# 4.1 Related Rates

Goal:
1. Understand how variables change with respect to time.
2. Understand "with respect to".

Study 4.1, # 1-7, 11, 17, 19, 23, 25, 29, 35

water drop ripples

pour cement high rise

water drop ripples

pour cement high rise



Calculus Home Page

Homework

Extra Problems

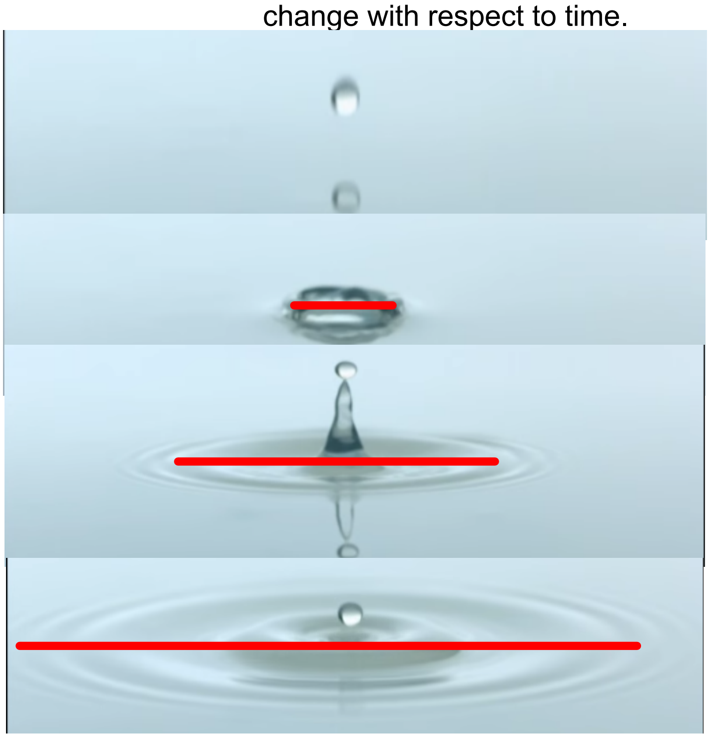
# 4.1 Related Rates

Goal: Understand how variables change with respect to time.

water drop ripples

What variables are changing?

with respect to what?



water drop ripples

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework

Extra Problems

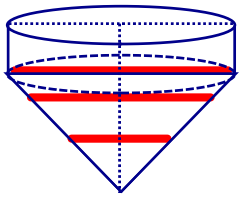
## 4.1 Related Rates

Goal: Understand how variables change with respect to time.

pour cement high rise

What variables are changing?

with respect to what?



pour cement high rise

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework

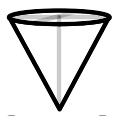
Homework

Extra Problems

## 4.1 Related Rates

Refer to problems # 24 - 27

For the following exercises, consider a right cone that is leaking water. The dimensions of the conical tank are a height of 16 ft and a radius of 5 ft.



**What is common to all these problems?**

25. How fast does the depth of the water change when the water is 10 ft high if the cone leaks water at a rate of 10 $ft^3$/min?

26. Find the rate at which the surface area of the water changes when the water is 10 ft high if the cone leaks water at a rate of 10 $ft^3$/min.

27. If the water level is decreasing at a rate of 3 in./min when the depth of the water is 8 ft, determine the rate at which water is leaking out of the cone.

## 4.1 Related Rates

Refer to problems # 24 - 27

What is common to all these problems?

geogebra: Related Rates

$$\frac{d\boxed{\phantom{x}}}{dt}$$

rate of change of distance with respect to time $\frac{dx}{dt}$ $\frac{dy}{dt}$

rate of change of volume with respect to time $\frac{dV}{dt}$

rate of change of area with respect to time $\frac{dA}{dt}$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework

Extra Problems

## 4.1 Related Rates

PREVIOUS DISTANCE, VELOCITY, ACCELERATION
were **functions of time** s(t), v(t), a(t)

**Definition** Ch. 3.4 Rates of Change

Let $s(t)$ be a function giving the position of an object at time $t$.

The velocity of the object at time $t$ is given by $v(t) = s'(t)$.

