

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Goals:

1. Understand how the sign of the derivative of a function relates to the behavior of the function, re: **increasing or decreasing**.
2. Use the **First Derivative Test** to determine relative extrema.
3. Understand how the sign of the 2nd derivative of a function relates to the behavior of the function, re: **concave up or concave down**.
4. Determine intervals where a function is **concave up or concave down**.
5. Find **Inflection Points** of a curve.
6. Use the **Second Derivative Test** to determine relative extrema.

Study 4.5 # 195, 197, 201, 203, 209-213, 221-229, 234, 237

Calculus Prof G. Battaly, Westchester Community College Homework

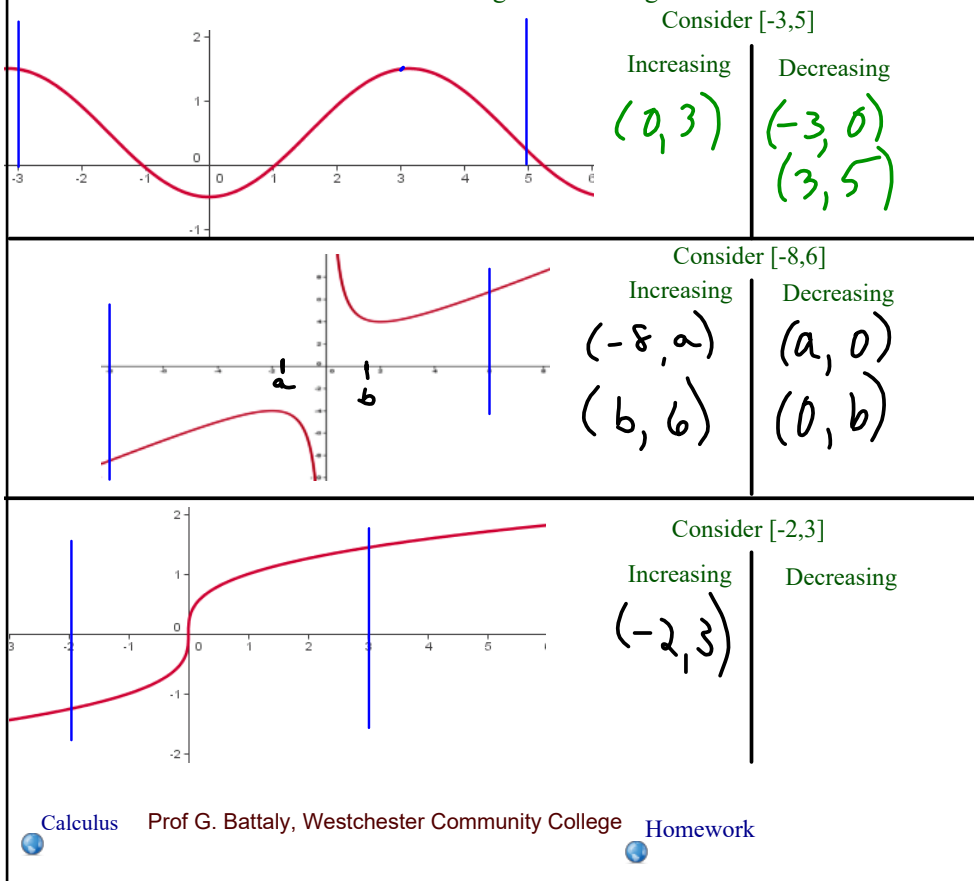
4.5 Increasing, Decreasing, Concavity, and Tests for Extrema
Find: Intervals where function is increasing or decreasing.

| | | | |
|--|--------------------|------------|------------|
| | Consider $[-3, 5]$ | Increasing | Decreasing |
| | Consider $[-8, 6]$ | Increasing | Decreasing |
| | Consider $[-2, 3]$ | Increasing | Decreasing |

Calculus Prof G. Battaly, Westchester Community College Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Find: Intervals where function is increasing or decreasing.

**4.5 Increasing, Decreasing, Concavity, and Tests for Extrema**

Can we tell if a function is increasing or decreasing, if we do not see its graph? How?

Is there a way to test for increasing or decreasing?

Hint: Consider the slope of the tangent line.

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Conclude:

Yes. We can tell if a function is increasing or decreasing, if we consider the slope of the tangent line. In particular we need to look at the sign of the slope. Is it positive or negative?

How can we examine the sign of slope of the tangent line?



Calculus

Prof G. Battaly, Westchester Community College



Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Conclude:

Yes. We can tell if a function is increasing or decreasing, if we consider the slope of the tangent line. In particular we need to look at the sign of the slope. Is it positive or negative?

How can we examine the sign of slope of the tangent line?

1. Find the **derivative**.
2. Determine the **intervals where** the derivative is **positive**. and where it is **negative**.



