4.3 Critical Values and Extrema

Goals:

1. Understand critical numbers and how to find them.
2. Understand the difference between relative extrema and absolute extrema.
3. Understand how critical numbers relate to the relative extrema of the function.
4. Find absolute extrema on a closed interval.

Study 4.3 # 109-123

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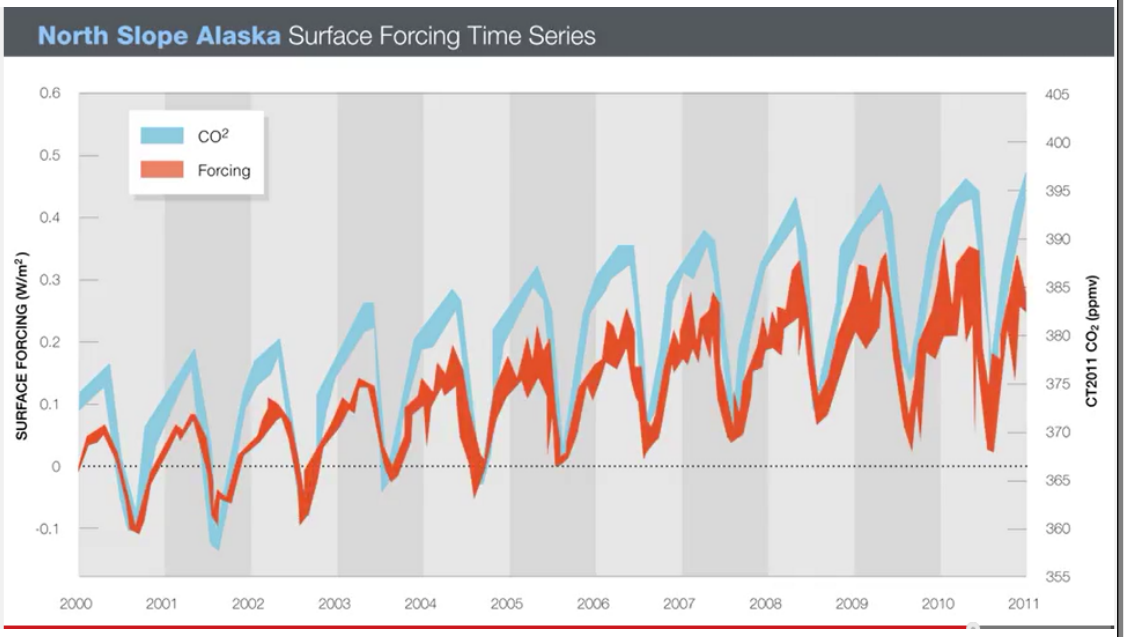
Moving Shadow problem

4.3 Critical Values and Extrema

CO_2 time series



First Direct Observation of Carbon Dioxide's Increasing Greenhouse Effect at the Earth's Surface

radiative forcing: difference between insolation (energy from sun) and energy radiated to space

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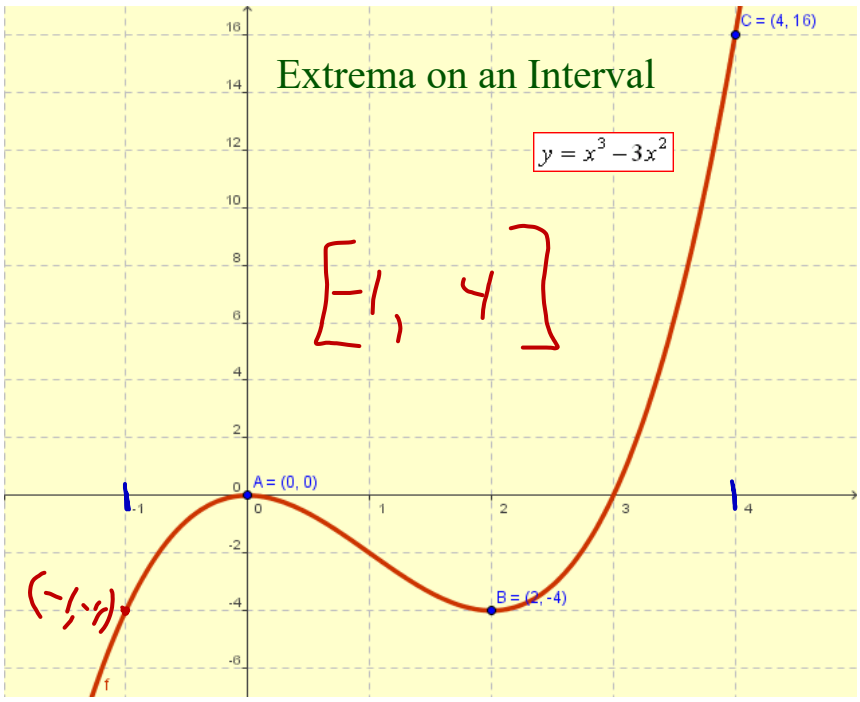
4.3 Critical Values and Extrema