The speed of the object at time $t$ is given by $|v(t)|$.

The acceleration of the object at $t$ is given by $a(t) = v'(t) = s''(t)$.

**RELATED RATES** ARE DIFFERENT FROM PREVIOUS
-**NOT** functions of time
-**ARE DERIVATIVES WITH RESPECT TO TIME**
of other variables

geogebra: Related Rates

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework

Extra Problems

4.1 Related Rates

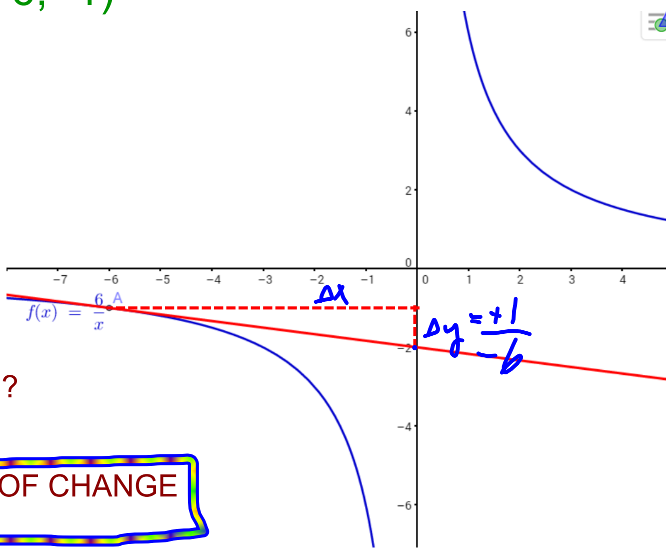
**HOW** do we find **DERIVATIVES WITH RESPECT TO TIME** of other variables? From 3.8: **implicit diff. with Chain Rule**

G: $xy = 6$ **F: dy/dx** at (-6, -1)

$$x \cdot \frac{dy}{dx} + y \cdot 1 = 0$$

$$\frac{dy}{dx} = -\frac{y}{x} \qquad = -\frac{-1}{-6}$$

$$\left.\frac{dy}{dx}\right|_{(-6,-1)} = -\frac{1}{6}$$




How can we interpret dy/dx = -1/6?

slope of tangent line at (-6, -1)

$\boxed{\text{ALSO, INSTANTANEOUS RATE OF CHANGE of y with respect to x !}}$

Related Rates: DERIVATIVES **WITH RESPECT TO TIME** INSTEAD OF WITH RESPECT TO X

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page Homework Extra Problems

4.1 Related Rates

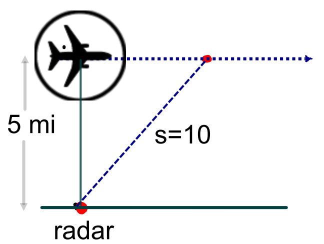
Related Rates: DERIVATIVES **WITH RESPECT TO TIME** INSTEAD OF WITH RESPECT TO X

When rate of change is with respect to time instead, how do you decide **what variables** to use?

How do you proceed? Example:

An airplane is flying at an altitude of 5 miles and passes directly over a radar antenna. When the plane is 10 miles away, the radar detects that the distance S is changing at a rate of 240 miles per hour. What is the speed of the plane?

**G: altitude 5 mi;**
**when plane 10 mi away, S changing at 240 mph**




**F: speed of plane**

rate of change with respect to time

geogebra: Related Rates

6. Check:
Did you include units? Yes
Does the answer make sense? Yes. dH/dt is directly proportional to S and inversely prop. to H, so it should be larger than dS/dt.
If there were angles involved, did you use radians? No angles

Homework

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Extra Problems

## 4.1 Related Rates

step-by-step

### Related Rates

1. Identify: a "given" rate, a "to find" rate, other conditions.
2. Determine the relationship (equation) between the "given" and the "to find". Use a diagram, if possible, or known formula.
3. If possible, use the given information to reduce the number of variables.
4. Differentiate implicitly with respect to time. This **always involves the chain rule**. For example,

$$\frac{dV}{dt} = \frac{dV}{dr}\,\frac{dr}{dt} \qquad or \qquad \frac{dy}{dt} = \frac{dy}{dx}\,\frac{dx}{dt}$$

5. Substitute the given values and solve for the unknown rate.
6. Check: * Did you include units?
   * does the answer make sense?
   * If there were angles involved, did you use radians?