Calculus

Prof G. Battaly, Westchester Community College



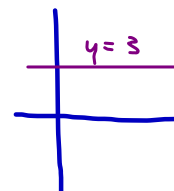
Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Test for Increasing & Decreasing Functions

Let f be a function that is continuous on $[a,b]$, and differentiable on (a,b) , then:

1. If $f'(x) > 0$ for all x on (a,b) ,
then f is **increasing** on $[a,b]$
2. If $f'(x) < 0$ for all x on (a,b) ,
then f is **decreasing** on $[a,b]$
3. If $f'(x) = 0$ for all x on (a,b) ,
then f is **constant** on $[a,b]$



Calculus

Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Find the intervals where y is increasing and intervals where y is decreasing. $y = -(x - 1)^2$

Calculus

Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Find the intervals where y is increasing and intervals where y is decreasing. $y = -(x - 1)^2$

$$\frac{dy}{dx} = -2(x - 1)(1)$$

CN:

1. y defined?
2. x when $y' = 0$?
3. x when y' dne?

CNs: ① y def. ② $\frac{dy}{dx} = 0$: $x - 1 = 0$
 $x = 1$ CN. ③ $\frac{dy}{dx}$ exists $\forall x$

| | | | | |
|-----------------------------|----------------|-----|---------------|--|
| | $(-\infty, 1)$ | 1 | $(1, \infty)$ | |
| test | x | 0 | 2 | |
| | $x - 1$ | $-$ | $+$ | |
| $\frac{dy}{dx} = -2(x - 1)$ | $+$ | | $-$ | |
| y | incr | | decr | |

y incr. on $(-\infty, 1)$
 y decr. on $(1, \infty)$

Tabular representation of the number line, the sign of the derivative, and the incr or decr status of the function

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Find the intervals where y is increasing and intervals where y is decreasing. $y = x^4 - 2x^2$

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Find the intervals where y is increasing and intervals where y is decreasing. $y = x^4 - 2x^2$

$$1 - 2 = -1$$

$$\frac{dy}{dx} = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x+1)(x-1)$$

$$\text{cns: } x = -1, 0, 1$$

other CNs? No.
dy/dx exists for all x

| | $(-\infty, -1)$ | $(-1, 0)$ | $(0, 1)$ | $(1, \infty)$ |
|-----------------|-----------------|-----------|----------|---------------|
| x | -2 | $-1/2$ | $1/2$ | 2 |
| $4x$ | $-$ | $-$ | $+$ | $+$ |
| $x+1$ | $-$ | $+$ | $+$ | $+$ |
| $x-1$ | $-$ | $-$ | $-$ | $+$ |
| $\frac{dy}{dx}$ | $-$ | $+$ | $-$ | $+$ |
| y | decr | incr | decr | incr |

y decr. $(-\infty, -1)$ $(0, 1)$
 incr $(-1, 0)$ $(1, \infty)$
 rel. min. $(-1, 1)$ rel. max. $(1, 1)$ rel. min. $(1, 1)$

Calculus

Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Wow! We can use this approach to determine max and mins!

The First Derivative Test for Relative Extrema

Let c be a **Critical Number** of the function f that is continuous on the open interval I containing c . If f is differentiable on the interval, except possibly at c , then $f(c)$ can be classified as:

1. a **relative min**, if $f'(x)$ changes from **negative to positive** at c . $\backslash /$
2. a **relative maximum**, if $f'(x)$ changes from **positive to negative** at c . $/ \backslash$
3. **neither a max nor a min** if $f'(x)$ is **positive on both sides of c** $//$ or **negative on both sides of c** $\backslash \backslash$

Calculus

Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Find **Relative Extrema** of a continuous function using **intervals** and the **First Derivative Test**

1. Find **critical numbers** [f defined and $f'(c) = 0$ or $f'(c)$ undefined]
2. **Determine intervals** for evaluation of f' and begin the interval table:
 - a) Locate the **critical numbers along a number line** containing the domain of the function
 - b) Determine the **intervals, using the critical numbers as endpoints**.
3. Continue the interval table by:
 - a) Selecting a **test value** for each interval.
 - b) **Write $f'(x)$ in factored form** in the first column.
 - c) For each interval, find the **sign of $f'(x)$** by determining the number of negative factors.
4. Determine whether $f(x)$, the original function, is **increasing** (when $f'(x) > 0$) or **decreasing** (when $f'(x) < 0$) on each interval.
5. The critical value for which $f(x)$ is **increasing to the left** and **decreasing to the right** is a **relative max**. \wedge
6. The critical value for which $f(x)$ is **decreasing to the left** and **increasing to the right** is a **relative min**. \vee
7. Find the **corresponding f or y value** for each critical value determined to be a relative max or min, and write the ordered pair $(c, f(c))$.



