4.10 Antiderivatives: Basic Concepts

Goals:

1. Understand that antiderivatives are the functions from which the present derivative was found.
2. The process of finding an antiderivative or indefinite integral requires the reverse process of finding a derivative.
3. Understand that solving a differential equation means to find the antiderivative of the function.

Study 4.10 #465, 471, 475-483, 487, 491-499, 503-511, 515, 517, 521, 523

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4.10 Antiderivatives: Basic Concepts

$$Find\ \frac{dy}{dx}:\ y = x^5 - 2x^3$$

1. multiply coefficient by exponent
2. subtract 1 from the exponent

$\frac{dy}{dx} = 5x^4 - 6x^2$

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4.10 Antiderivatives: Basic Concepts

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4.10 Antiderivatives: Basic Concepts

Suppose you are given:

$$f'(x) = 5x^4$$

How would you find *f(x)*?

Inverse of finding the derivative:
1. multiply coefficient by exponent
2. subtract 1 from the exponent

Therefore:
1. Add 1 to exponent
2. Divide by new exponent.

Is there only one *f(x)*?

$$\frac{5x^{4+1}}{4+1} = x^5 + C$$

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4.10 Antiderivatives: Basic Concepts

Suppose you are given:

$\int f'(x) = \int 5x^4 \, dx$

How would you find *f(x)*?

$= \frac{5x^{4+1}}{4+1} + C$

$= \frac{5x^5}{5} + C = x^5 + C$

Is there only one *f(x)*?

$\frac{5x^{4+1}}{4+1} = x^5 + C$

Inverse of finding the derivative:
1. multiply coefficient by exponent
2. subtract 1 from the exponent

Therefore:
1. Add 1 to exponent
2. Divide by new exponent.

4.10 Antiderivatives: Basic Concepts

Definition, **Antiderivative**:

A function *F* is an **antiderivative** of *f* on an interval **I** if ***F ' (x) = f(x)*** for all x in **I**.

A **Differential Equation** in x and y is an equation that involves x, y, and the derivative of y.

eg: $\frac{dy}{dx} = 5x^4 - 6x^2$

4.10 Antiderivatives: Basic Concepts

So far, we have **created** differential equations by finding derivatives. Now, we reverse the process.
start with the definition of the differential of y

Definition of dy:

Let $y = f(x)$ represent a function that is differentiable on an open interval containing x.
The differential of x (dx) is any nonzero real number.
The differential of y (dy) is:

$$dy = f'(x)\,dx \qquad dy = \frac{dy}{dx}\,dx$$

Proceed to **solve a differential equation** to find y.

4.10 Antiderivatives: Basic Concepts

Antidifferentiation or **Indefinite Integration**

The **operation of finding all the solutions** to the differential equation, $dy = f(x)\,dx$, is called antidifferentiation or indefinite integration and is denoted by the integral sign $\int$. The solution when $F'(x) = f(x)$ is:

$$\boxed{y = \int \underbrace{f(x)}_{\text{integrand}}\, d\underbrace{x}_{\substack{\text{variable}\\ \text{of integration}}} = F(x) + \underbrace{c}_{\substack{\text{constant}\\ \text{of integration}}}}$$

[looking for the function, F(x), for which the integrand is the derivative, F'(x) = f(x)]

4.10 Antiderivatives: Basic Concepts

Solve the differential equation:

$$\frac{dy}{dx} = 5x^4$$

$\int$

1.	$dy = \frac{dy}{dx}\, dx$	1. Write definition of differential.
2,3	$dy = \qquad dx$	2. Substitute given function as the derivative. 3. Insert integral signs (function is now the integrand)
4	$y =$	4. Integrate. Note: this has NO integral sign and NO differentials
5	$y =$	5. Simplify.

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4.10 Antiderivatives: Basic Concepts

Solve the differential equation:

$$\frac{dy}{dx} = 5x^4$$

$\int$

1.	$dy = \frac{dy}{dx}\, dx$	1. Write definition of differential.
2,3	$\int dy = \int 5x^4\, dx$	2. Substitute given function as the derivative. 3. Insert integral signs (function is now the integrand)
4	$y = \frac{5x^5}{5} + C$	4. Integrate. Note: this has NO integral sign and NO differentials
5	$y = x^5 + C$	5. Simplify.

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4.10 Antiderivatives: Basic Concepts

$$\boxed{y = \int \underbrace{f(x)}_{\text{integrand}}\underbrace{dx}_{\text{variable of integration}} = F(x) + \underbrace{c}_{\text{constant of integration}}}$$

Note: **Operators $\int$ and dx are no longer present after the operation of integration is performed.**

The integral sign $\int$ and dx indicate the operation of integration the same way that a plus sign indicates the operation of addition.

For the division problem, $12 \div 2 = 6$, the result no longer has the operator, $\div$ Instead, it contains only the result, 6.