Calculus Home Page

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Homework

Extra Problems

## 4.1 Related Rates

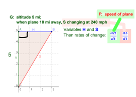
geogebra: Related Rates

An airplane is flying at an altitude of 5 miles and passes directly over a radar antenna. When the plane is 10 miles away, the radar detects that the distance S is changing at a rate of 240 miles per hour. What is the speed of the plane?

G: ___________________________

F: ___________________________

1. Identify: a "given" rate, a "to find" rate, other variable
2. Determine the relationship (equati between the 'given' and the 'to find Use a diagram, if possible, or know
3. If possible, use the given informa to reduce the number of variable (Not needed here.)
4. Differentiate implicitly with resp This always involves the **chain**
5. Substitute the given values and solve for the unknown rate.



6. Check:
Did you include units? Yes
Does the answer make sense? Yes. dH/dt is directly proportional to S and inversely prop. to H, so it should be larger than dS/dt.
If there were angles involved, did you use radians? No angles

Homework

Calculus Home Page

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Extra Problems

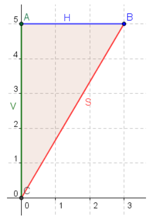
## 4.1 Related Rates

geogebra: Related Rates

An airplane is flying at an altitude of 5 miles and passes directly over a radar antenna. When the plane is 10 miles away, the radar detects that the distance S is changing at a rate of 240 miles per hour. What is the speed of the plane?

G: altitude 5 mi;
when plane 10 mi away, S changing at 240 mph

F: speed of plane




Variables **H** and **S**
Then rates of change: $\frac{dH}{dt}$ $\frac{dS}{dt}$

1. Identify: a "given" rate, a "to find" rate, other variables.

$$H^2 + 5^2 = S^2$$

2. Determine the relationship (equation) between the 'given' and the 'to find'. Use a diagram, if possible, or known formula.

3. If possible, use the given information to reduce the number of variables. (Not needed here.)

$$2H \cdot \frac{dH}{dt} + 0 = 2S \cdot \frac{dS}{dt}$$

4. Differentiate implicitly with respect to time. This always involves the **chain rule**.

remember F: speed of plane $\frac{dH}{dt}$

at $s = 10$,
$\frac{ds}{dt} = 240 m/h.$

$$2H \cdot \frac{dH}{dt} = 2(10mi) \cdot 240\ mi/h$$

5. Substitute the given values and solve for the unknown rate.

$$\frac{dH}{dt} = \frac{\left[(10mi) \cdot 240 \frac{mi}{h}\right]}{H}$$

need H.
(from above)

$$H^2 + 5^2 = S^2$$
$$H^2 = 10^2 - 5^2 = 75$$
$$H = \sqrt{75} = \sqrt{(25)(3)} = 5\sqrt{3}$$

$$\frac{dH}{dt} = \frac{\left[(10mi) \cdot 240 \frac{mi}{h}\right]}{5\sqrt{3}\ mi}$$

$$\frac{dH}{dt} = \frac{\left[(10mi) \cdot 240 \frac{mi}{h}\right]}{5\sqrt{3}\ mi} = \frac{480}{\sqrt{3}} \frac{mi}{h} = 277.1 \frac{mi}{h}$$

6. Check:
Did you include units? Yes
Does the answer make sense? Yes. dH/dt is directly proportional to S and inversely prop. to H, so it should be larger than dS/dt.
If there were angles involved, did you use radians? No angles $\frac{dH}{dt} = \frac{S}{H}\frac{dS}{dt} > \frac{dS}{dt}$ since $S > H$ and $\frac{S}{H} > 1$

Class Notes: Prof. G. Battaly, Westchester Community College, NY
Calculus Home Page
Homework
Extra Problems

