3.9 Exponential & Logarithmic Functions: Derivatives

GOALS:
1. Recognize exponential and logarithmic functions and how they relate to each other.
2. Use rules of exponents and rules of logarithms as needed to solve equations or find derivatives
3. Solve equations for variables that occur in the exponent
4. Find derivatives of exponential functions
5. Find derivatives of logarithmic functions

Homework Video: Inverse Functions

Study 3.9 # 331-335 all, 338, 341, 347, 354, 357
not posted yet

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3.9 Exponential & Logarithmic Functions: Derivatives

Exponential		Logarithmic
$10^2 = 100$	⟺	$\log_{10}100 = 2$ $\log 100 = 2$
$2^4 = 16$	⟺	$\log_2 16 = 4$
$5^3 = 125$	⟺	$\log_5 125 = 3$
$e^x = y$	⟺	$\log_e y = x$ $\ln y = x$

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3.9 Exponential & Logarithmic Functions: Derivatives

Natural Exponential Function

The LOG is the EXPONENT

IFF

$y = e^x \longleftrightarrow x = \ln y$

To convert from exponential to logarithmic

Take log of each side and simplify

$\ln y = \ln e^x$

$\ln y = x \ln e$

$\ln y = x$

Direct translation

The LOG is the EXPONENT

$\ln \quad = \quad x$

The LOG of WHAT ?

$\ln y = \quad x$

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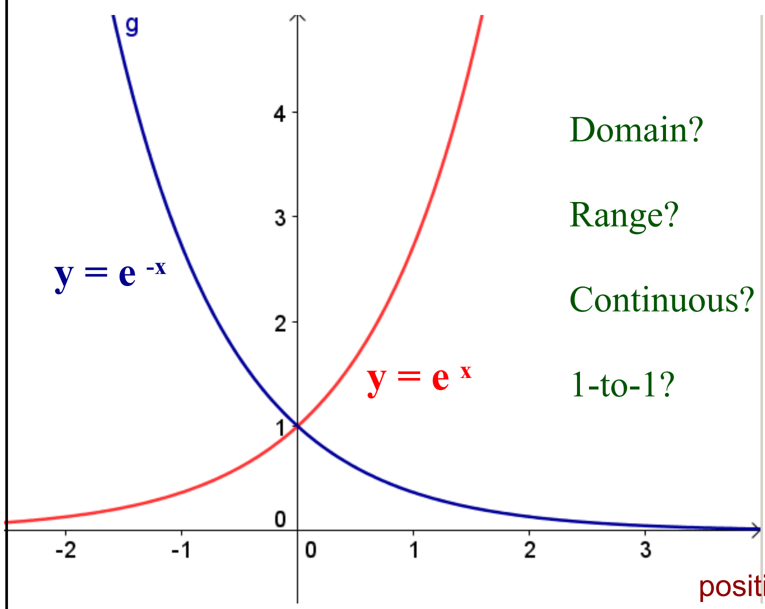
3.9 Exponential & Logarithmic Functions: Derivatives

Properties of the Natural Exponential Function



Domain?

Range?

Continuous?

1-to-1?

$y = e^{x}$ $\qquad$ $y = e^{-x}$

positive exponent: increasing
negative exponent: decreasing

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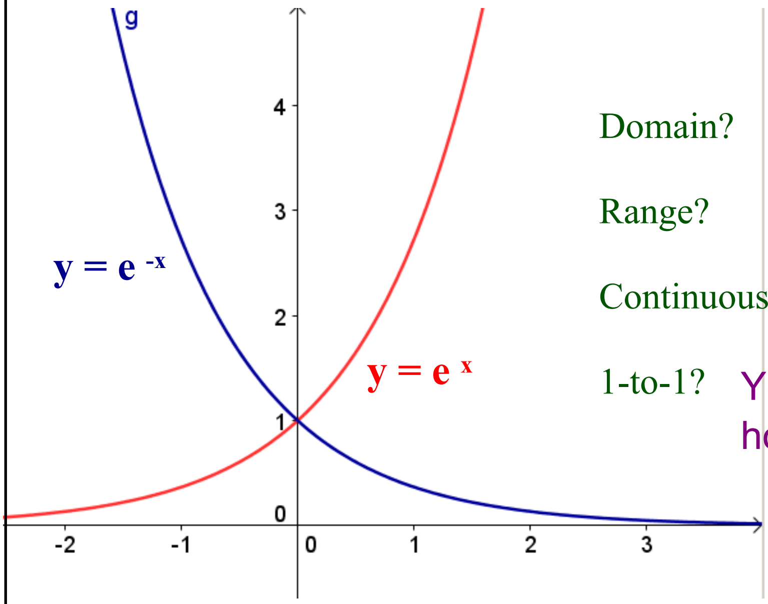
3.9 Exponential & Logarithmic Functions: Derivatives

