4.2 Approximations and Differentials

Goals:

1. Learn the definition of a differential.
2. Find differentials of functions.
3. Use differentials for approximations.
4. Recognize that tangent lines can be used for approximations of function values.

Study 4.2 # 69-83

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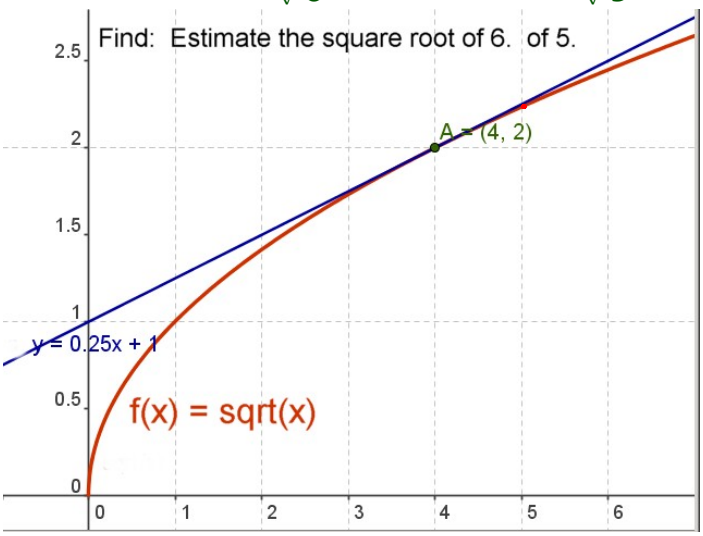
4.2 Approximations and Differentials

Approximations using Tangent Lines

Estimate $\sqrt{6}$ **Estimate $\sqrt{5}$**



One method is to **use the tangent line**
This can be a good estimate when the value of x is close to a value that is easy to compute.

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Another method is to use differentials:

Definition:

Let $y = f(x)$ represent a function that is differentiable on an open interval containing x.

The differential of x, dx, is any nonzero real number.

The differential of y, dy, is:

$$dy = f'(x)\,dx$$

$$dy = \frac{dy}{dx}\,dx$$

4.2 Approximations and Differentials

Approximations using Differentials

Find $\sqrt{5}$ Use $y = \sqrt{x}$. Let $x = 4$ and $dx = +1$

$$dy = f'(x)\,dx$$

$$y = x^{1/2}$$
$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

Calculator ~ 2.24

4.2 Approximations and Differentials

Approximations using Differentials

Find $\sqrt{5}$ Use $y = \sqrt{x}$. Let $x = 4$ and $dx = +1$

$$dy = f'(x)\,dx$$
$$dy = 1/(2\sqrt{x})\,dx$$
$$dy = 1/(2\sqrt{4})\,(1)$$
$$dy = 1/4$$

$y = x^{1/2}$
$\frac{dy}{dx} = \frac{1}{2}x^{-1/2}$
$= \frac{1}{2\sqrt{x}}$

Therefore, $\sqrt{5} \approx \sqrt{4} + dy = 2 + 1/4 = 2.25$

Calculator ~ 2.24

4.2 Approximations and Differentials

Finding Differentials of Functions using Definition of the Differential

G: $y = 3\,x^{2/3}$ F: differential dy

$dy = f'(x)\,dx$

4.2 Approximations and Differentials

Finding Differentials of Functions using Definition of the Differential

G: $y = 3\,x^{2/3}$ F: differential dy

$$dy = f'(x)\,dx$$

$$dy = 3\,(2/3)\,x^{-1/3}\,dx$$

$$dy = \frac{2}{x^{1/3}}\,dx$$

4.2 Approximations and Differentials

72. $y = 3x^2 - x + 6, \quad x = 2, \quad dx = 0.1$

F: dy

$$dy = \frac{dy}{dx}\,dx$$

4.2 Approximations and Differentials

72. $y = 3x^2 - x + 6, \quad x = 2, \quad dx = 0.1$

F: dy

$dy = \frac{dy}{dx}\, dx$

$dy = (6x-1)\,dx$

$dy\big|_{\substack{x=2 \\ dx=0.1}} = (6(2)-1)\,0.1 = 11(0.1) = 1.1$

So, to approximate y when x=2.1

$\approx f(2)+dy = 3(4)-2+6+dy = 16+1.1 = 17.1$

Actual: f(2.1)=17.13

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Finding Differentials of Functions using Definition of the Differential

G: $y = x\cos x$ F: differential dy

$dy = \frac{dy}{dx}\, dx$

4.2 Approximations and Differentials

Finding Differentials of Functions using Definition of the Differential

G: $y = x\,cosx$ F: differential dy

$$dy = \frac{dy}{dx}\,dx$$

$$dy = (-x\,sinx + cosx)\,dx$$

4.2 Approximations and Differentials

78. dV if the sides of a cube change from 10 to 10.1.

F: differential dy

$$dy = \frac{dy}{dx}\,dx$$

4.2 Approximations and Differentials

78. dV if the sides of a cube change from 10 to 10.1.

