4.8 Indeterminate Forms and L'Hopital's Rule

Goals:
1. Apply L'Hopital's Rule to simplify and find limits
   a) Identify Indeterminate Forms
   b) Recognize the forms needed for L'Hopital's
   c) If needed, algebraically transform expressions to the correct form:
      rational expression w limit → 0/0 or ∞/∞

Study 4.8 # 359, 365-371, 377, 379, 383-391, 399, 401, 405

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4.8 Indeterminate Forms and L'Hopital's Rule

Consider:

$$\lim_{x\to 1}\frac{x^2-x}{(x^2+1)(x-1)} \longrightarrow ?$$

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4.8 Indeterminate Forms and L'Hopital's Rule

Consider:

$$\lim_{x\to 1}\frac{\overset{x(x-1)}{x^2 - x}}{(x^2+1)(x-1)} \longrightarrow ?\ \frac{0}{0}$$

$$= \lim_{x\to 1}\frac{x}{x^2+1} = \frac{1}{2}$$

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4.8 Indeterminate Forms and L'Hopital's Rule

What about:

$$\lim_{x\to 0}\frac{1-e^x}{x} \rightarrow \frac{0}{0}$$

No easy algebraic approach

$\downarrow$

L'Hopital's Rule

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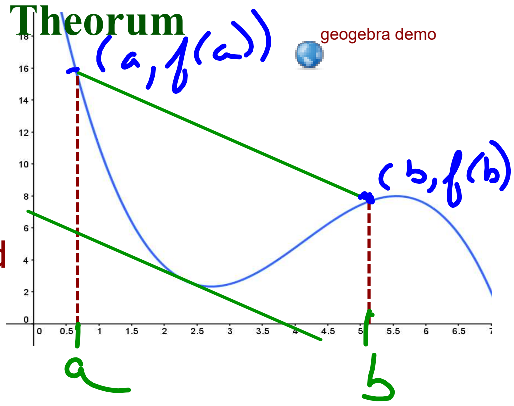
4.4 Rolle's Theorum and the Mean Value Theorum

geogebra demo



Mean Value Theorum

Let f be:

1. continuous on closed interval [a,b] and
2. differentiable on open interval (a,b)

then at least one c (a,b)

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Interpretation:

There exists at least one *c* on the interval from *a* to *b* such that the derivative at *c* equals the slope of the secant line joining the endpoints.

ALSO: There exists at least one c on the interval where the instantaneous rate of change equals the average value.

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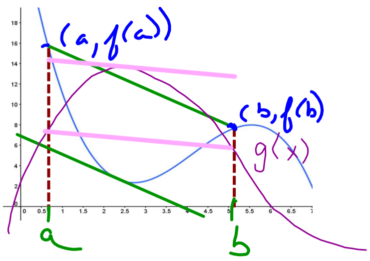
4.8 Indeterminate Forms and L'Hopital's Rule

Mean Value Theorum



Let f be:

1. continuous on closed interval [a,b] and
2. differentiable on open interval (a,b)

then at least one c (a,b)

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Extended MVT: Given that both f(x), g(x)

1 continuous on [a,b],

2. differentiable on (a,b)

and 3. $g'(x) \neq 0$ for any x on (a,b)

Then there exists c on (a,b) such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

Used to prove L'Hopital'sRule

$$g'(c) = \frac{g(b)-g(a)}{b - a}$$

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4.8 Indeterminate Forms and L'Hopital's Rule

L'Hopital's Rule Let f(x), g(x) be

1. **differentiable on (a,b)** containing c, except possibly at c itself.
2. **g ' (x) $\neq$ 0** for any x on (a,b), except possibly at c itself.

If :
$$\lim_{x \to c} \frac{f(x)}{g(x)} \Rightarrow \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

Then:
$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

provided the limit exists or is infinite.

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4.8 Indeterminate Forms and L'Hopital's Rule

Indeterminate Forms:

$$\frac{0}{0} \quad \frac{\infty}{\infty} \quad \infty - \infty$$

$$0\infty \quad 0^0 \quad 1^\infty \quad \infty^0$$

Determinate Forms:

$$\infty + \infty \Rightarrow \infty$$

$$-\infty - \infty \Rightarrow -\infty$$

$$0^\infty \Rightarrow 0$$

$$0^{-\infty} \Rightarrow \infty$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$

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4.8 Indeterminate Forms and L'Hopital's Rule

Indeterminate Forms:

$\boxed{\frac{0}{0} \quad \frac{\infty}{\infty}}$ $\quad \infty - \infty$

$0\infty \quad 0^0 \quad 1^\infty \quad \infty^0$

Determinate Forms:

$\infty + \infty \Rightarrow \infty$

$-\infty - \infty \Rightarrow -\infty$

$0^\infty \Rightarrow 0$

$0^{-\infty} \Rightarrow \infty$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

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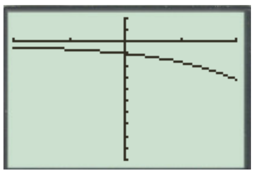
4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x\to 0}\frac{1-e^x}{x} \rightarrow \frac{0}{0}$$

① cont
② $g'(x) \neq 0$

$$\overset{LH}{=} \lim_{x\to 0}\frac{-e^x}{1} = -1$$



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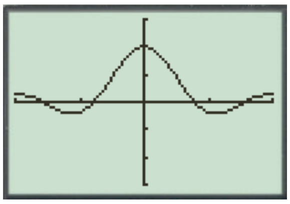
4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x \to 0} \frac{\sin 4x}{2x} \to \frac{0}{0}$$

4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x \to 0} \frac{\sin 4x}{2x} \to \frac{0}{0}$$

$$\overset{LH}{=} \lim_{x \to 0} \frac{4\cos 4x}{2} = 2$$



4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x \to 2^-} \frac{\sqrt{4-x^2}}{x-2} \to \frac{0}{0} \qquad -2 \le x \le 2$$

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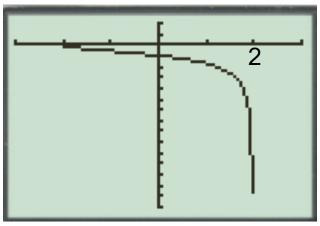
4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x \to 2^-} \frac{\sqrt{4-x^2}}{x-2} \to \frac{0}{0} \qquad -2 \le x \le 2$$

$$\stackrel{LH}{=} \lim_{x \to 2^-} \frac{\frac{1}{\cancel{2}} \frac{-\cancel{2}x}{\sqrt{4-x^2}}}{1} \to \frac{-2}{0} = -\infty$$



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4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x \to 1} \frac{\ln x^2}{x^2 - 1} \to \frac{0}{0}$$

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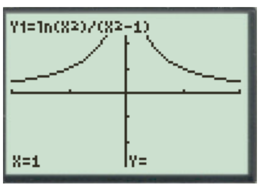
4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x \to 1} \frac{\ln x^2}{x^2 - 1} \to \frac{0}{0}$$

$$= \lim_{x \to 1} \frac{2 \ln x}{x^2 - 1} \stackrel{LH}{=} \lim_{x \to 1} \frac{2 \cdot \frac{1}{x}}{2x}$$

$$= \lim_{x \to 1} \frac{1}{x^2} = 1$$



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4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x \to 0^+} \frac{e^x - (1+x)}{x^3} \to \frac{1-1}{0} = \frac{0}{0}$$

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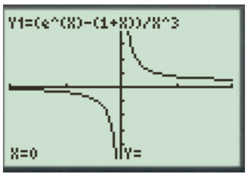
4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x \to 0^+} \frac{e^x \overbrace{- (1+x)}^{-1-x}}{x^3} \to \frac{1-1}{0} = \frac{0}{0}$$

$$\overset{LH}{=} \lim_{x \to 0^+} \frac{e^x - 1}{3x^2} \to \frac{1-1}{0} = \frac{0}{0}$$

$$\overset{LH}{=} \lim_{x \to 0^+} \frac{e^x}{6x} \to \frac{1}{+0} = \infty$$



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4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x\to\infty} \frac{x-1}{x^2+2x+3} \longrightarrow \frac{\infty}{\infty}$$

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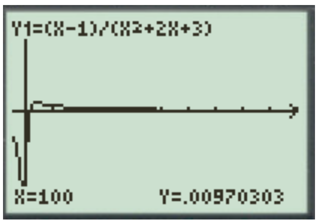
4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x\to\infty} \frac{x-1}{x^2+2x+3} \longrightarrow \frac{\infty}{\infty}$$

$$\overset{LH}{=} \lim_{x\to\infty} \frac{1}{2x+2} \longrightarrow \frac{1}{\infty} = 0$$



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4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x \to \infty} \frac{x^3}{x+2} \to \frac{\infty}{\infty}$$

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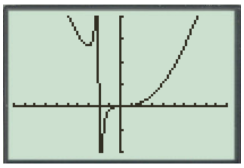
4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x \to \infty} \frac{x^3}{x+2} \to \frac{\infty}{\infty}$$

$$\overset{LH}{=} \lim_{x \to \infty} \frac{3x^2}{1} = +\infty$$



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4.8 Indeterminate Forms and L'Hopital's Rule