Describe what happens to the graph on [-1, 4]

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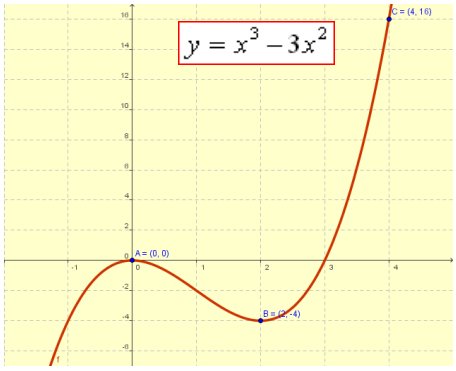
4.3 Critical Values and Extrema

Extrema on an Interval



Describe what happens to the graph on [-1, 4]

Looking at y values as x changes,

Is there a maximum y value on [-1,4]?

Is there a minimum y value on [-1,4]?

Would the minimum y value be different on [0,4]?

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4.3 Critical Values and Extrema

Definition of Extrema: Let *f* be defined on an interval *I* containing c. *(Note that I can be open or closed.)*

1. f(c) is a **minimum of f** on *I* if $f(c) \leq f(x)$ for all x in *I*.
2. f(c) is a **maximum of f** on *I* if $f(c) \geq f(x)$ for all x in *I*.

The maximum and minimum values are **extreme values**.
Note that these are values of f (or y), not values of x.

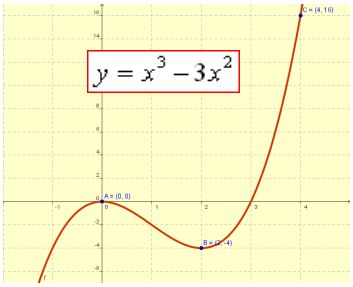
The minimum and maximum on the interval are called the **absolute** minimum and absolute maximum.

Extreme Value Theorem

If *f* is continuous on a **closed** interval [a, b], then f has both a minimum and a maximum on the interval.

4.3 Critical Values and Extrema

Extrema on an Interval



Describe what happens to the graph on [-1, 4]

We need a way to describe the points
where the graph of f turns:
from up to down
from down to up

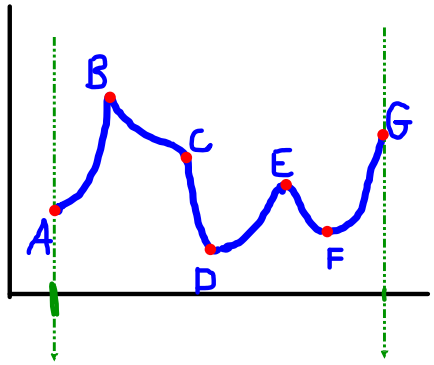
Restrict the interval to an open interval containing the point. Then,

Definition of **Relative Extrema**:

1. If there is an **open interval** containing *c* on which *f(c)* is a maximum**,** then *f(c)* is called a **relative maximum** of f. Or ***f* has a relative max at *(c,f(c))***.
2. If there is an **open interval** containing *c* on which *f(c)* is a minimum**,** then *f(c)* is called a **relative minimum** of f. Or ***f* has a relative min at *(c,f(c))***.

4.3 Critical Values and Extrema

Problem 1. At each labeled point, determine if there is a relative min or max, or an absolute min or max.



A

B

C

D

E

F

G

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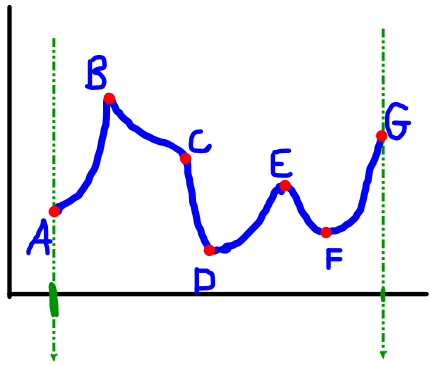
4.3 Critical Values and Extrema

Problem 1. At each labeled point, determine if there is a relative min or max, or an absolute min or max.



