4.7 Optimization Problems

Goal:
Solve Optimization Problems
a) Interpret and Set Up Word Problems
b) Use tests for relative extrema to find maximum or minimum values.

Study 4.7 # 311, 313, 317-321, 329, 333, 349

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4.7 Optimization Problems

Applications of Relative Extrema

Requires finding derivatives and performing either test for relative extrema:
1st Derivative Test
2nd Derivative Test

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4.7 Optimization Problems

Applications of Relative Extrema - examples

Maximum profit for making and selling a tire pump

Minimum force required to move a 200 lb box up an incline to a loading platform

Minimum cost to fill an order of 100 computers

Maximum area using a fixed amount of fencing

Minimum cost for construction of a box with fixed volume

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***How to Do Optimization Problems*:**

Read the problem carefully, and identify what's given and what you need to find.

Organize the info: draw a diagram, construct a table, etc.

Identify the unknown variables; add to diagram or table.

Write an equation to relate the given and the to find.

Reduce the number of variables to 2.

Find the derivative and Critical Numbers.

Test the critical numbers for max or min, using 1st derivative or 2nd derivative test, and state solution.

**Check the solution: Is "to find" found?
Does solution make sense? Do numbers fit?**

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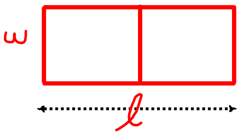
4.7 Optimization Problems

1st Deriv Test, or 2nd Deriv Test

G: **400 ft of fencing** to construct two adjacent corrals.

F: The dimensions which will **maximize the area** for grazing.



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4.7 Optimization Problems

1st Deriv Test, or 2nd Deriv Test

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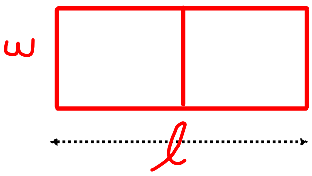

$3w + 2\ell = 400$

$A = \ell w$

$\ell = \frac{400 - 3w}{2}$

$\ell = 200 - \frac{3}{2}w$

$A = \left(200 - \frac{3}{2}w\right)w = 200w - \frac{3}{2}w^2$

Test CNs: 2nd Deriv. Test

$\frac{d^2A}{dw^2} = -3 < 0$ all x

Find CNs: dA/dw = 0 or DNE

$\frac{dA}{dw} = 200 - 3w$

CN: $200 - 3w = 0$ or ~~DNE~~

$w = \frac{200}{3}$ ft. CN

$\therefore$ $w = \frac{200}{3}$ ft results in max A.

$\ell = 200 - \frac{3}{2}w = 200 - \frac{3}{2}\cdot\frac{200}{3} = 200 - 100 = 100$ ft. $= \ell$

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4.7 Optimization Problems

1st Deriv Test, or 2nd Deriv Test

G: Two positive numbers. **The sum of the square of the 1st number and the 2nd number is 54**, and the **product is maximum**.

F: the numbers

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4.7 Optimization Problems

1st Deriv Test, or 2nd Deriv Test

G: Two positive numbers. **The sum of the 1st number squared and the 2nd number is 54**, and the **product is maximum**.

F: the numbers let x be 1^{st} #, y 2^{nd}

$x^2 + y = 54$

$P = xy$ max P

$y = 54 - x^2$

$P = x(54 - x^2)$

$P = 54x - x^3$

$\frac{dP}{dx} = 54 - 3x^2$

CN: $\frac{dP}{dx} = 0$ or ~~DNE~~

$\frac{dP}{dx} = 54 - 3x^2$

$3(18 - x^2) = 0$

$x = \pm\sqrt{18} = \cancel{\pm} 3\sqrt{2}$ need pos. #

$x = 3\sqrt{2}$

Does this result in max P?

Use 2nd Derivative Test

$\frac{d^2P}{dx^2} = -6x$

$\left.\frac{d^2P}{dx^2}\right|_{x=3\sqrt{2}} < 0$ ∴ C.D

$x = 3\sqrt{2}$ results in max Prod.