Calculus

Prof G. Battaly, Westchester Community College



Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

G: $f(x) = x^2 + 8x + 10$ F: a) CNs, b) interv inc, decr, c) rel.extrema

step-by-step: 1st Deriv Test



Calculus

Prof G. Battaly, Westchester Community College



Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

G: $f(x) = x^2 + 8x + 10$ F: a) CNs, b) inter inc, decr, c) rel. extrema

$$f(x) = x^2 + 8x + 10$$

$$f'(x) = 2x + 8 = 2(x + 4)$$

$$\text{CN: } x = -4$$

F: a) CNs. ✓
 b) inter. ↑ ↓ ✓
 c) rel. extr.

other CNs? No.
 dy/dx exists for all x

| | $(-\infty, -4)$ | $(-4, \infty)$ |
|------------------|-----------------|----------------|
| x | -5 | 0 |
| $f'(x) = 2(x+4)$ | - | + |
| $f(x)$ | decr. | incr. |

$(-4, -6)$ rel. min

$$f(-4) = (-4)^2 + 8(-4) + 10$$

$$= 16 - 32 + 10$$

$$= -6$$

Calculus Prof G. Battaly, Westchester Community College Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

$$f(x) = x^3 - 6x^2 + 15$$

F: intervals where f is incr, decr
 relative extrema

Calculus Prof G. Battaly, Westchester Community College Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

$$f(x) = x^3 - 6x^2 + 15$$

$$f(4) = 64 - 96 + 15 = -17$$

$$f'(x) = 3x^2 - 12x$$

$$\text{CNS } f'(x) = 0 \Rightarrow x = 0, 4$$

$$3x(x-4) = 0$$

$$3x = 0 \quad | \quad x - 4 = 0$$

$$x = 0 \quad | \quad x = 4 \text{ (CNS)}$$

F: intervals where f is incr, decr
relative extrema

| | $(-\infty, 0)$ | $(0, 4)$ | $(4, \infty)$ |
|---------|----------------|---------------------|----------------------|
| x | -1 | 1 | 5 |
| $3x$ | - | + | + |
| $x-4$ | - | - | + |
| $f'(x)$ | + | - | + |
| $f(x)$ | incr | decr | incr |
| | | (0, 15) rel. MAX | (4, -17) rel. MIN |

Calculus

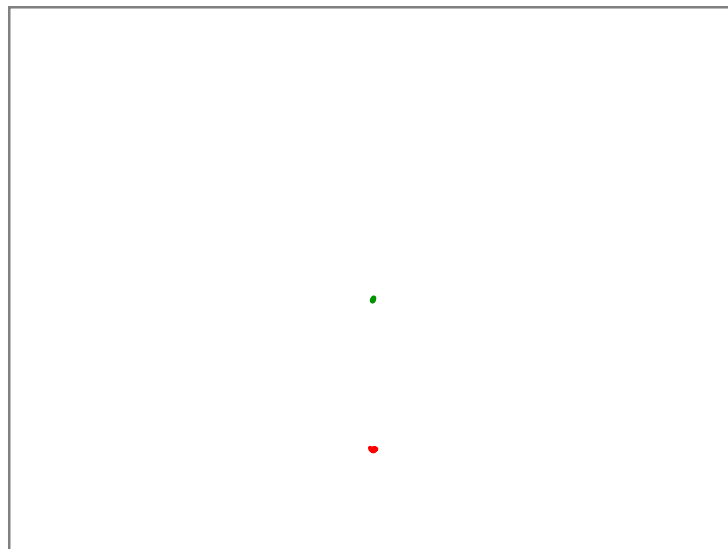
Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Compare $f(x)$, $f'(x)$, $f''(x)$ Consider: $y = x^3 - x$

Graph of function

1st Derivative: graph, slope of, relate to y ?2nd derivative: graph, relate to y ?

Calculus

Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

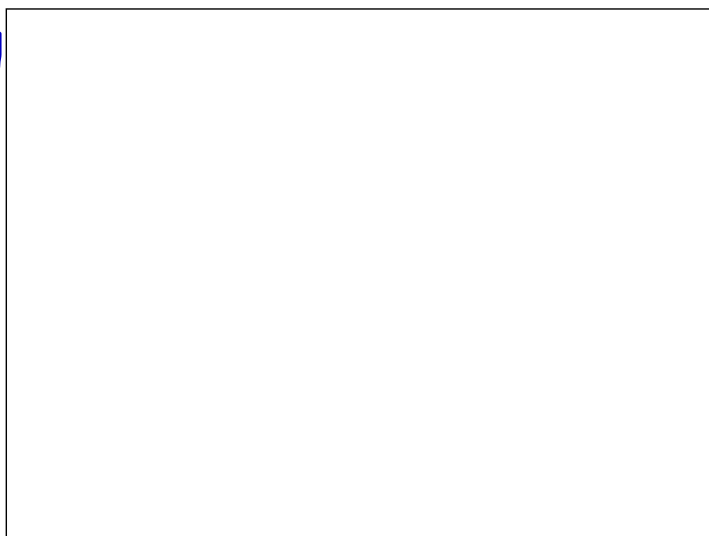
Consider: $y = x^3 - x$

Graph of function

1st Derivative: graph, slope of, relate to y?