## 4.1 Related Rates

2. Find $\frac{dx}{dt}$ at $x = -2$ and $y = 2x^2 + 1$ if $\frac{dy}{dt} = -1$.

Class Notes: Prof. G. Battaly, Westchester Community College, NY
Calculus Home Page
Homework
Extra Problems

## 4.1 Related Rates

2. Find $\frac{dx}{dt}$ at $x = -2$ and $y = 2x^2 + 1$ if $\frac{dy}{dt} = -1$.

$$\frac{dy}{dt} = 4x\frac{dx}{dt}$$

$$-1 = 4(-2)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{-1}{-8} = \frac{1}{8}$$

Calculus Home Page

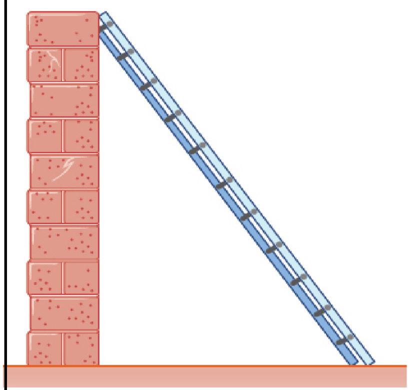
Class Notes: Prof. G. Battaly, Westchester Community College, NY

Homework

Extra Problems

## 4.1 Related Rates

6. A 25-ft ladder is leaning against a wall. If we push the ladder toward the wall at a rate of 1 ft/sec, and the bottom of the ladder is initially 20 ft away from the wall, how fast does the ladder move up the wall 5 sec after we start pushing?



Calculus Home Page

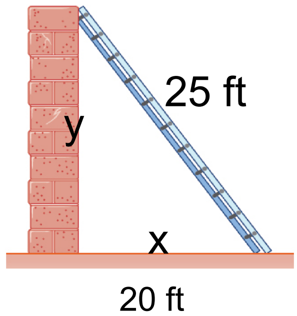
Class Notes: Prof. G. Battaly, Westchester Community College, NY

Homework

Extra Problems

## 4.1 Related Rates

6. A 25-ft ladder is leaning against a wall. If we push the ladder toward the wall at a rate of 1 ft/sec, and the bottom of the ladder is initially 20 ft away from the wall, how fast does the ladder move up the wall 5 sec after we start pushing?




G: 25 ft ladder, $dx/dt=-1$ ft/s, $x=20$ft when $t=0$sec

F: $dy/dt$ when $t=5$ sec

$$z^2 = x^2 + y^2 = 25^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

$$15(-1) + 20\frac{dy}{dt} = 0$$

$dx/dt=-1$ft/s, for $t=5$s,

$x=20-5 = 15$ ft

then $25^2=15^2 + y^2$

and $y^2=400$ and $y=20$

$$\frac{dy}{dt} = 15/20 = 3/4 \text{ ft/sec}$$



## 4.1 Related Rates

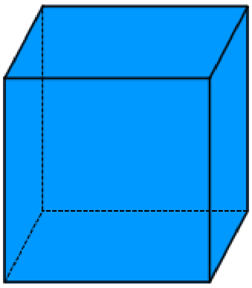
16. The side of a cube increases at a rate of $\frac{1}{2}$ m/sec. Find the rate at which the volume of the cube increases when the side of the cube is 4 m.

## 4.1 Related Rates

16. The side of a cube increases at a rate of $\frac{1}{2}$ m/sec. Find the rate at which the volume of the cube increases when the side of the cube is 4 m.

G: cube, side incr 1/2 m/s
F: rate of V change, side = 4m

G: dx/dt = 1/2 m/s
F: dV/dt when x = 4m




$$V = x^3$$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

$$\frac{dV}{dt} = 3(4^2)\left(\frac{1}{2}\right) = 24 \text{ m}^3\text{/sec}$$

Calculus Home Page

Homework
Extra Problems

## 4.1 Related Rates

G: $y = 4(x^2 - 5x)$ $\frac{dy}{dt} = 2$

a) F: $\left.\frac{dy}{dt}\right|_{x=3}$

Calculus Home Page
Class Notes: Prof. G. Battaly, Westchester Community College, NY
Homework
Extra Problems