$V = x^3$

$dV = 3x^2\,dx$

F: differential dy

$dy = \frac{dy}{dx}\,dx$

$x = 10$

$dx = 0.1$

$dV = 3(10)^2(0.1)$

$= 300(0.1) = 30.0$ cubic units

4.2 Approximations and Differentials

Finding Differentials of Functions using Definition of the Differential

G: $y = 2 - x^4$ F: differential dy

$dy = \frac{dy}{dx}\,dx$

4.2 Approximations and Differentials

Finding Differentials of Functions using Definition of the Differential

G: $y = 2 - x^4$ F: differential dy

$$dy = \frac{dy}{dx}\, dx$$

$$dy = -4x^3\, dx$$

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Finding Differentials of Functions using Definition of the Differential

G: $y = 3x^5 - 2x^2 + 1$ F: differential dy

$$dy = \frac{dy}{dx}\, dx$$

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Finding Differentials of Functions using Definition of the Differential

G: $y = 3x^5 - 2x^2 + 1$ F: differential dy

$$dy = \frac{dy}{dx}\,dx$$

$$dy = (15x^4 - 4x)\,dx$$

4.2 Approximations and Differentials

70. G: $y = \sqrt{1+x}$ F: dy

$dy = \frac{dy}{dx}\,dx$

4.2 Approximations and Differentials

70. G: $y = \sqrt{1+x}$ F: dy

$y = (1+x)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(1+x)^{-1/2}(1)$

$\frac{dy}{dx} = \frac{1}{2\sqrt{1+x}}$

$dy = \frac{dy}{dx}\,dx$

$dy = \frac{1}{2\sqrt{1+x}}\,dx$

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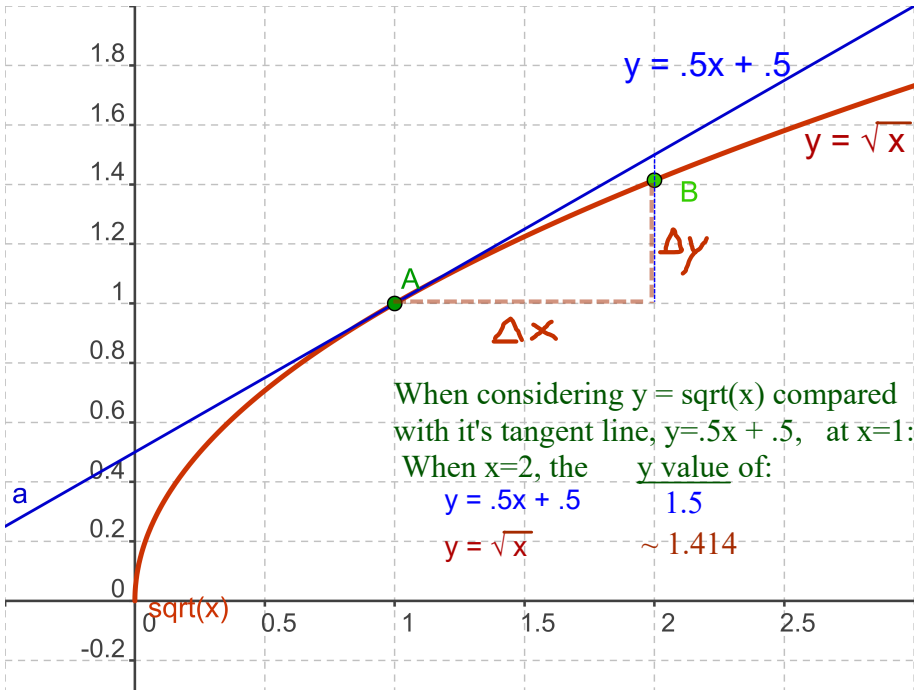
EXTRA BACKGROUND



When considering y = sqrt(x) compared with it's tangent line, y=.5x + .5, at x=1:

When x=2, the

	y value of:
y = .5x + .5	1.5
$y = \sqrt{x}$	~ 1.414

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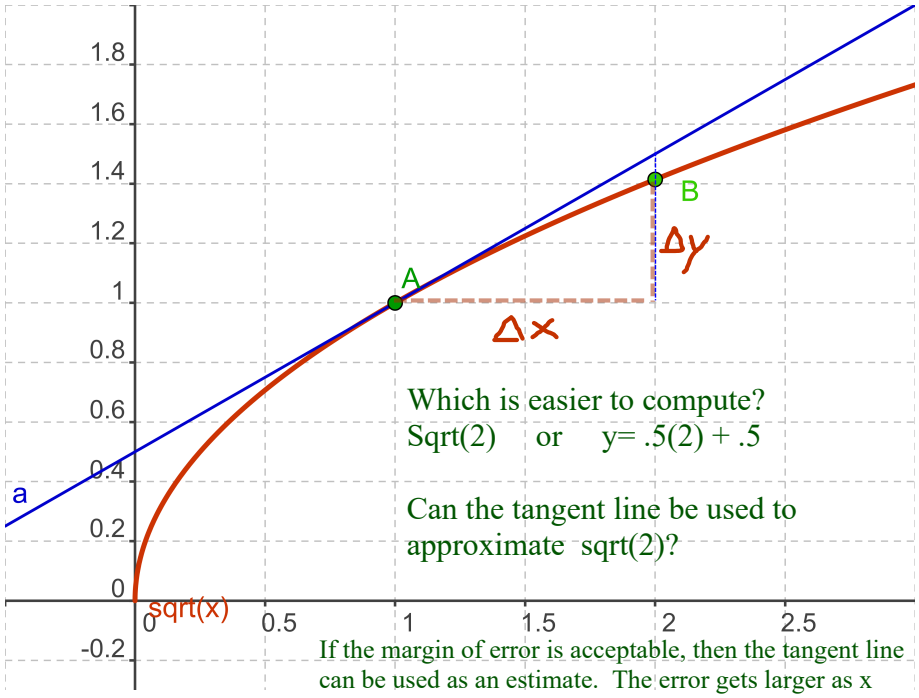
EXTRA BACKGROUND



Which is easier to compute?
Sqrt(2) or y= .5(2) + .5

Can the tangent line be used to approximate sqrt(2)?

If the margin of error is acceptable, then the tangent line can be used as an estimate. The error gets larger as x gets further from the original value.

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