$y = 54 - x^2 = 54 - 18 = 36$

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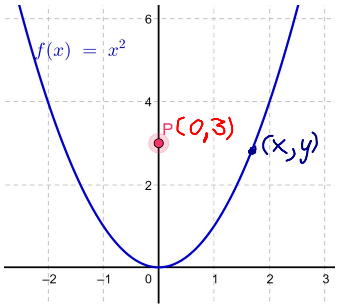
4.7 Optimization Problems

1st Deriv Test, or 2nd Deriv Test

350. **[T]** Where is the parabola $y = x^2$ closest to point $(0, 3)$?



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4.7 Optimization Problems

1st Deriv Test, or 2nd Deriv Test

350. **[T]** Where is the parabola $y = x^2$ closest to point $(0, 3)$?

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$d = \sqrt{(x-0)^2 + (y-3)^2}$

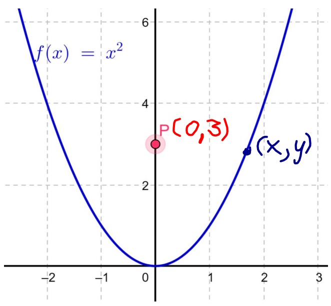
$d = \sqrt{x^2 + (x^2-3)^2}$

$d^2 = x^2 + (x^2-3)^2$

$2d \frac{dd}{dt} = 2x + 2(x^2-3)2x$

$d \frac{dd}{dt} = x + 2x^3 - 6x$

$\frac{dd}{dt} = \frac{2x^3 - 5x}{\sqrt{x^2+(x^2-3)^2}}$



CN: $\frac{dd}{dt} = 0$ or DNE

$2x^3 - 5x = 0$

$x(2x^2-5) = 0$

$x = 0 \quad | \quad x = \pm\sqrt{5/2}$

	$-\sqrt{5/2}$	0	$\sqrt{5/2}$	
x	-5	-1	1	5
$\frac{x(2x^2-5)}{\sqrt{}}$	$-$	$+$	$-$	$+$
d	decr	incr	decr	incr

$\left(-\sqrt{\tfrac{5}{2}}, -\right)$ MIN $\quad \frac{3}{4}$ $\quad \left(\sqrt{\tfrac{5}{2}}, -\right)$ MIN

On f(x), points closest to (0,3) are $(-\sqrt{5/2}, 5/2)$ and $(\sqrt{5/2}, 5/2)$

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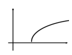
1st Deriv Test, or 2nd Deriv Test

G: $f(x) = \sqrt{x-8}$

F: point (x,y) on f closest to (12,0)



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4.7 Optimization Problems

1st Deriv Test, or 2nd Deriv Test

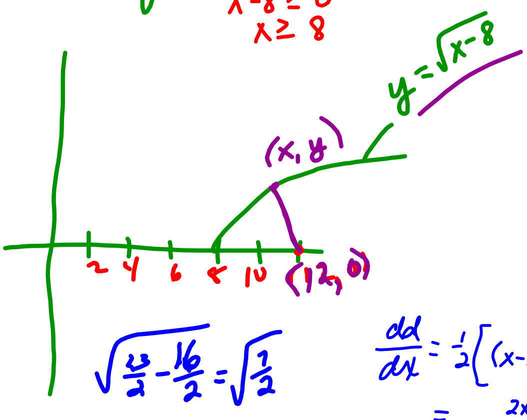
G: $f(x) = \sqrt{x-8}$

$x-8 \geq 0$

$x \geq 8$

F: pt. (x,y) on f closest to (12, 0)