A none

B Absolute Max

C none

D Absolute Min

E relative max

F relative min

G none

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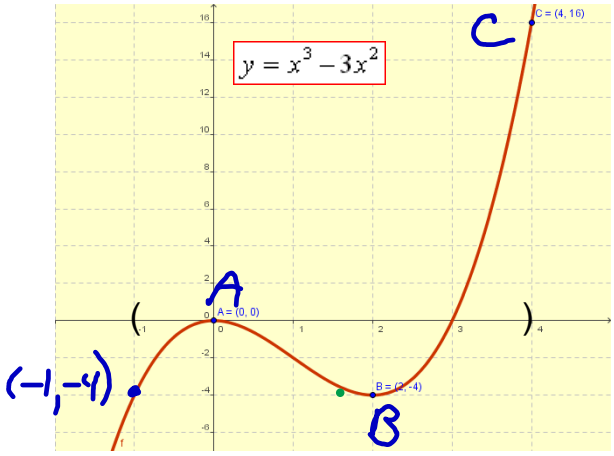
4.3 Critical Values and Extrema



On [-1,4]

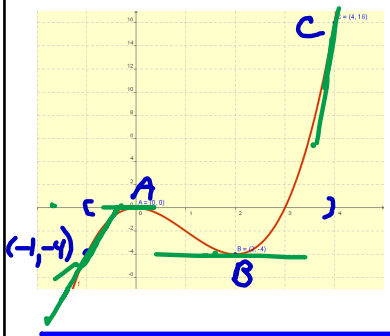
ABS. MIN =
" MAX =

rel min =
rel max =

What is the slope of the tangent line at these extrema?
Can the slope help us to find the extrema?
If so, for which extrema is it helpful?

3.1 Extrema on an Interval

4.3 Critical Values and Extrema



ABS. MIN = -4
" MAX = 16

rel min = -4
rel max = 0

Definition of **Critical Number:** **CN**

Let **f be defined at c,** c an interior point.
If ***f ' (c) = 0*, or**
if *f* is not differentiable at *c*,

then ***c* is a critical number** of *f*.

CN

① f defined
② f'(c) = 0
③ f'(c) DNE

c is X value

CN

4.3 Critical Values and Extrema

$g(x) = x^2(x^2-4) = x^4-4x^2$ F: CNs

CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

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4.3 Critical Values and Extrema

$g(x) = x^2(x^2-4) = x^4-4x^2$

① g defined for all x

② $g'(x) = 4x^3-8x$

CN: $g'(x)=0$

$4x(x^2-2)$

$x=0 \quad x=\pm\sqrt{2}$

There are 3 Critical Numbers

③ $g'(x)$ undef.

CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

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4.3 Critical Values and Extrema

$y = |x|$ F: CN

CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

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4.3 Critical Values and Extrema

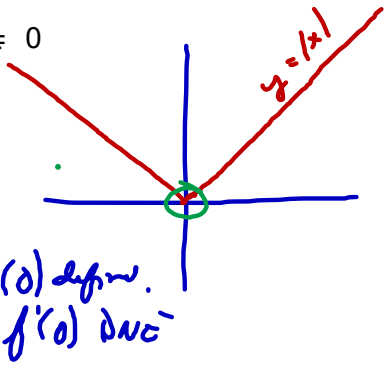
$y = |x|$ F: CN

CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

① y defined for all x

② $y' = \begin{cases} +1, & x>0 \\ -1, & x<0 \end{cases}$ -> $y' \neq 0$

③ not differentiable at $x=0$



$\neq$

is CN bec. f(0) defined but f'(0) DNE

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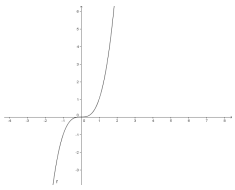
4.3 Critical Values and Extrema

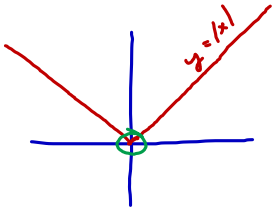
Relationship between extrema and CN?

Do all extrema occur at CNs?

Do all CNs occur at extrema?