Properties of the Natural Exponential Function

ln y = x



	$y = e^{x}$	$y = e^{-x}$
Domain?	All real numbers	
Range?	y > 0	
Continuous?	Yes	
1-to-1?	Yes, passes vertical and horizontal line tests	

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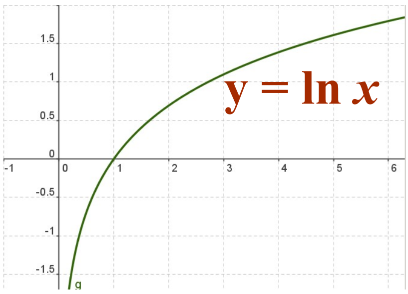
3.9 Exponential & Logarithmic Functions: Derivatives

Properties of $y = \ln x$

1. Domain: $x > 0$ Range: all y

2. continuous, increasing over all domain

3. concave down



Inverse Functions

At www.geogebra.org/classic copy the url into File/Open:
http://www.battaly.com/calc/geogebra/inverse_lnx/inverse_lnx.ggb

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3.9 Exponential & Logarithmic Functions: Derivatives

Properties of Exponents	Properties of Logarithms
1. $e^0 = 1$ or $x^0 = 1$, $x \neq 0$	1. $\ln(1) = 0$
2. $e^1 = e$ or $x^1 = x$	2. $\ln(e) = 1$
3. $x^a x^b = x^{a+b}$ ⟷	3. $\ln(a\,b) = \ln(a) + \ln(b)$
4. $\frac{x^a}{x^b} = x^{a-b}$ ⟷	4. $\ln \frac{a}{b} = \ln(a) - \ln(b)$
5. $(x^a)^n = x^{an}$ ⟷	5. $\ln a^n = n \ln(a)$

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3.9 Exponential & Logarithmic Functions: Derivatives

skip - algebra topic

Properties of Exponents: Property # 3

let $e^x = A$ then $\ln A = x$

and $e^y = B$ and $\ln B = y$

$$e^x e^y = e^{x+y} = AB$$

$$\ln(AB) = x + y$$

$$\ln(AB) = \ln A + \ln B$$

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3.9 Exponential & Logarithmic Functions: Derivatives

Derivatives

Exponential

$$\frac{d\,(e^x)}{dx} = e^x$$

Logarithmic

$$\frac{d\,(\ln x)}{dx} = \frac{1}{x}$$

For $u = f(x)$

$$\frac{d\,(e^u)}{dx} = e^u \frac{du}{dx}$$

$$\frac{d\,(\ln u)}{dx} = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u}$$

$$\frac{d\,(\ln x)}{dx} = \frac{d}{dx}\left[\int_1^x \frac{1}{t}\,dt\right] = \frac{1}{x}$$

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Homework Part 1

Homework Part 2

3.9 Exponential & Logarithmic Functions: Derivatives

Practice Properties First ...

$\ln(xyz)$

$\ln\sqrt{xy}$

$$y = \ln\sqrt{\frac{x+1}{x-1}}$$

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3.9 Exponential & Logarithmic Functions: Derivatives

Practice Properties First ...

ln (xyz) = ln x + ln y + ln z

ln √xy = ½ ln (xy) = ½ [ln x + ln y]

$$y = \ln\sqrt{\frac{x+1}{x-1}} = \ln\left(\frac{x+1}{x-1}\right)^{1/2} = \frac{1}{2}\ln\frac{x+1}{x-1} = \frac{1}{2}[\ln(x+1) - \ln(x-1)]$$

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3.9 Exponential & Logarithmic Functions: Derivatives

G: 3 ln x + 2 ln y - 4 ln z

Write as the natural log of a single expression.

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $3 \ln x + 2 \ln y - 4 \ln z$

Write as the natural log of a single expression.

