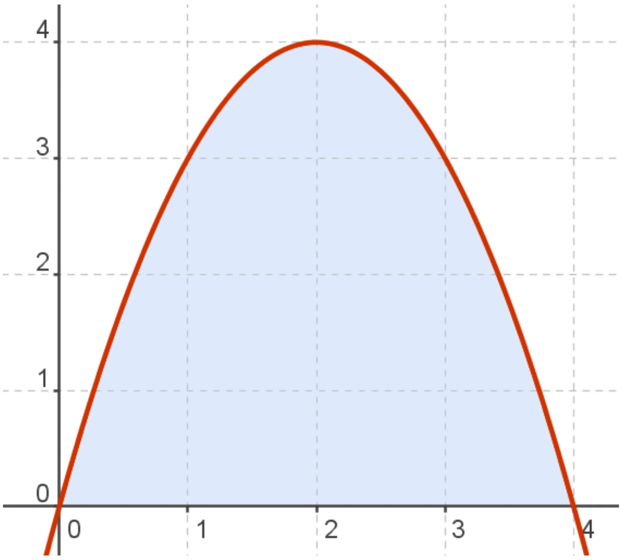
5.1 Area

What about the shaded area below?



Can we find the area with a formula
or other tools from algebra or geometry?

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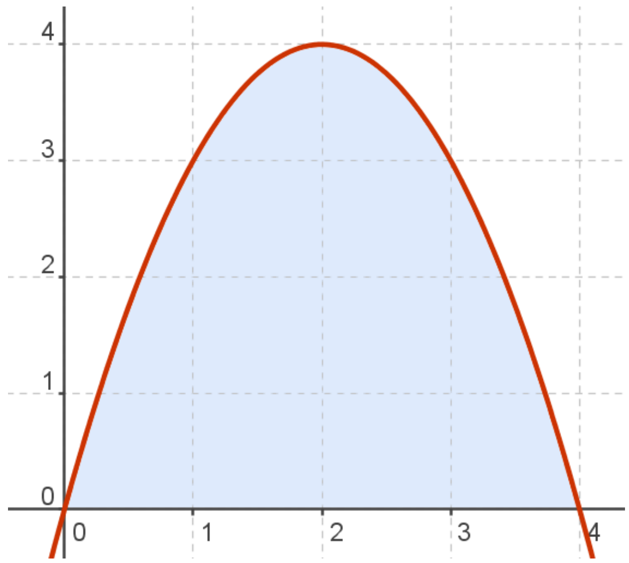
5.1 Area

How can we estimate the area?



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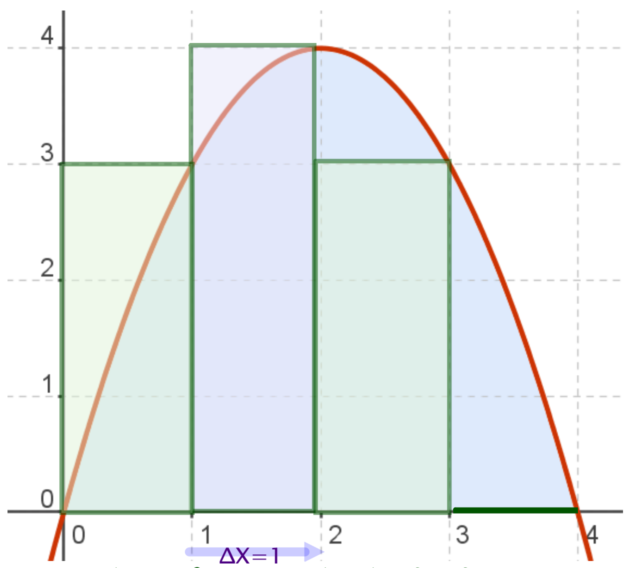
5.1 Area

Using 4 rectangles:



Area of rectange 1 = 1 x 3 = 3

Area of rectange 2 = 1 x 4 =4

Area of rectange 3 = 1 x 3 = 3

Area of rectange 4 = 1 x 0 = 0

$\Sigma = 10$

Actual area = 10.67

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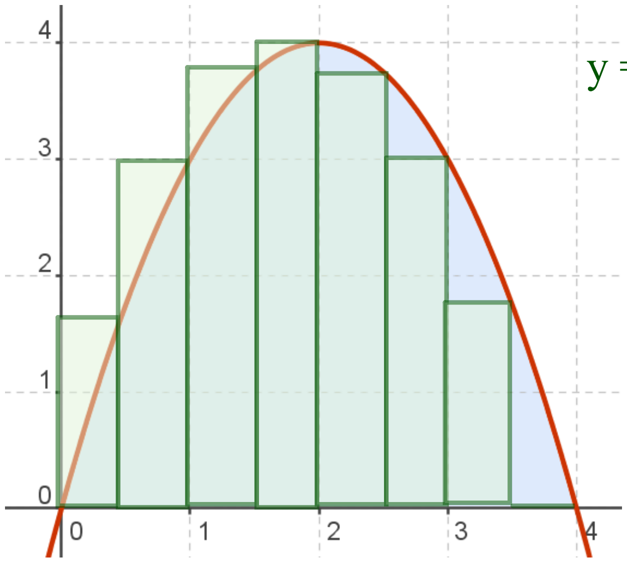
5.1 Area

Using 8 rectangles:



y = -x(x-4)

$$\sum_{i=1}^{8} \Delta x_i f(c_i)$$

c_i	$f(c_i)$	$\Delta x f(c_i)$
0	0	0
.5	1.75	.875
1	3	1.5
1.5	3.75	1.875
2	4	2
2.5	3.75	1.875
3	3	1.5
3.5	1.75	.875
		10.5

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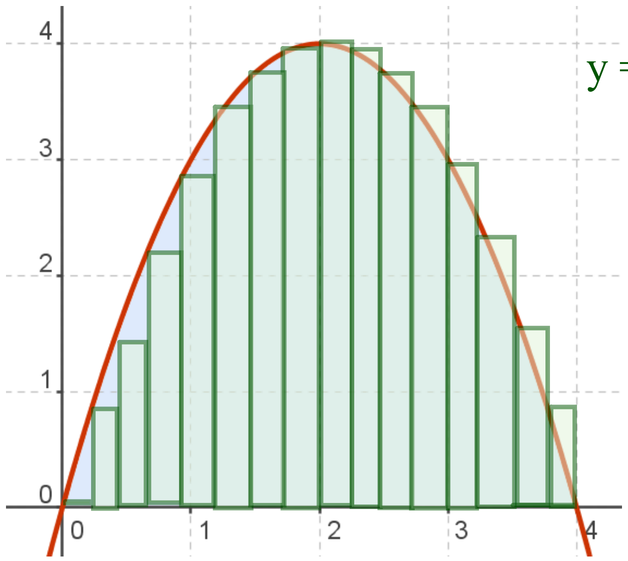
5.1 Area

Using 16 rectangles:



y = -x(x-4)

10.62

$$\sum_{i=1}^{16} \Delta x_i f(c_i)$$

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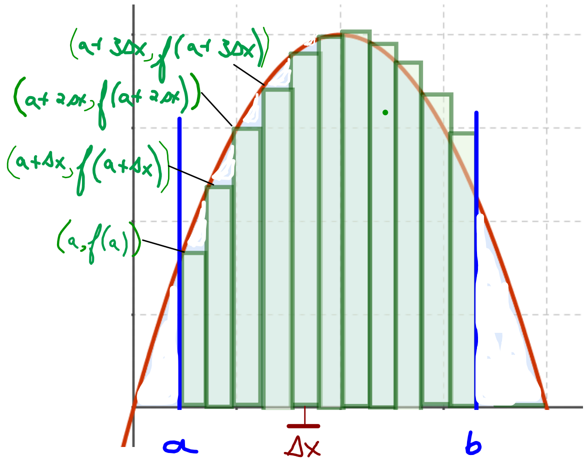
5.1 Area

To find the **area under the curve** over a specified interval, we would need to divide it into lots of little polygons, such as rectangles, for which we can find the area with known formulas.