Extrema ⟷ ? CN



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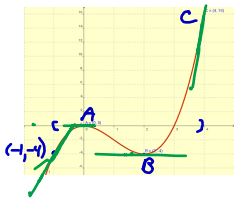
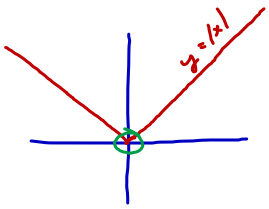
4.3 Critical Values and Extrema

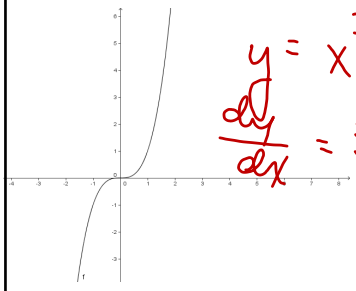
3.1 Extrema on an Interval





At extrema, f '(x)=0
or f '(x) dne

Extrema ⟶ CN



$y = x^3$

$\frac{dy}{dx} = 3x^2$

F: CN

$3x^2 = 0$

$x = 0$ CV

NOT extrema

Extrema ⟶// CN

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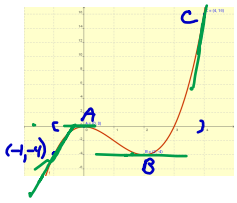
4.3 Critical Values and Extrema

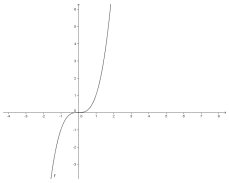
Relationship between extrema and CN?

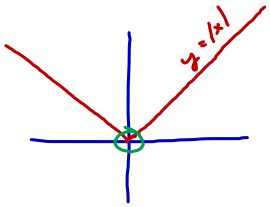
Do all extrema occur at CNs? Yes.

Do all CNs occur at extrema? Not always, but sometimes.

3.1 Extrema on an Interval







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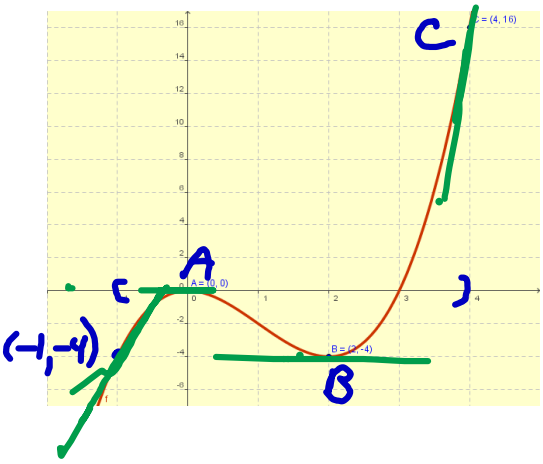
4.3 Critical Values and Extrema

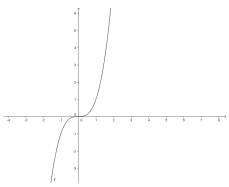
How Can We Find Absolute Extrema on a Closed Interval?

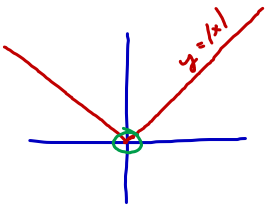


Can occur at:

1) _____ or

2) _____





1) CV
2) end points

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4.3 Critical Values and Extrema

Find Absolute Extrema, Closed Interval

1. Find CV's for the interval and corresponding y values
2. Evaluate to find y at the endpoints
3. Compare all y values for both the CV's and the endpts.

Largest - ABSOLUTE MAX

Smallest - ABSOLUTE MIN

at CN or endpt

	x	y
CNs		
Endpts		

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4.3 Critical Values and Extrema

G: $f(x) = x^2 + 2x - 4$

F: abs. extr. on $[-1, 1]$

at CN or endpt

	x	y
CNs		
Endpts		

CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

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4.3 Critical Values and Extrema