$\ln x^3 + \ln y^2 - \ln z^4$

$\ln \frac{x^3 y^2}{z^4}$

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3.9 Exponential & Logarithmic Functions: Derivatives

Derivative of the Natural Logarithm

$$\frac{d(\ln x)}{dx} = \frac{1}{x}$$

For $u = f(x)$

$$\frac{d(\ln u)}{dx} = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u}$$

$$\frac{d(\ln x)}{dx} = \frac{d}{dx}\left[\int_1^x \frac{1}{t}\,dt\right] = \frac{1}{x}$$

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $y = \ln x$ F: dy/dx

$$\frac{d(\ln u)}{dx} = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d(\ln u)}{dx} = \frac{u'}{u}$$

G: $y = \ln 2x$ F: dy/dx

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $y = \ln x$ F: dy/dx

$$\frac{dy}{dx} = \frac{1}{x}$$

G: $y = \ln 2x$ F: dy/dx

$$\frac{dy}{dx} = \frac{1}{2x}(2) = \frac{1}{x}$$

OR:

$y = \ln 2 + \ln x$

$dy/dx = 0 + 1/x = 1/x$

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $h(x) = \ln(2x^2+1)$ F: $h'(x)$

$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}(\ln u) = \frac{u'}{u}$$

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $h(x) = \ln(2x^2+1)$ F: $h'(x)$

$$h'(x) = \frac{1}{2x^2+1}(4x)$$

$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$$

$$h'(x) = \frac{4x}{2x^2+1}$$

$$\frac{d}{dx}(\ln u) = \frac{u'}{u}$$

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $y = x \ln x$ F: dy/dx

G: $y = \ln \sqrt{x^2-4}$ F: dy/dx

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $y = x \ln x$ F: dy/dx

$$\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x = 1 + \ln x$$

G: $y = \ln \sqrt{x^2-4}$ F: dy/dx

$$y = \ln(x^2-4)^{1/2} = \frac{1}{2}\ln(x^2-4)$$

$u = x^2-4$

$\frac{du}{dx} = 2x$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{u}\frac{du}{dx}\right] = \frac{1}{2}\left[\frac{1}{x^2-4} \cdot 2x\right]$$

$$= \frac{x}{x^2-4}$$

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $y = e^x$ F: dy/dx

$\ln y = \ln e^x = x \ln e$

$\ln y = x$

$\frac{d(\ln y)}{dx} = \frac{d(x)}{dx} = 1$ implicit differentiation

$\frac{1}{y}\frac{dy}{dx} = 1$

$\frac{dy}{dx} = y = e^x$

$\frac{d(e^x)}{dx} =$?

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $y = xe^x$ F: dy/dx

$\frac{d(e^x)}{dx} = e^x$

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3.9 Exponential & Logarithmic Functions: Derivatives

$$\frac{d(e^x)}{dx} = e^x$$

G: $y = xe^x$ F: dy/dx

$$\frac{dy}{dx} = x\frac{d(e^x)}{dx} + e^x\frac{dx}{dx}$$

$$\frac{dy}{dx} = x e^x + e^x$$

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3.9 Exponential & Logarithmic Functions: Derivatives

Derivative of Exponential Function

For $u = f(x)$

$$\frac{d(e^x)}{dx} = e^x \qquad \frac{d(e^u)}{dx} = e^u\frac{du}{dx}$$

Chain Rule

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3.9 Exponential & Logarithmic Functions: Derivatives

$\frac{d(e^x)}{dx} = e^x$

G: $y = e^{x^2}$ F: dy/dx

G: $y = e^{-x^2+1}$ F: dy/dx

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3.9 Exponential & Logarithmic Functions: Derivatives

$\frac{d(e^x)}{dx} = e^x$

G: $y = e^{x^2}$ F: dy/dx

$\frac{dy}{dx} = e^{x^2}(2x) = 2xe^{x^2}$

$u = x^2$
$du/dx = 2x$

G: $y = e^{-x^2+1}$ F: dy/dx

$u = -x^2 + 1$
$du/dx = -2x$

$\frac{dy}{dx} = e^{-x^2+1}(-2x) = -2xe^{-x^2+1}$

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3.9 Exponential & Logarithmic Functions: Derivatives

$\frac{d}{dx}(e^x) = e^x$

G: $y = \ln(1+e^x)$ F: dy/dx

G: $y = e^{\ln x}$ F: dy/dx

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3.9 Exponential & Logarithmic Functions: Derivatives