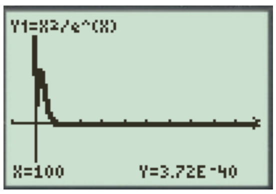
$$\lim_{x \to \infty} \frac{x^2}{e^x} \to \frac{\infty}{\infty}$$

4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x \to \infty} \frac{x^2}{e^x} \to \frac{\infty}{\infty}$$

$$\overset{LH}{=} \lim_{x \to \infty} \frac{2x}{e^x} \to \frac{\infty}{\infty}$$

$$\overset{LH}{=} \lim_{x \to \infty} \frac{2}{e^x} \to \frac{2}{\infty} = 0$$



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4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x\to\infty}(1+x)^{1/x} \to \infty^0$$

$$y = 2^x$$
$$\ln y = x\ln 2$$
$$\frac{\ln y}{\ln 2} = x$$

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4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x\to\infty}(1+x)^{1/x} \to \infty^0$$

$$y = 2^x$$
$$\ln y = x\ln 2$$
$$\frac{\ln y}{\ln 2} = x$$

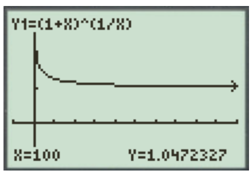
$$y = \lim_{x\to\infty}(1+x)^{1/x}$$

$$\ln y = \lim_{x\to\infty}\ln(1+x)^{1/x}$$

$$= \lim_{x\to\infty}\frac{1}{x}\ln(1+x) \to \frac{\infty}{\infty}$$

$$\overset{LH}{=} \lim_{x\to\infty}\frac{1}{1+x} = \frac{1}{\infty} = 0$$

$$\ln y = 0 \qquad y = e^0 = 1$$



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4.8 Indeterminate Forms and L'Hopital's Rule

$$\lim_{x\to 2^+}\left(\frac{1}{x^2-4}-\frac{\sqrt{x-1}}{x^2-4}\right)\to \infty-\infty$$

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4.8 Indeterminate Forms and L'Hopital's Rule

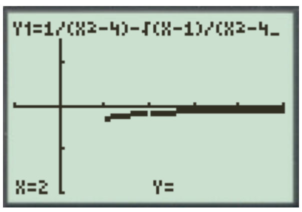
$$\lim_{x\to 2^+}\left(\frac{1}{x^2-4}-\frac{\sqrt{x-1}}{x^2-4}\right)\to \infty-\infty$$

$$=\lim_{x\to 2^+}\frac{1-\sqrt{x-1}}{x^2-4}\to\frac{0}{0}$$

$x-1\geq 0$

$x\geq 1$

$$\overset{LH}{=}\lim_{x\to 2^+}\frac{-\frac{1}{2}\frac{1}{\sqrt{x-1}}}{2x}=\frac{-\frac{1}{2}\cdot\frac{1}{1}}{4}=-\frac{1}{8}$$



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4.8 Indeterminate Forms and L'Hopital's Rule

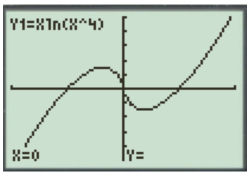
385. $\lim_{x \to 0^+} x \ln(x^4)$

387. $\lim_{x \to \infty} x^2 e^{-x}$

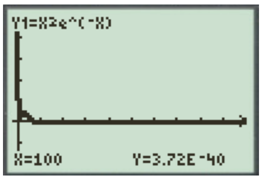
382. $\lim_{x \to 0^+} x^{2x}$

4.8 Indeterminate Forms and L'Hopital's Rule

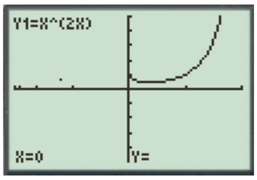
385. $\lim_{x \to 0^+} x \ln(x^4)$



387. $\lim_{x \to \infty} x^2 e^{-x}$



382. $\lim_{x \to 0^+} x^{2x}$



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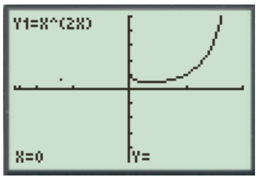
382. $\lim_{x \to 0^+} x^{2x} = y \to 0^0$

$$\ln y = \lim_{x \to 0^+} 2x \ln x = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{2x}} \to \frac{-\infty}{\infty}$$

$$\overset{LH}{=} \lim_{x \to 0^+} \frac{\frac{1}{x}}{-(2x)^{-2}(2)} = \lim_{x \to 0^+} \frac{-4x^2}{2x} = \lim_{x \to 0^+} -2x = 0$$

$\therefore \ln y = 0$ or $e^0 = y$

$y = 1$



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