## 4.1 Related Rates

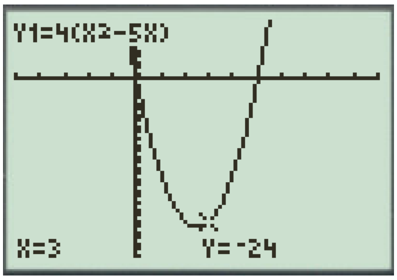
$$y = 4(x^2 - 5x)$$

F: $\left.\dfrac{dy}{dt}\right|_{x=3}$

G: $\dfrac{dx}{dt} = 2$

$$\boxed{y = 4x^2 - 20x}$$

$$\frac{dy}{dt} = 8x\frac{dx}{dt} - 20\frac{dx}{dt}$$




$$\left.\frac{dy}{dt}\right|_{x=3,\ dx/dt=2} = 8(3)(2) - 20(2) = 48 - 40 = 8$$

Calculus Home Page

Class Notes: Prof. G. Battaly, Westchester Community College, NY

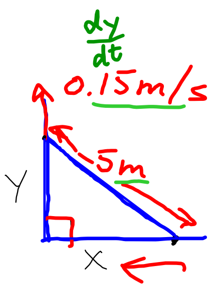
Homework

Extra Problems

## 4.1 Related Rates

G: Using a rope, a worker pulls a 5 m plank up the side of a building at a rate of 0.15 m/s.

F: How fast sliding on ground when 2.5 m from building? F: $\left.\dfrac{dx}{dt}\right|_{x=2.5}$




$x, y,$ F: $\dfrac{dx}{dt}$, G: $\dfrac{dy}{dt}$

1. Identify: a "given" rate, a "to find" rate, other variables.
2. Determine the relationship (equation) between the 'given' and the 'to find'. Use a diagram, if possible, or known formula.
3. If possible, use the given information to reduce the number of variables. (Not needed here.)
4. Differentiate implicitly with respect to time. This always involves the **chain rule**.
5. Substitute the given values and solve for the unknown rate.

6. Check:
Did you include units? Yes
Does the answer make sense? Yes. x is decreasing
If there were angles involved, did you use radians? No angles

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

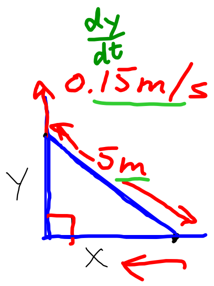
Homework

Extra Problems

## 4.1 Related Rates

F: How fast sliding on ground when 2.5 m from building?

F: $\frac{dx}{dt}$ | $x = 2.5$




F: G:

$x, y, \frac{dx}{dt}, \frac{dy}{dt}$

1. Identify: a "given" rate, a "to find" rate, other variables.

$$x^2 + y^2 = 5^2$$

2. Determine the relationship (equation) between the 'given' and the 'to find'. Use a diagram, if possible, or known formula.

3. If possible, use the given information to reduce the number of variables. (Not needed here.)

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

4. Differentiate implicitly with respect to time. This always involves the **chain rule**.

$$x \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt} = 0$$

need y:
(from above)

$$2.5^2 + y^2 = 5^2$$

$$y^2 = 5^2 - 2.5^2 = 25 - 6.25$$

$$y = \sqrt{25 - 6.25} = 4.33$$

$$2.5m \cdot \frac{dx}{dt} + y \cdot (0.15\, m/s) = 0$$

5. Substitute the given values and solve for the unknown rate.

$$\frac{dx}{dt} = -y \cdot \frac{\left(0.15 \frac{m}{s}\right)}{2.5m}$$

$$\frac{dx}{dt} = -(4.33m) \cdot \frac{\left(0.15 \frac{m}{s}\right)}{2.5m} = -0.26\, m/s$$

6. Check:
Did you include units? Yes
Does the answer make sense? Yes. x is decreasing
If there were angles involved, did you use radians? No angles

Calculus Home Page

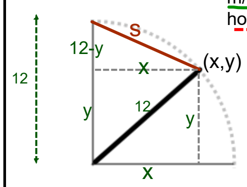
Class Notes: Prof. G. Battaly, Westchester Community College, NY

Homework

Extra Problems

## 4.1 Related Rates

A winch at the top of a 12-m building pulls a 12 m pipe to a vertical position. The winch pulls in the rope at a rate of -0.2 m/sec. Find the rate of vertical change and the rate of horizontal change at the end of the pipe when y = 6.