Likewise, for integration, the result no longer has either the integral sign $\int$ or the dx. Therefore, to continue **to write the $\int$ or the dx** after the operation of integration has been performed **is not correct**.

4.10 Antiderivatives: Basic Concepts

Rules of:

Differentiation	Integration
$\frac{d(c)}{dx} = 0$	$\int 0\, dx = c$
$\frac{d(kx)}{dx} = k$	$\int k\, dx = kx + c$
$\frac{d(x^n)}{dx} = nx^{n-1}$	$\boxed{\int x^n dx = \frac{x^{n+1}}{n+1} + c}$
$\frac{d(\sin x)}{dx} = \cos x$	$\int \cos x dx = \sin x + c$
$\frac{d(\cos x)}{dx} = -\sin x$	$\int \sin x dx = -\cos x + c$

$$-\int -\sin x dx = -\cos x + c$$

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$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

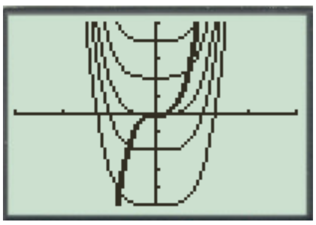
$$\int 8x^3\, dx$$

4.10 Antiderivatives: Basic Concepts

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int 8x^3\, dx = 8\frac{x^4}{4} + C$$

$$= 2x^4 + C$$



check: $\frac{d(2x^4 + c)}{dx}$

$= 8x^3 + 0$

4.10 Antiderivatives: Basic Concepts

$$\int \left(8x^3 + \frac{1}{2x^2}\right) dx$$

4.10 Antiderivatives: Basic Concepts

$$\int \left(8x^3 + \frac{1}{2x^2}\right) dx = \int \left(8x^3 + \frac{1}{2}x^{-2}\right) dx$$

$$= \frac{8x^4}{4} + \frac{1}{2}\frac{x^{-1}}{-1} + C$$

$$= 2x^4 - \frac{1}{2x} + C$$

$$= 2x^4 - \frac{1}{2x} + C$$

check: $\frac{d\left(2x^4 - \frac{1}{2}x^{-1} + C\right)}{dx}$

$$= 8x^3 + \frac{1}{2}x^{-2} + 0$$

$$= 8x^3 + \frac{1}{2x^2} \checkmark$$

4.10 Antiderivatives: Basic Concepts

$$\int \frac{1}{4x^2}\,dx$$

$$\int \frac{1}{x^3}\,dx$$

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4.10 Antiderivatives: Basic Concepts

$$\int \frac{1}{4x^2}\,dx = \frac{1}{4}\int x^{-2}\,dx$$

$$= \frac{1}{4}\,\frac{x^{-1}}{-1} + c = -\frac{1}{4x} + c$$

$$\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = \frac{x^{-2}}{-2} + c$$

$$= -\frac{1}{2x^2} + c$$

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4.10 Antiderivatives: Basic Concepts

$$\int (8x^3 - 9x^2 + 4)\,dx$$

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4.10 Antiderivatives: Basic Concepts

$$\int (8x^3 - 9x^2 + 4)\,dx$$

$$= \frac{8x^4}{4} - \frac{9x^3}{3} + 4x + C$$

$$2x^4 - 3x^3 + 4x + C$$

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4.10 Antiderivatives: Basic Concepts

$$\int \cos x\,dx =$$

$$\int \sin x\,dx =$$

$$\int \sec^2 x\,dx =$$

$$\int \sec x \tan x\,dx =$$

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4.10 Antiderivatives: Basic Concepts

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

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4.10 Antiderivatives: Basic Concepts

$$\int \frac{\sin x}{1-\sin^2 x}\,dx$$

4.10 Antiderivatives: Basic Concepts

$$\int \frac{\sin x}{1-\sin^2 x}\,dx = \int \frac{\sin x}{\cos^2 x}\,dx$$

$$= \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\,dx$$

$$= \int \tan x \sec x\,dx = \int \sec x \tan x\,dx$$

$$= \sec x + C$$

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4.10 Antiderivatives: Basic Concepts

G: $f''(x) = x^2$, $f'(0) = 8$, $f(0) = 4$

F: Solve the diff. eq. F: $f(x)$

1. Start with 2nd Derivative. Integrate
2. Substitute given to find c.
3. State 1st Derivative
4. Integrate 1st Derivative
5. Substitute given to find c_2.
6. State function.