$\Delta x = \frac{b-a}{n}$

where *n* is the number of rectangles

as $n \to \infty$
$\Delta x \to 0$

Areas of small rectangles

$\Delta x\, f(a)$ $\Delta x\, f(a+2\Delta x)$
$\Delta x\, f(a+\Delta x)$ $\Delta x\, f(a+3\Delta x)$

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5.1 Area

Definition of Area of a Region in a Plane

Let $\boxed{\text{f be continuous}}$ and $\boxed{\text{non-negative}}$ on [a,b].
The **area** of the region bounded by the graph of f, the x-axis, and the vertical lines x = a and x = b is:

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

$x_{i-1} < c_i < x_i$

$\Delta x = \frac{b-a}{n}$

as $n \to \infty$, $\Delta x \to 0$

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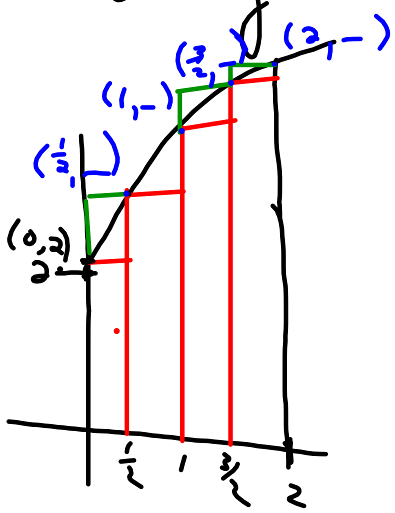
5.1 Area

G: $y = \sqrt{x} + 2$ F: upper & lower sums



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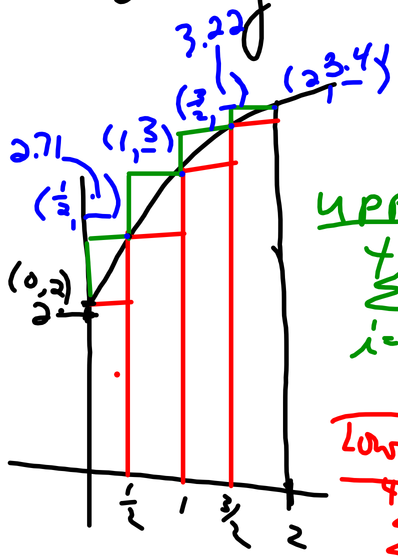
5.1 Area

G: $y = \sqrt{x} + 2$ F: upper & lower sums



$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

Upper Sum:

$\sum_{i=1}^{4} \Delta x\, f(c_i) = \frac{1}{2}(2.71 + 3 + 3.22 + 3.41)$

$= 6.17$

Lower Sum

$\sum_{i=1}^{4} \Delta x\, f(c_i) = \frac{1}{2}(2 + 2.71 + 3 + 3.22)$

$= 5.47$

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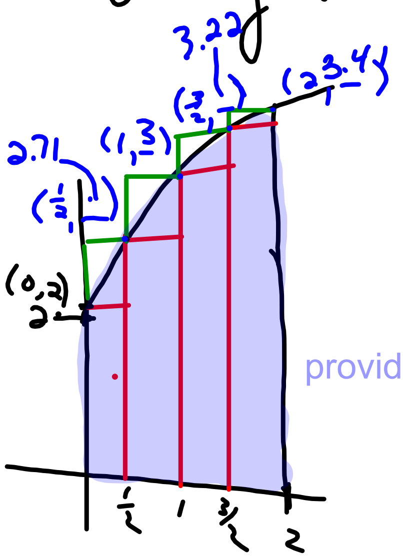
5.1 Area

G: $y=\sqrt{x}+2$ F: upper & lower sums



$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

upper Sum:

$\sum_{i=1}^{4} \Delta x\, f(c_i) = \frac{1}{2}(2.71+3+3.22+3.41)$
$= 6.17$

Lower Sum

$\sum_{i=1}^{4} \Delta x\, f(c_i) = \frac{1}{2}(2+2.71+3+3.22)$
$= 5.47$

provides actual value of area

$$\boxed{A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x}$$

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