1. Identify: a "given" rate, a "to find" rate, other variables.

2. Determine the relationship (equation) between the 'given' and the 'to find'. Use a diagram, if possible, or known formula.

3. If possible, use the given information to **reduce the number of variables**.

4. Differentiate implicitly with respect to time. This always involves the **chain rule**.

5. Substitute the given values and solve for the unknown rate.

6. Check:
Did you include units? Yes
Does the answer make sense? Yes.
x decreasing, y increasing; so dx/dt <0 and dy/dt >0
If there were angles involved, did you use radians? No

Calculus Home Page

Class Notes: Prof. G. Battaly, Westchester Community College, NY

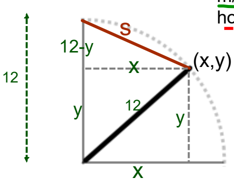
Homework

Extra Problems

## 4.1 Related Rates

A winch at the top of a 12-m building pulls a 12 m pipe to a vertical position. The winch pulls in the rope at a rate of -0.2 m/sec. Find the rate of vertical change and the rate of horizontal change at the end of the pipe when y = 6.

$\frac{ds}{dt}$




$s, x, y,$ F: $\left.\frac{dx}{dt}\right|_{y=6}$ $\left.\frac{dy}{dt}\right|_{y=6}$

1. Identify: a "given" rate, a "to find" rate, other variables.

bottom right triangle:

$$x^2 + y^2 = 12^2$$

top left triangle:

$$x^2 + (12-y)^2 = s^2$$

2. Determine the relationship (equation) between the 'given' and the 'to find'. Use a diagram, if possible, or known formula.

solve bottom right for $x^2$ and substitute in top left to get just 2 variables:

$$12^2 - y^2 + (12-y)^2 = s^2$$

simplify: $12^2 - y^2 + 12^2 - 24y + y^2 = s^2$

$$2(12^2) - 24y = s^2$$

3. If possible, use the given information to reduce the number of variables.

$$-24\frac{dy}{dt} = 2s\frac{ds}{dt}$$

4. Differentiate implicitly with respect to time. This always involves the **chain rule**.

$$-24\frac{dy}{dt} = 2s(-0.2)\frac{m}{s} = -0.4s\ mps$$

5. Substitute the given values and solve for the unknown rate.

need s: (from above)

$s^2 = 2(12^2) - 24(6) = 144$

s=12

$$\frac{dy}{dt} = \frac{-0.4s}{-24}\ mps = \frac{s}{60}\ mps$$

$$\frac{dy}{dt} = \frac{12}{60}\ mps = \frac{1}{5}\ mps$$

Still need *dx/dt* From equation for bottom right triangle:

$$2x\cdot\frac{dx}{dt} + 2y\cdot\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{2y}{2x}\frac{dy}{dt} = -\frac{y}{x}\frac{dy}{dt} = -\frac{6}{x}\frac{1}{5}\ mps$$

$x^2 + 6^2 = 12^2$
$x^2 = 144 - 36$
$x^2 = 108 = 36(3)$
$x = 6\sqrt{3}$

Still need *x* Use bottom right equation above to get $x = 6\sqrt{3}$

$$\frac{dx}{dt} = -\frac{6}{6\sqrt{3}}\frac{1}{5} = -\frac{1}{5\sqrt{3}}\ mps$$

6. Check:
Did you include units? Yes
Does the answer make sense? Yes.
x decreasing, y increasing; so dx/dt <0 and dy/dt >0
If there were angles involved, did you use radians? No angles

Calculus Home Page
Class Notes: Prof. G. Battaly, Westchester Community College, NY
Homework
Extra Problems

## 4.1 Related Rates

$G: y = 2(x^2 - 3x)$

$F:$ a) $\left.\frac{dy}{dt}\right|_{x=3},$

b) $\left.\frac{dx}{dt}\right|_{x=1};\ \frac{dy}{dt} = 5$

Calculus Home Page
Class Notes: Prof. G. Battaly, Westchester Community College, NY
Homework
Extra Problems