2nd derivative: graph, relate to y?

$$\frac{dy}{dx} = 3x^2 - 1$$

Compare $f(x)$, $f'(x)$, $f''(x)$

Calculus

Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Consider: $y = x^3 - x$

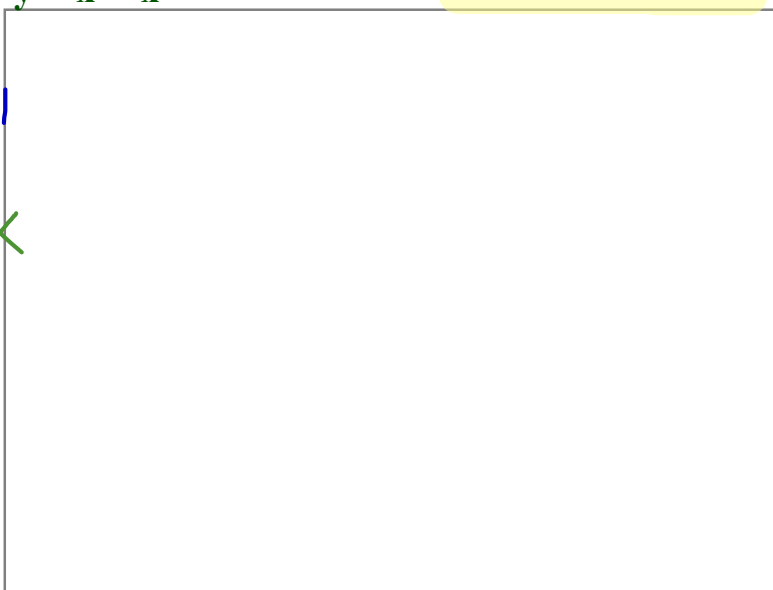
Graph of function

1st Derivative: graph, slope of, relate to y?

2nd derivative: graph, relate to y?

$$\frac{dy}{dx} = 3x^2 - 1$$

$$\frac{d^2y}{dx^2} = 6x$$

Compare $f(x)$, $f'(x)$, $f''(x)$

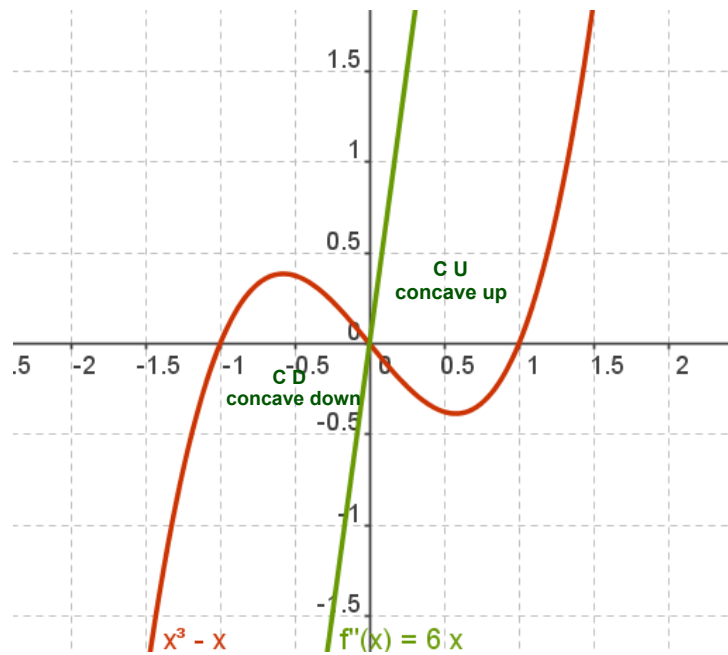
Calculus

Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

How does the 2nd derivative relate to original function?



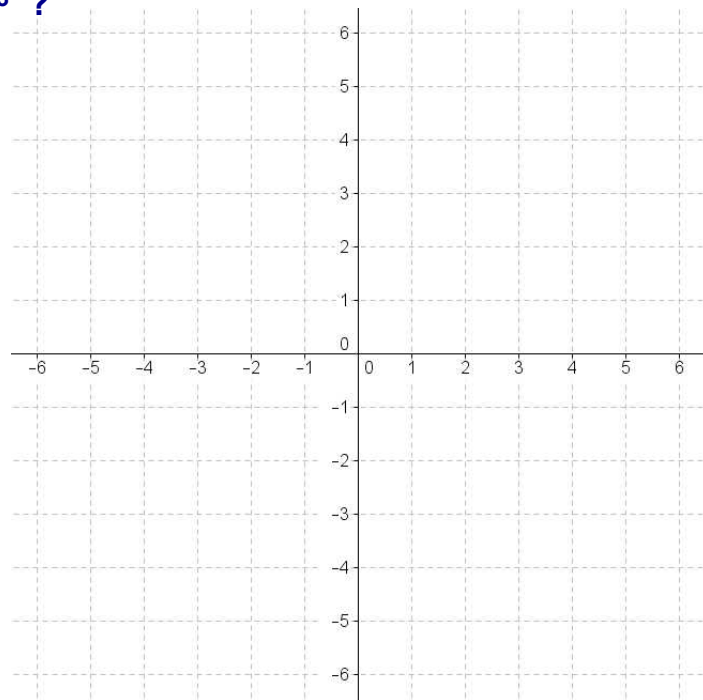
Calculus Prof G. Battaly, Westchester Community College Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

What about $y = x^3$?