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4.10 Antiderivatives: Basic Concepts

G: $f''(x) = x^2$, $f'(0) = 8$, $f(0) = 4$

F: Solve the diff. eq. F: $f(x)$

1. Start with f '' (x). Integrate

$$f'(x) = \int f''(x)\,dx = \int x^2\,dx = \frac{x^3}{3} + c$$

$$f'(x) = \frac{1}{3}x^3 + c$$

3. State f ' (x).

2. Substitute given to find c.

$$f'(0) = 0 + c = 8 \quad \therefore c = 8$$

$$\boxed{f'(x) = \frac{1}{3}x^3 + 8}$$

half way there

4. Integrate f ' (x).

$$f(x) = \int f'(x)\,dx = \int \left(\frac{1}{3}x^3 + 8\right) dx$$

$$= \frac{1}{3}\frac{x^4}{4} + 8x + c_2 = \frac{1}{12}x^4 + 8x + c_2$$

5. Substitute given to find c_2.

$$f(0) = \frac{1}{12}\cdot 0 + 8(0) + c_2 = 4 \quad \therefore c_2 = 4$$

6. State f (x).

$$f(x) = \frac{1}{12}x^4 + 8x + 4$$

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4.10 Antiderivatives: Basic Concepts

Find: Indefinite Integral & Check by Differentiation

$$\int \frac{x+6}{\sqrt{x}}\,dx$$

4.10 Antiderivatives: Basic Concepts

Find: Indefinite Integral & Check by Differentiation

$$\int \frac{x+6}{\sqrt{x}}\,dx = \int \frac{x+6}{x^{1/2}}\,dx = \int \left(\frac{x^1}{x^{1/2}} + \frac{6}{x^{1/2}}\right)dx$$

$$= \int \left(x^{1/2} + 6x^{-1/2}\right)dx$$

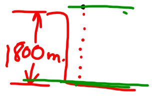
$$= \frac{x^{3/2}}{3/2} + \frac{6x^{1/2}}{1/2} + c = \frac{2}{3}x^{3/2} + 12x^{1/2} + c$$

check: $\dfrac{d(\qquad\qquad)}{dx} = x^{1/2} + 6x^{-1/2}$

4.10 Antiderivatives: Basic Concepts

$G: a(t) = -9.8\,m/s^2 \quad F: a)\ h(t)$

b) How long to hit bottom?
ie: find t when h = 0



$v(t) = h'(t)$

$a(t) = v'(t) = h''(t)$

$a(t) = -9.8\,m/s^2$

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1. Start with 2nd Derivative. Integrate
2. Substitute given to find c.
3. State 1st Derivative
4. Integrate 1st Derivative
5. Substitute given to find c_2.
6. State function.

b) How long to hit bottom?   At bottom when h(t) = 0

4.10 Antiderivatives: Basic Concepts

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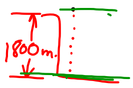

$v(t) = h'(t)$

$a(t) = v'(t) = h''(t)$

$a(t) = -9.8\,m/s^2$

$v(t) = \int a(t)\,dt = \int -9.8\,dt$
$= -9.8t + c_1$

1. Start with h "(t) = a(t). Integrate

2. Substitute given to find c.

$t=0,\ v(0) = -9.8(0) + c_1 \quad \therefore c_1 = v_0 = 0$

$\therefore v(t) = -9.8t\ m/s$

3. State h ' (t) = v(t)

$h(t) = \int v(t)\,dt$

4. Integrate h '(t)=v(t)

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$h(t) = \int -9.8t\,dt$

$h(t) = -\frac{9.8t^2}{2} + c_2 = -4.9t^2 + c_2$

5. Substitute given to find c_2.

$t=0: \ h(0) = -4.9(0) + c_2$

$c_2 = h_0 = 1800m.$

6. State h(t).

$\boxed{h(t) = -4.9t^2 + 1800 \ m.}$

b) How long to hit bottom?   At bottom when h(t) = 0

$-4.9t^2 + 1800 = 0$

$+4.9t^2 = +1800$

$t^2 = \frac{1800}{4.9}$

$t = \pm 10\sqrt{\frac{18}{4.9}} = \boxed{19.17sec}$

negative meaningless

4.10 Antiderivatives: Basic Concepts

Think Questions: True or False?

1. The antiderivative of $f(x)$ is unique.

2. Each antiderivative of an nth-degree polnomial function is an $(n+1)$th degree polynoial function.

3. If $p(x)$ is a polynomial function,then p has exactly one antiderivative whose graph contains the origin.

4. If $F(x)$ and $G(x)$ are antiderivatives of $f(x)$, then

$F(x) = G(x) + c$

5. If $f'(x) = g(x)$, then $\int g(x)\,dx = f(x) + c$

6. $\int f(x)g(x)dx = \int f(x)dx \int g(x)dx$

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4.10 Antiderivatives: Basic Concepts

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T 5. If $f'(x) = g(x)$, then $\int g(x)\,dx = f(x) + c$

F 6. $\int f(x)g(x)dx = \int f(x)dx \int g(x)dx$

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4.10 Antiderivatives: Basic Concepts

Find a function f such that the graph of f has a horizontal tangent at (2,0) and f "(x) = 2x

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