## 4.1 Related Rates

G: $y = 2(x^2 - 3x)$ F: a) $\left.\frac{dy}{dt}\right|_{x=3}$

b) $\left.\frac{dx}{dt}\right|_{x=1}$ ; $\frac{dy}{dt} = 5$

$$\frac{dy}{dt} = 2\left[2x\frac{dx}{dt} - 3\frac{dx}{dt}\right] = 4x\frac{dx}{dt} - 6\frac{dx}{dt}$$

$$\frac{dy}{dt} = (4x-6)\frac{dx}{dt} = 6\frac{dx}{dt}$$

b) $$\frac{dx}{dt} = \frac{dy/dt}{4x-6} = \frac{5}{4-6} = \frac{5}{-2}$$

Calculus Home Page Class Notes: Prof. G. Battaly, Westchester Community College, NY Homework Extra Problems

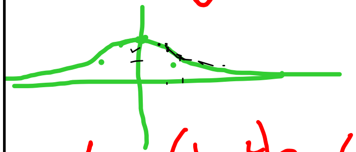
## 4.1 Related Rates

G: $y = \frac{1}{1+x^2}$ , $\frac{dx}{dt} = 2$ cm/s. F: $\left.\frac{dy}{dt}\right|_{x=-2}$

Calculus Home Page Class Notes: Prof. G. Battaly, Westchester Community College, NY Homework Extra Problems

## 4.1 Related Rates

G: $y = \frac{1}{1+x^2}$, $\frac{dx}{dt} = 2$ cm/s. F: $\boxed{\frac{dy}{dt}}\Big|_{x=-2}$



$$\frac{dy}{dt} = \frac{(1+x^2)0 - (2x\frac{dx}{dt})}{(1+x^2)^2}$$

$$\frac{DdN - NdD}{D^2}$$

$$= \frac{-2x}{(1+x^2)^2}\frac{dx}{dt}$$

$$\frac{dy}{dt}\Big|_{x=-2} = \frac{-2(-2)}{(1+4)^2} \cdot 2 \text{ cm/s.} = \frac{8}{25} \text{ cm/s.}$$

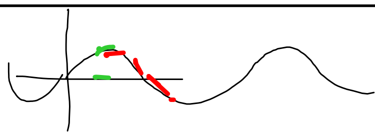
Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework

Extra Problems

## 4.1 Related Rates



G: $y = \sin x$ $\frac{dx}{dt} = 2$ cm/sec. F: $\frac{dy}{dt}\Big|_{x=\frac{\pi}{6}}$

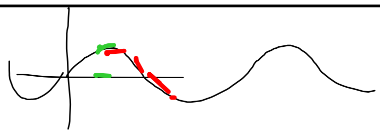
Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework

Extra Problems

## 4.1 Related Rates



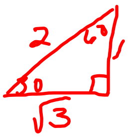
G: $y = \sin x$ $\quad \frac{dx}{dt} = 2$ cm/sec.

F: $\left.\frac{dy}{dt}\right|_{x=\frac{\pi}{6}}$

$y = \sin x$

$\frac{dy}{dt} = (\cos x)\frac{dx}{dt}$

$\left.\frac{dy}{dt}\right|_{x=\frac{\pi}{6}} = \left(\cos\frac{\pi}{6}\right) 2\ \text{cm/s.}$



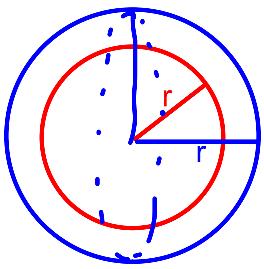

$\sin 30 = \frac{1}{2}$

$= \frac{\sqrt{3}}{2} \cdot 2\ \text{cm/s} = \sqrt{3}\ \text{cm/s.}$

$\frac{dy}{dt} = 2\cos x$ cm/s

Calculus Home Page Class Notes: Prof. G. Battaly, Westchester Community College, NY Homework Extra Problems