$$\frac{dy}{dx} =$$

$$\frac{d^2y}{dx^2} =$$



Change $f(x)$ to x^3

C D concave down C U concave up

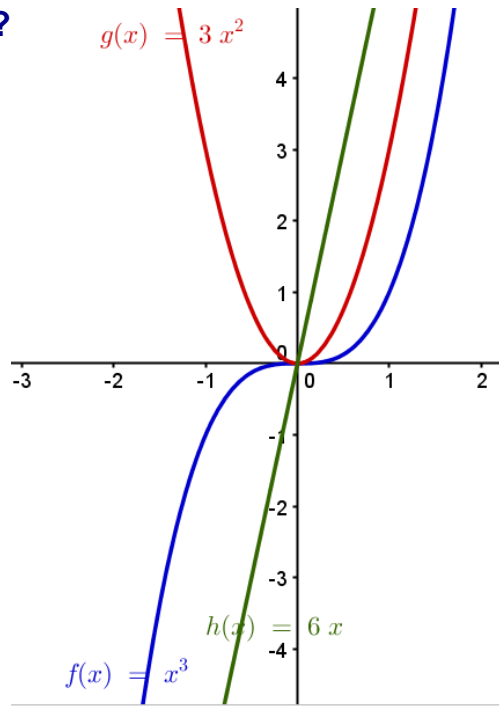
Calculus Prof G. Battaly, Westchester Community College Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

What about $y = x^3$?

$$\frac{dy}{dx} =$$

$$\frac{d^2y}{dx^2} =$$

Compare $f(x)$, $f'(x)$, $f''(x)$ C D C U
concave down concave up

Calculus

Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

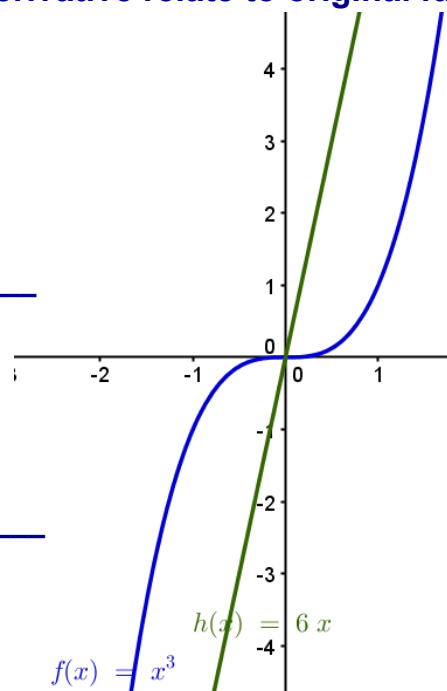
How does the 2nd derivative relate to original function?

What about $y = x^3$?When $\frac{d^2y}{dx^2} < 0$?

then y is _____

When $\frac{d^2y}{dx^2} > 0$?

then y is _____

C D
concave downC U
concave up

Calculus

Prof G. Battaly, Westchester Community College

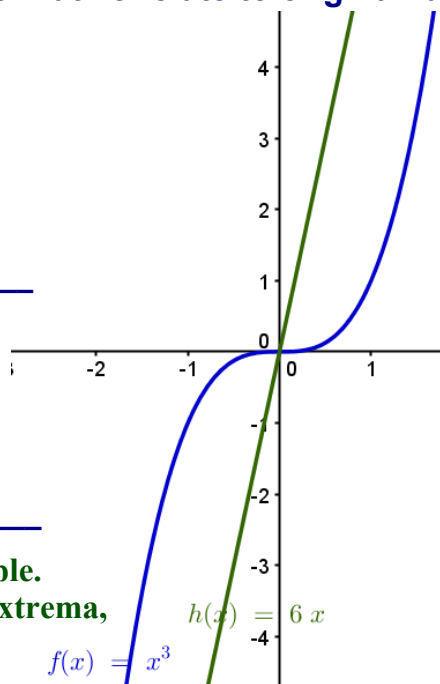
Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

How does the 2nd derivative relate to original function?

What about $y = x^3$?When $\frac{d^2y}{dx^2} < 0$?then y is
C D
concave downWhen $\frac{d^2y}{dx^2} > 0$?then y is
C U
concave up

Similar to previous example.
Even though no relative extrema,
it is still c.u. when $x > 0$
and c.d. when $x < 0$.



4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Definition of Concavity

Let f be differentiable on an open interval I . The graph of f is:

concave upward on I if f' is increasing on I

concave downward on I if f' is decreasing on I

or when $f''(x) > 0$

or when $f''(x) < 0$

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Step by step: on-line

2nd Derivative Test for Relative Extrema

Let f be a function, with

$f'(c) = 0$ and

(horizontal slope)

 $f''(x)$ continuous on open interval containing c 1. If $f''(c) > 0$, then f has a local **Min** at $(c, f(c))$ 2. If $f''(c) < 0$, then f has a local **Max** at $(c, f(c))$ 1. If $f''(c) = 0$, then the **test fails**

Calculus

Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Step by step: on-line

2nd Derivative Test for Relative Extrema: Step by Step

1. Find $f'(x)$ and values of c where $f'(c) = 0$ [not quite critical numbers - does not include $f'(c)$ undefined]
2. Find $f''(x)$. Is it continuous at c ? Test only valid when continuous.
3. Find the sign of $f''(c)$ for all c .
4. Determine the relative extrema using the Second Derivative Test:
 - a) If $f''(c) > 0$, then f is concave up and $f(c)$ is a **relative min**
 - b) If $f''(c) < 0$, then f is concave down and $f(c)$ is a **relative max**
 - c) If $f''(c) = 0$, then the **test fails**. (consider an Inflection Point - a point where concavity changes)