G: $f(x) = x^2 + 2x - 4$

F: abs. extr. on $[-1, 1]$

① f defined for all x

CNs ② $f'(x) = 0$ or ③ DNE

$f'(x) = 2x + 2$

$2x + 2 = 0$

$x + 1 = 0$

$x = -1$

at CN or endpt

	x	y	
CNs	-1	-5	ABS MIN
Endpts	-1	$1-2-4 = -5$	
	1	$1+2-4 = -1$	ABS MAX

CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

4.3 Critical Values and Extrema

$y = |x|$

F: abs. extr. on $[-2, 3]$

at CN or endpt

	x	y
CNs		
Endpts		

CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

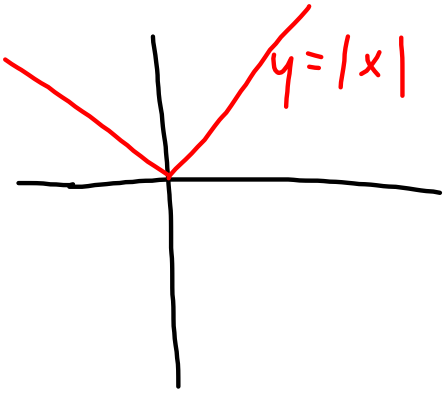
4.3 Critical Values and Extrema

F: abs. extr. on $[-2,3]$

$y = |x|$

③

CN: $x = 0$ $\frac{dy}{dx}$ DNE



at CN or endpt

	x	y	
CNs	0	0	ABS MIN
Endpts	-2	2	
	3	3	ABS MAX

CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

4.3 Critical Values and Extrema

G: $f(x) = \frac{2x}{x^2+1}$ F: Abs Extr $[-2,2]$

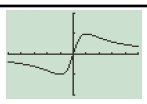
CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

at CN or endpt

	x	y
CNs		
Endpts		

4.3 Critical Values and Extrema

G: $f(x) = \frac{2x}{x^2+1}$ F: Abs Extr $[-2, 2]$



CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

① f defined for all x

CN

$f'(x) = \frac{(x^2+1)(2) - 2x(2x)}{(x^2+1)^2}$

$= \frac{2x^2+2-4x^2}{(x^2+1)^2}$

$= \frac{2-2x^2}{(x^2+1)^2} = \frac{2(1-x^2)}{(x^2+1)^2}$

② $f'(x) = 0$: $\frac{2(1-x^2)}{(x^2+1)^2} = 0$

$2(1-x^2) = 0$

$1-x^2 = 0$

$(1-x)(1+x) = 0$

$1-x=0 \mid 1+x=0$

$x=1 \mid x=-1$

$x = -1, 1$ $f'(x)$ def. all x

at CN or endpt

	x	y
CNs	-1	$-2/2 = -1$
	1	$2/2 = +1$
Endpts	-2	$-4/5$
	2	$4/5$

CN $\frac{dy}{dx} \neq 0$ nt def.

$\frac{a}{b} = 0$ $a = 0$ $\frac{dy}{dx} = 0$

$\frac{1}{b} = 0$ $\frac{dy}{dx}$ DNE

③ f' defined for all x (den $\neq 0$)

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4.3 Critical Values and Extrema

G: $h(t) = \frac{t}{t-2}$ F: Abs. Extr. $[3, 5]$

$t \neq 2 \notin [3, 5]$

CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

at CN or endpt

	x	y
CNs		
Endpts		

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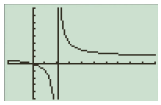
4.3 Critical Values and Extrema

G: $h(t) = \frac{t}{t-2}$ F: Abs. Extr. $[3,5]$ $t \neq 2 \notin [3,5]$



CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

① h defined for all x

$$h'(t) = \frac{(t-2)(1) - t(1)}{(t-2)^2}$$

$$= \frac{t-2-t}{(t-2)^2} = \frac{-2}{(t-2)^2} \quad \therefore \text{No CNs}$$

② CN: not for $h'(t) = 0$
$h'(t)$ DNE ③ $t = 2$ & h not def.

x	y
CN	
endpt. 3	3/1 = 3
5	5/3

abs MAX

4.3 Critical Values and Extrema

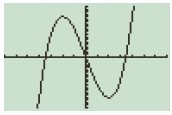
G: $f(x) = x^3 - 12x$ F: Abs. Extr. $[0, 4]$

CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

at CN or endpt

x	y
CNs	
Endpts	

4.3 Critical Values and Extrema



G: $f(x) = x^3 - 12x$ F: Abs. Etr. $[0, 4]$

① f defined for all x

$f'(x) = 3x^2 - 12 = 3(x^2 - 4)$

CN:

② $3(x^2 - 4) = 0$

$x = \pm 2$ $\quad -2 \notin [0, 4]$

③ $f'(x)$ exists all x

	x	y	
CN	2	$8 - 24 = -16$	ABS MIN
endpts	0	0	
	4	$64 - 48 = 16$	ABS MAX

CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

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