## 4.1 Related Rates



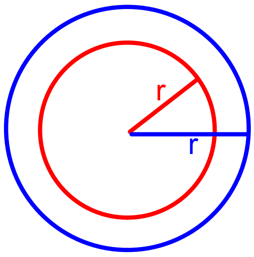

16. G: radius of sphere increases 3 in/min $= \frac{dr}{dt}$

F: rate of change of V when r =9 in, r =36 in

$\left.\frac{dV}{dt}\right|_{r=9\text{in},\ r=36\text{in.}}$

Calculus Home Page Class Notes: Prof. G. Battaly, Westchester Community College, NY Homework Extra Problems

## 4.1 Related Rates




16. G: radius of sphere increases 3 in/min

$\frac{dr}{dt}$

F: rate of change of V when r =9 in, r =36 in

$\frac{dV}{dt}$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\left(3\pi r^2\right)\frac{dr}{dt} = 4\pi r^2(3) \text{ in}^3/\text{min}.$$

$$= 12\pi r^2 \text{ in}^3/\text{min}.$$

$$\left.\frac{dV}{dt}\right|_{r=9\text{ in}} = 12\pi(9\text{ in})^2 \text{ in}/\text{in} = 81(12)\pi \text{ in}^3/\text{min}.$$

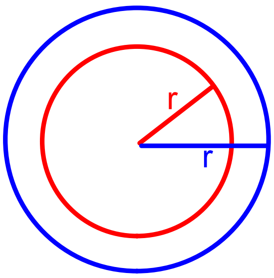
Calculus Home Page

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Homework

Extra Problems

## 4.1 Related Rates




G: *dr/dt* is constant

F: Is dA/dt constant? Why?

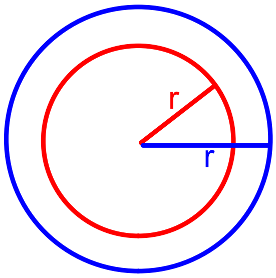
Calculus Home Page

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Homework

Extra Problems

## 4.1 Related Rates




G: *dr/dt* is constant

F: Is dA/dt constant? Why?

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi k \underline{r}$$

Therefore, ***dA/dt*** is NOT constant.
It varies directly as ***r*** varies.

Homework

Calculus Home Page

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Extra Problems

## 4.1 Related Rates

G: $xy = 4$

F: $\left.\underline{\frac{dy}{dt}}\right|_{x=8}$ and $\frac{dx}{dt} = 10$

Homework

Calculus Home Page

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Extra Problems

## 4.1 Related Rates

G: $xy = 4$

F: $\left.\frac{dy}{dt}\right|_{x=8}$ and $\frac{dx}{dt} = 10$

$$x\frac{dy}{dt} + y\frac{dx}{dt} = 0$$

$$8\frac{dy}{dt} + y(10) = 0$$

$$xy = 4$$
$$y = \frac{4}{x} = \frac{4}{8}$$
$$y = 1/2$$

$$8\frac{dy}{dt} + \frac{1}{2}\cdot 10 = 0$$

$$8\frac{dy}{dt} = -5$$

$$\frac{dy}{dt} = -5/8$$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework

Extra Problems

## 4.1 Related Rates

Use Implicit Differentiation

| $y = 2x + x^3 - z$ | Differentiate with respect to: |
|---|---|
| | |
| | x |
| | |

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework

Extra Problems

4.1 Related Rates Use Implicit Differentiation

| $y = 2x + x^3 - z$ | Differentiate with respect to: |
|---|---|
| $\frac{dy}{dt} = 2\frac{dx}{dt} + 3x^2\frac{dx}{dt} - \frac{dz}{dt}$ | t |
| $\frac{dy}{dx} = 2 + 3x^2 - \frac{dz}{dx}$ | x |
| $\frac{dy}{dz} = 2\frac{dx}{dz} + 3x^2\frac{dx}{dz} - 1$ | z |
| $1 = 2\frac{dx}{dy} + 3x^2\frac{dx}{dy} - \frac{dz}{dy}$ | y |

Calculus Home Page

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Homework
Extra Problems