Calculus

Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Step by step: on-line

$$y = x^3 - x \quad \text{F: Rel. extr, 2nd derivative test}$$

Calculus

Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

$$G: y = x^3 - x \quad \text{F: Rel. extr, 2nd deriv. test}$$

$$y = x^3 - x$$

$$\frac{dy}{dx} = 3x^2 - 1$$

$$\frac{d^2y}{dx^2} = 3(2x) = 6x$$

$$\text{C.N.s: } 3x^2 = 1$$

$$x^2 = \frac{1}{3}$$

$$x = \pm \sqrt{\frac{1}{3}}$$

$$\text{at } x = \frac{1}{\sqrt{3}}, \frac{d^2y}{dx^2} \geq 0 \therefore \text{c.d.}$$

$$\left(\frac{1}{\sqrt{3}}, -\right) \text{ rel. min.}$$

$$x = -\frac{1}{\sqrt{3}}, \frac{d^2y}{dx^2} < 0 \therefore \text{c.d.}$$

$$\left(-\frac{1}{\sqrt{3}}, -\right) \text{ rel. max.}$$



$$y = x(x^2 - 1)$$

$$x = \frac{1}{\sqrt{3}}: \frac{1}{\sqrt{3}} \left(\frac{1}{3} - 1\right) = -\frac{2}{3\sqrt{3}}$$

Calculus

Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Example: G: $h(x) = x^5 - 5x + 2$
 F: open interval where c.u. and c.d.

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

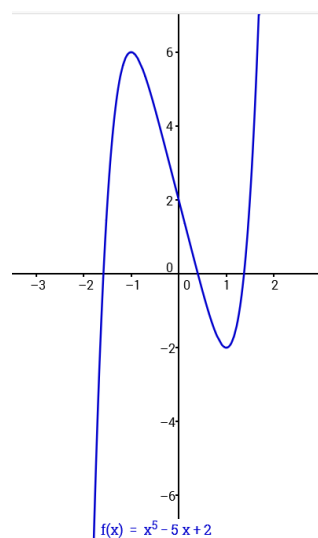
Example: G: $h(x) = x^5 - 5x + 2$ $-1 + 5 + 2$
 F: open interval where c.u. and c.d. need $h''(x)$

$$h'(x) = 5x^4 - 5 = 5(x^4 - 1) = 5(x^2 + 1)(x^2 - 1)$$

$$h''(x) = 20x^3$$

$$x > 0, h''(x) > 0; \text{c.u.}$$

$$x < 0, h''(x) < 0; \text{c.d.}$$



NOT ASKED FOR

rel. extr: c.u.: $x = \pm 1$
 $h''(-1) < 0 \therefore \text{c.d. } (-1, 6) \text{ rel MAX}$
 $h''(1) > 0 \therefore \text{c.u. } (1, -2) \text{ rel MIN}$

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Definition of Point of Inflection

Let f be a function that is **continuous** on an open interval and let c be a point on the interval. If the graph of f has a **tangent line** at this point $(c, f(c))$, then this point is a **point of inflection** of the graph of f if the **concavity of f changes** from upward to downward (or downward to upward) at the point

Inflection Point at $(c, f(c))$

1. f continuous
2. f has a tangent line
3. concavity changes
(f'' changes sign)

If $(c, f(c))$ is a point of inflection of the graph of f , then either $f''(c) = 0$ or $f''(c)$ does not exist at $x = c$.



Prof G. Battaly, Westchester Community College

Class Notes: Prof. G. Battaly, Westchester Community College,

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Example: $G: f(x) = 2x^3 - 3x^2 - 12x + 5$
F: IP

Inflection Point at $(c, f(c))$

1. f continuous
2. f has a tangent line
3. concavity changes
(f'' changes sign)

Compare $f(x)$, $f'(x)$, $f''(x)$



Prof G. Battaly, Westchester Community College



Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Example: G: $f(x) = 2x^3 - 3x^2 - 12x + 5$
 F: IP $\xrightarrow{\text{cont.}} \text{tang. line exists, conc. change.}$ $\xrightarrow{\text{at poly.}}$ f''

$$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x-2)(x+1)$$

CN: $x = -1, 2$ but not required for IP

$$f''(x) = 12x - 6 = 6(2x - 1)$$

poss IP when $f''(x) = 0$

$$2x - 1 = 0$$

$$x = \frac{1}{2}$$

$x = 1/2$ is possible IP
 need to show change in concavity
 [change in sign of $f''(x)$]

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 - 12\left(\frac{1}{2}\right) + 5$$

$$= \frac{2}{8} - \frac{3}{4} - 6 + 5$$

$$= \frac{2-6}{8} - 1 = -\frac{4}{8} - 1 = -\frac{3}{2}$$

| | | |
|--------|-------------------|---------|
| | $x = \frac{1}{2}$ | $x = 2$ |
| x | 0 | 1 |
| f'' | - | + |
| $f(x)$ | c.d | c.u |