$\frac{d}{dx}(e^x) = e^x$

G: $y = \ln(1+e^x)$ F: dy/dx

$$\frac{dy}{dx} = \frac{e^x}{1+e^x}$$

G: $y = e^{\ln x}$ F: dy/dx

$\ln y = \ln e^{\ln x} = \ln x \ln e = \ln x$

So, $y = x$ and $dy/dx = 1$

$$\frac{dy}{dx} = e^{\ln x}\,\frac{1}{x}$$

$$\frac{dy}{dx} = 1$$

$y = x$

$dy/dx = 1$

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3.9 Exponential & Logarithmic Functions: Derivatives

$\frac{d\,(e^x)}{dx} = e^x$

$$e^{\ln x} = x = \ln e^x$$

$f(x) = e^x$ and $g(x) = \ln x$

are inverse functions
1 - to - 1
$f(g(x)) = g(f(x)) = x$

Video: Inverse Functions

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3.9 Exponential & Logarithmic Functions: Derivatives

$\frac{d\,(e^x)}{dx} = e^x$

G: $y = ln\frac{1+e^x}{1-e^x}$ F: dy/dx

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3.9 Exponential & Logarithmic Functions: Derivatives

$\frac{d}{dx}(e^x) = e^x$

G: $y = ln\frac{1+e^x}{1-e^x}$ F: dy/dx

$y = \ln(1+e^x) - \ln(1-e^x)$

$\frac{dy}{dx} = \frac{e^x}{1+e^x} + \frac{+e^x}{1-e^x}$

$\frac{dy}{dx} = \frac{e^x(1-e^x)+e^x(1+e^x)}{(1+e^x)(1-e^x)}$

$\frac{dy}{dx} = \frac{e^x[(1-e^x)+(1+e^x)]}{(1+e^x)(1-e^x)} = \frac{2e^x}{1-e^{2x}}$

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3.9 Exponential & Logarithmic Functions: Derivatives

$\frac{d}{dx}(e^x) = e^x$

G: $-6 + 3e^x = 8$ F: solve for x

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3.9 Exponential & Logarithmic Functions: Derivatives

$\frac{d(e^x)}{dx} = e^x$

G: $-6 + 3e^x = 8$ F: solve for x

$3e^x = 14$

$e^x = 14/3$

$\ln e^x = \ln 14/3$

$x = \ln 14/3$

Use logarithms to solve equations with variable in the exponent.

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $y = x^x$ F: dy/dx

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $y = x^x$

F: dy/dx

$\ln y = \ln x^x = x \ln x$

$\frac{dy/dx}{y} = x \frac{1}{x} + \ln x = 1 + \ln x$

$\frac{dy}{dx} = x^x (1 + \ln x)$

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $y = (\ln x)^x$

F: dy/dx

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $y = (\ln x)^x$ F: dy/dx

$\ln y = \ln (\ln x)^x$

$\ln y = x \ln (\ln x)$

$\frac{dy/dx}{y} = x \frac{1}{\ln x} \frac{1}{x} + \ln (\ln x) \; 1$

$\frac{dy/dx}{y} = \frac{1}{\ln x} + \ln (\ln x)$

$\frac{dy}{dx} = \left[\frac{1}{\ln x} + \ln (\ln x) \right] (\ln x)^x$

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3.9 Exponential & Logarithmic Functions: Derivatives

348. $y = (\ln x)^{\ln x}$ F: dy/dx

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3.9 Exponential & Logarithmic Functions: Derivatives

348. $y = (\ln x)^{\ln x}$ F: dy/dx

$\ln y = \ln [(\ln x)^{\ln x}]$

$\ln y = \ln x \ln (\ln x)$

$\frac{dy/dx}{y} = \ln x \frac{1}{\ln x} \frac{1}{x} + \ln (\ln x) \frac{1}{x}$

$\frac{dy/dx}{y} = \frac{1}{x} + \frac{\ln (\ln x)}{x}$

$\frac{dy}{dx} = \frac{1}{x} [\, 1 + \ln(\ln x)\,] (\ln x)^{\ln x}$

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3.9 Exponential & Logarithmic Functions: Derivatives

$\frac{d\,(e^x)}{dx} = e^x$

G: 350. $y = (x^2 - 1)^{\ln x}$ F: dy/dx

Use logarithms to find derivatives that involve variables in exponents.