dist to be minimize



$d = \sqrt{(x-x_1)^2+(y-y_1)^2}$

$= \sqrt{(x-12)^2+(y-0)^2}$

$= \sqrt{(x-12)^2+y^2}$

$= \sqrt{(x-12)^2+(\sqrt{x-8})^2}$

$d = \sqrt{(x-12)^2+(x-8)}$

$d^2 = (x-12)^2 + (x-8)$

$2d\frac{dd}{dx} =$

$\sqrt{\frac{23}{2}-\frac{16}{2}} = \sqrt{\frac{7}{2}}$

$\frac{dd}{dx} = \frac{1}{2}\left[(x-12)^2+(x-8)\right]^{-1/2}(2(x-12)+1)$

$= \frac{2x-24+1}{2\sqrt{(x-12)^2+(x-8)}} = \frac{2x-23}{2\sqrt{(x-12)^2+(x-8)}}$ $\not\ni$ 0 denom.

CN: $\frac{dd}{dx} = 0$: $2x-23=0$ $x=\frac{23}{2}$

$x < \frac{23}{2}$, $\frac{dd}{dx} < 0$ \ d decr.

$x > \frac{23}{2}$. $\frac{dd}{dx} > 0$ / d incr.

$\therefore \left(\frac{23}{2}, \sqrt{\frac{7}{2}}\right)$ results in min dist.

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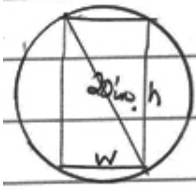
4.7 Optimization Problems

1st Deriv Test, or 2nd Deriv Test

G: wooden beam, strength, S, is directly proportional to the width, w, and the square of the height, h.



F: dimensions of strongest beam cut from a round log 20 in. in diameter

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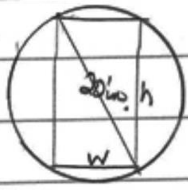
4.7 Optimization Problems

1st Deriv Test, or 2nd Deriv Test

37. G: wooden beam, strength, S, is directly proportional to the width, w, and the square of the height, h.



F: dimensions of strongest beam cut from a round log 20 in. in diameter

$w^2+h^2=20^2$

$h^2=20^2-w^2$

maximize $S=kwh^2$

$S=kw(400-w^2)=400kw-kw^3$

$\frac{dS}{dw}=400k-3kw^2$

$\frac{d^2S}{dw^2}=-6kw$ c.d. all w.

CNs: $w^2=\frac{400k}{3k}=\frac{400}{3}$

$\boxed{w=\pm\frac{20}{\sqrt{3}}=\frac{20}{\sqrt{3}}\text{ in}}$

$\therefore$ CN $\Rightarrow$ rel. max

$h^2=400-\frac{400}{3}=\frac{1200-400}{3}=\frac{800}{3}$

$\boxed{h=20\sqrt{\frac{2}{3}}\text{ in.}}$

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1st Deriv Test, or 2nd Deriv Test

F: 2 pos. #'s whose prod. is 192 and whose sum is minimum.

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4.7 Optimization Problems

1st Deriv Test, or 2nd Deriv Test

1. F: 2 pos. #'s whose prod. is 192 and whose sum is minimum.

Let x be one # and y be other #.

∴ G: $x, y \ni xy = 192$ F: $x, y \ni S = x + y$ is min

Write S in terms of one variable: $S = x + \frac{192}{x}$

To minimize, find $\frac{dS}{dx} = 1 - 192x^{-2} = 1 - \frac{192}{x^2} = \frac{x^2 - 192}{x^2}$

CNs: $x = 0$, $x = \pm\sqrt{192} = \pm\sqrt{64}\sqrt{3} = \pm 8\sqrt{3}$ x pos.

$y = \frac{192}{x} = \frac{192}{\sqrt{192}} = 8\sqrt{3}$ ∴ $x = y = 8\sqrt{3}$

$\frac{d^2S}{dx^2} = +384x^{-3} = \frac{384}{x^3} > 0$ all $x > 0$ ∴ c.u.

∴ $x = y = 8\sqrt{3}$ results in min S.

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