$\therefore \left(\frac{1}{2}, -\frac{3}{2}\right)$ IP

Calculus

Prof. G. Battaly, Westchester Community College

Homework

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Problems for 3.4

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Example: G: $f(x) = 2x^3 - 3x^2 - 12x + 5$
 F: ~~IP~~ Rel. Extr. 2nd Deriv. Test

$$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x-2)(x+1)$$

$$f''(x) = 12x - 6 = 6(2x - 1)$$

$$\therefore \left(\frac{1}{2}, -\frac{3}{2}\right) \text{ IP}$$

CN: $x = -1, 2$
 but not required for IP

$$6(x-2)(x+1) = 0$$

$$(x-2)(x+1) = 0$$

$$x-2=0 \quad | \quad x+1=0$$

$$x=2 \quad | \quad x=-1$$

Calculus

Prof. G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Rel. Extr. 2nd Deriv. Test

Example: G: $f(x) = 2x^3 - 3x^2 - 12x + 5$
 F:

$$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$$

$$f''(x) = 12x - 6 = 6(2x - 1)$$

$$\therefore \left(\frac{1}{2}, -\frac{3}{2}\right) \text{ IP}$$

CN: $x = -1, 2$
 but not required for IP

$f''(2) > 0$: c.w. $(2, -15)$ rel. min

$f''(-1) < 0$: c.d. $(-1, 12)$ rel. max

$$6(x-2)(x+1) = 0$$

$$(x-2)(x+1) = 0$$

$$x-2=0 \quad | \quad x+1=0$$

$$x=2 \quad | \quad x=-1$$

$$f(2) = 2(2^3) - 3(2^2) - 12(2) + 5 = 16 - 12 - 24 + 5 = -15$$

$$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 5 = -2 - 3 + 12 + 5 = 12$$

Prof G. Battaly, Westchester Community College

Calculus

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Theorem: Points of Inflection

If $(c, f(c))$ is a point of inflection of the graph of f ,
 then either:

$$f''(c) = 0 \quad \text{or} \quad f'' \text{ does not exist at } x = c$$

how?

Consider: $y = x^{1/3}$ and $y = x^{2/3}$
 has IP no IP

Change $f(x)$ to $x^{1/3}$

Calculus

Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Theorem: Points of Inflection

If $(c, f(c))$ is a point of inflection of the graph of f ,
then either:

$$f''(c) = 0 \quad \text{or} \quad f'' \text{ does not exist at } x = c$$

how?

$$\begin{aligned} y &= x^{1/3} \\ \frac{dy}{dx} &= \frac{1}{3} x^{-2/3} \\ \frac{d^2y}{dx^2} &= -\frac{2}{9} x^{-5/3} \\ &= -\frac{2}{9x^{5/3}} \end{aligned}$$

Consider:

$$y = x^{1/3}$$

$$(0,0) \text{ is IP } \begin{cases} x < 0, \frac{d^2y}{dx^2} > 0 \text{ c.u.} \\ x > 0, \frac{d^2y}{dx^2} < 0 \text{ c.d.} \end{cases}$$

change in concavity from
concave up to concave down



Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

Theorem: Points of Inflection

If $(c, f(c))$ is a point of inflection of the graph of f ,
then either:

$$f''(c) = 0 \quad \text{or} \quad f'' \text{ does not exist at } x = c$$

how?

$$\begin{aligned} y &= x^{2/3} \\ \frac{dy}{dx} &= \frac{2}{3} x^{-1/3} \\ \frac{d^2y}{dx^2} &= -\frac{2}{9} x^{-4/3} \\ \frac{d^2y}{dx^2} &= -\frac{2}{9x^{4/3}} \end{aligned}$$

Consider:

$$y = x^{2/3}$$

$$(0,0) \text{ not IP } \begin{cases} x < 0, \frac{d^2y}{dx^2} < 0 \text{ c.d.} \\ x > 0, \frac{d^2y}{dx^2} < 0 \text{ c.d.} \end{cases}$$

no change in concavity



Prof G. Battaly, Westchester Community College

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

226. $f(x) = x^4 - 6x^3$

For the following exercises, determine

- intervals where f is increasing or decreasing,
- local minima and maxima of f ,
- intervals where f is concave up and concave down, and
- the inflection points of f .