$\ln y = \ln (x^2 - 1)^{\ln x}$

$\ln y = (\ln x) \ln(x^2 - 1)$

$\frac{dy/dx}{y} = (\ln x) \frac{2x}{x^2 - 1} + \ln(x^2 - 1) \frac{1}{x}$

$dy/dx = \left[\frac{2x \ln x}{x^2 - 1} + \frac{\ln(x^2 - 1)}{x}\right] (x^2 - 1)^{\ln x}$

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3.9 Exponential & Logarithmic Functions: Derivatives

$y = \ln(\ln x^2)$ F: $\frac{dy}{dx}$

$y = \ln(2\ln x)$

$u = 2\ln x$

$\frac{du}{dx} = 2 \cdot \frac{1}{x}$

$\frac{dy}{dx} = \frac{1}{2\ln x} \cdot 2 \cdot \frac{1}{x}$

$= \frac{1}{x} \cdot \frac{1}{\ln x} = \frac{1}{x\ln x}$

$\frac{dy}{dx}$

3.9 Exponential & Logarithmic Functions: Derivatives

G: $f(x) = \ln\sqrt{1+\sin^2 x}$ F: eq. tang. line at $\left(\frac{\pi}{4}, \ln\sqrt{\frac{3}{2}}\right)$

$f(x) = \ln(1+\sin^2 x)^{1/2}$

$f(x) = \frac{1}{2}\ln(1+\sin^2 x)$

$f(x) = \frac{1}{2}\ln u$

$u = 1+\sin^2 x$

$\frac{du}{dx} = 2\sin x\cos x$

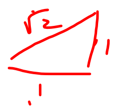
$f'(x) = \frac{1}{2} \cdot \frac{1}{u} \cdot \frac{du}{dx}$

$f'(x) = \frac{1}{2} \cdot \frac{2\sin x\cos x}{1+\sin^2 x} = \frac{\sin x\cos x}{1+\sin^2 x}$

$y - y_1 = m_T(x - x_1)$

$y - \ln\sqrt{\frac{3}{2}} = m_T\left(x - \frac{\pi}{4}\right)$

$f'\left(\frac{\pi}{4}\right) = \frac{\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}}{1+\left(\frac{1}{\sqrt{2}}\right)^2} = \frac{\frac{1}{2}}{1+\frac{1}{2}} = \frac{1}{3}$



$y - \ln\sqrt{\frac{3}{2}} = \frac{1}{3}x - \frac{\pi}{4}\cdot\frac{1}{3}$

$y = \frac{1}{3}x - \frac{\pi}{12} + \ln\sqrt{\frac{3}{2}}$

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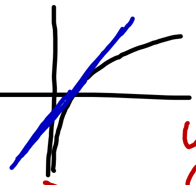
3.9 Exponential & Logarithmic Functions: Derivatives

G: $y = \ln x^{3/2}$ F: eq. tang. $(1,0)$

$y = \frac{3}{2}\ln x$



$m = \frac{dy}{dx} = \frac{3}{2}\cdot\frac{1}{x}$

$y = mx + b$

$y - y_0 = m_T(x - x_0)$

at $(1,0) = \frac{3}{2}\cdot\frac{1}{1} = \frac{3}{2}$

$y - 0 = \frac{3}{2}(x-1)$

$y = \frac{3}{2}x - \frac{3}{2}$

Point slope form of straight line

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$y_2 - y_1 = m(x_2 - x_1)$

$y - y_1 = m(x - x_1)$

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3.9 Exponential & Logarithmic Functions: Derivatives

$\ln\left(\frac{x^2-1}{x^3}\right)^3$

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3.9 Exponential & Logarithmic Functions: Derivatives

$$\ln\left(\frac{x^2-1}{x^3}\right)^3$$

$$3\ln\left(\frac{x^2-1}{x^3}\right)$$

$$3\left[\ln(x^2-1) - \ln x^3\right]$$

$$3\left[\ln(x^2-1) - 3\ln x\right]$$

$$= 3\left[\ln\left[(x+1)(x-1)\right] - 3\ln x\right]$$

$$= 3\left[\ln(x+1) + \ln(x-1) - 3\ln x\right]$$

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $g(t) = \frac{\ln t}{t^2}$ F: $g'(t)$

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $g(t) = \frac{\ln t}{t^2}$ F: $g'(t)$

$$g'(t) = \frac{t^2 \frac{d(\ln t)}{dt} - (\ln t)(2t)}{t^4}$$

$$= \frac{t^2 \cdot \frac{1}{t} - 2t\ln t}{t^4} = \frac{t - 2t\ln t}{t^4}$$

$$= \frac{t(1 - 2\ln t)}{t^4} = \frac{1 - 2\ln t}{t^3}$$

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3.9 Exponential & Logarithmic Functions: Derivatives

$y = \ln\sqrt{\frac{x+1}{x-1}}$ F: dy/dx

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3.9 Exponential & Logarithmic Functions: Derivatives

$y = \ln\sqrt{\frac{x+1}{x-1}}$ F: dy/dx

$= \ln\left(\frac{x+1}{x-1}\right)^{1/2} = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$

$= \frac{1}{2}\left[\ln(x+1) - \ln(x-1)\right]$

$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x+1} - \frac{1}{x-1}\right] = \frac{1}{2}\left[\frac{x-1-(x+1)}{(x+1)(x-1)}\right]$

$= \frac{1}{2}\left[\frac{x-1-x-1}{(x+1)(x-1)}\right] = \frac{1}{2}\left[\frac{-2}{(x+1)(x-1)}\right]$

$= \frac{-1}{(x+1)(x-1)} = \frac{1}{1-x^2}$

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $x^2 - 3\ln y + y^2 = 10$ F: dy/dx

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $x^2 - 3\ln y + y^2 = 10$ F: dy/dx

$$2x - 3 \cdot \frac{1}{y} \cdot \frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\left[\frac{-3}{y} + 2y\right]\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{\frac{-3}{y} + 2y} \cdot \frac{y}{y} = \frac{-2xy}{-3 + 2y^2}$$

$$= \frac{2xy}{3 - 2y^2}$$

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $y = \ln x^3$
$= 3\ln x$

F: eq. tang. line at $(1, 0)$

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $y = \ln x^3 = 3\ln x$

$\frac{dy}{dx} = 3\,\frac{1}{x} = \frac{3}{x}$

F: eq. tang. line at $(1, 0)$

$y = mx + b$

$y - y_1 = m_T(x - x_1)$

$y - 0 = \left(\frac{3}{1}\right)(x - 1)$

$y = 3x - 3$

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $f(x) = \ln\sqrt{1+\sin^2 x}$

$f(x) = \ln(1+\sin^2 x)^{1/2}$

$f(x) = \frac{1}{2}\ln(1+\sin^2 x)$

$f(u) = \frac{1}{2}\ln u$

$\frac{d(\ln u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$u = 1 + (\sin x)^2$

$du = \frac{du}{dx}\cdot dx$

$\frac{du}{dx} = 2\sin x\cos x$

$f'(x) = \frac{1}{2}\cdot\frac{1}{u}\frac{du}{dx}$

$f'(x) = \frac{1}{2}\cdot\frac{1}{1+\sin^2 x}\cdot 2\sin x\cos x$

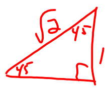
$f'(x) = \frac{\sin x\cos x}{1+\sin^2 x}$

$y = .33x - .059$

F: eq tang. line at $\left(\frac{\pi}{4}, \ln\sqrt{\frac{3}{2}}\right)$

$y - y_1 = m_T(x - x_1)$

$y - \ln\sqrt{\frac{3}{2}} = m_T\left(x - \frac{\pi}{4}\right)$



$f'\left(\frac{\pi}{4}\right) = \frac{\sin\frac{\pi}{4}\cos\frac{\pi}{4}}{1+\left(\sin\frac{\pi}{4}\right)^2} = \frac{\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}}{1+\left(\frac{1}{\sqrt{2}}\right)^2} = \frac{\frac{1}{2}}{1+\frac{1}{2}}\cdot\frac{2}{2} = \frac{1}{2+1} = \frac{1}{3}$

$y - \ln\sqrt{\frac{3}{2}} = m_T\left(x - \frac{\pi}{4}\right)$

$y - \ln\sqrt{\frac{3}{2}} = \frac{1}{3}\left(x - \frac{\pi}{4}\right)$

$= \frac{1}{3}x - \frac{\pi}{12}$

$y = \frac{1}{3}x - \frac{\pi}{12} + \ln\sqrt{\frac{3}{2}}$

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $x^2 - 3\ln y + y^2 = 10$ F: $\frac{dy}{dx}$

$$2x - 3\cdot\frac{1}{y}\cdot\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\left(-\frac{3}{y} + 2y\right)\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{-\frac{3}{y} + 2y}\cdot\frac{y}{y}$$

$$= \frac{-2xy}{-3 + 2y^2} = \frac{2xy}{3 - 2y^2}$$

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3.9 Exponential & Logarithmic Functions: Derivatives