Change $f(x)$ to $x^{1/3}$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Calculus

Prof G. Battaly, Westchester Community College

Problems for 3.4

Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

226. $f(x) = x^4 - 6x^3$

$$f'(x) = 4x^3 - 18x^2 = 2x^2(2x - 9)$$

$$f''(x) = 12x^2 - 36x = 12x(x - 3)$$

\vdots CN: $x = 0, 9/2$
 \vdots f has IP
 $x = 0, 3$

| | | | | |
|----------------------|-----------------------|-----------------------|----------------|------------------------------|
| | 0 | 3 | $4\frac{1}{2}$ | |
| test | -1 | 1 | 4 | 5 |
| $f'(x) = 2x^2(2x-9)$ | - | - | - | + |
| $f(x)$ | dec | dec | dec | incr |
| $f''(x) = 12x(x-3)$ | + | - | + | + |
| $f(x)$ | cu | cd | cu | cu |
| | (0, <u> </u>) IP | (3, <u> </u>) IP | | (9/2, <u> </u>) rel.MIN |
| | (0,0)IP | (3,-81)IP | | (9/2,-136.7) rel.MIN |

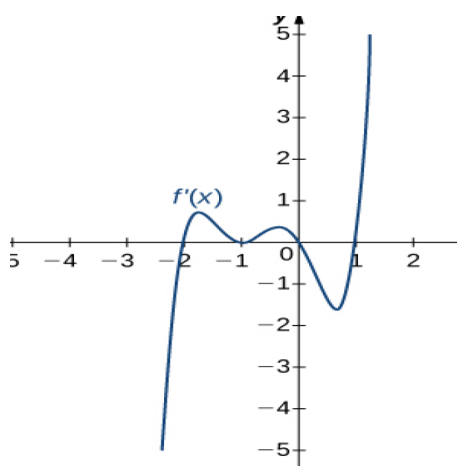
Change $f(x)$ to $x^{1/3}$

Calculus

Prof G. Battaly, Westchester Community College

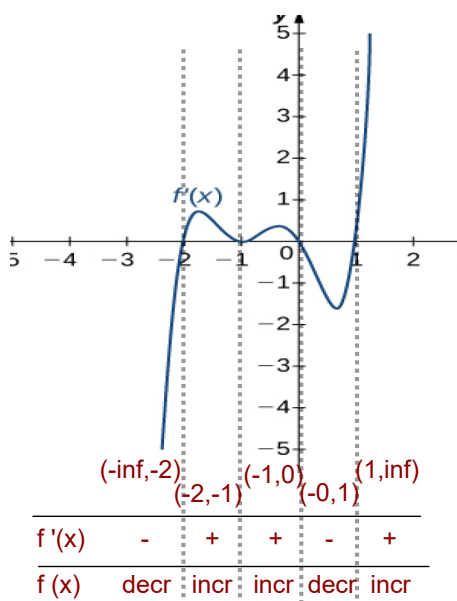
Homework

For the following exercises, analyze the graphs of f' , then list all intervals where f is increasing or decreasing.



Homework

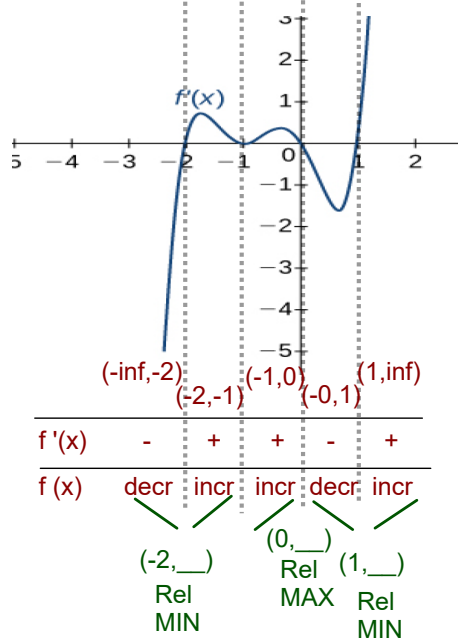
For the following exercises, analyze the graphs of f' , then list all intervals where f is increasing or decreasing.



Homework

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

For the following exercises, analyze the graphs of f' , then list all intervals where f is increasing or decreasing.



F: which represent relative extrema?

Do not know the y-values
Need the function itself to
find the y-values

Class Notes: Prof. G. Battaly, Westchester Community College, NY

[Calculus Home Page](#)

[Calculus](#)

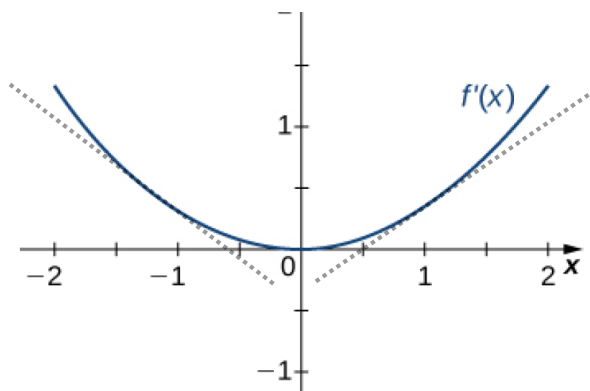
Prof G. Battaly, Westchester Community College

[Problems for 3.4](#)

[Homework](#)

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

For the following exercises, analyze the graphs of f' , then list all inflection points and intervals f that are concave up and concave down.



To determine the sign of $f''(x)$, need to look at slope of $f'(x)$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

[Calculus Home Page](#)

[Calculus](#)

Prof G. Battaly, Westchester Community College

[Problems for 3.4](#)

[Homework](#)

4.5 Increasing, Decreasing, Concavity, and Tests for Extrema

For the following exercises, analyze the graphs of f' , then list all inflection points and intervals f that are concave up and concave down.