G: $f(x) = \ln\left[\frac{x}{x^2+1}\right]$ F: $\frac{dy}{dx}$

$$f(x) = \ln x - \ln(x^2+1)$$

$$f'(x) = \frac{1}{x} - \frac{2x}{x^2+1} = \frac{x^2+1-2x(x)}{x(x^2+1)}$$

$$= \frac{x^2+1-2x^2}{x(x^2+1)} = \frac{1-x^2}{x(x^2+1)}$$

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3.9 Exponential & Logarithmic Functions: Derivatives

61. G: $y = \ln(\ln x^2)$

F: $\frac{dy}{dx}$ $\frac{2}{x(2\ln x)}$

$$\frac{dy}{dx} = \frac{1}{\ln x^2} \cdot \frac{1}{x^2} \cdot 2x$$

$$\frac{d(\ln u)}{dx} = \frac{1}{u} \cdot \frac{du}{dx}$$

$$= \frac{2x}{x^2 \ln x^2}$$

$$= \frac{2}{x \ln x^2}$$

$$= \frac{2}{x\,2\ln x} = \frac{1}{x \ln x}$$

$y = \ln(2\ln x)$

$= \ln 2 + \ln(\ln x) \rightarrow u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$

$= \ln 2 + \ln u$

$$\frac{dy}{dx} = 0 + \frac{1}{u}\frac{du}{dx} = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$$

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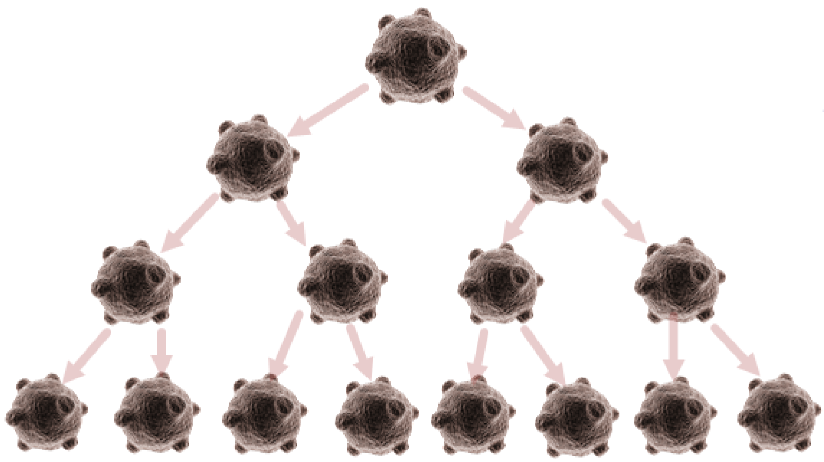
Appendix: Exponential Model in Biology

Bacteria reproduce by binary fission.
A single cell divides to form 2 cells.

#	$y = 2^x$



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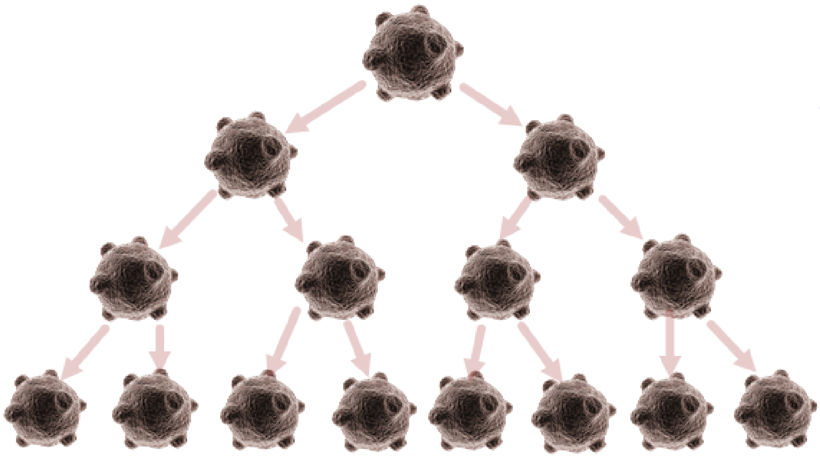
Bacteria reproduce by binary fission.
Consider a single cell:



#	2^x
1	2^0
2	2^1
4	2^2
8	2^3

$y = 2^x$ is exponential function.

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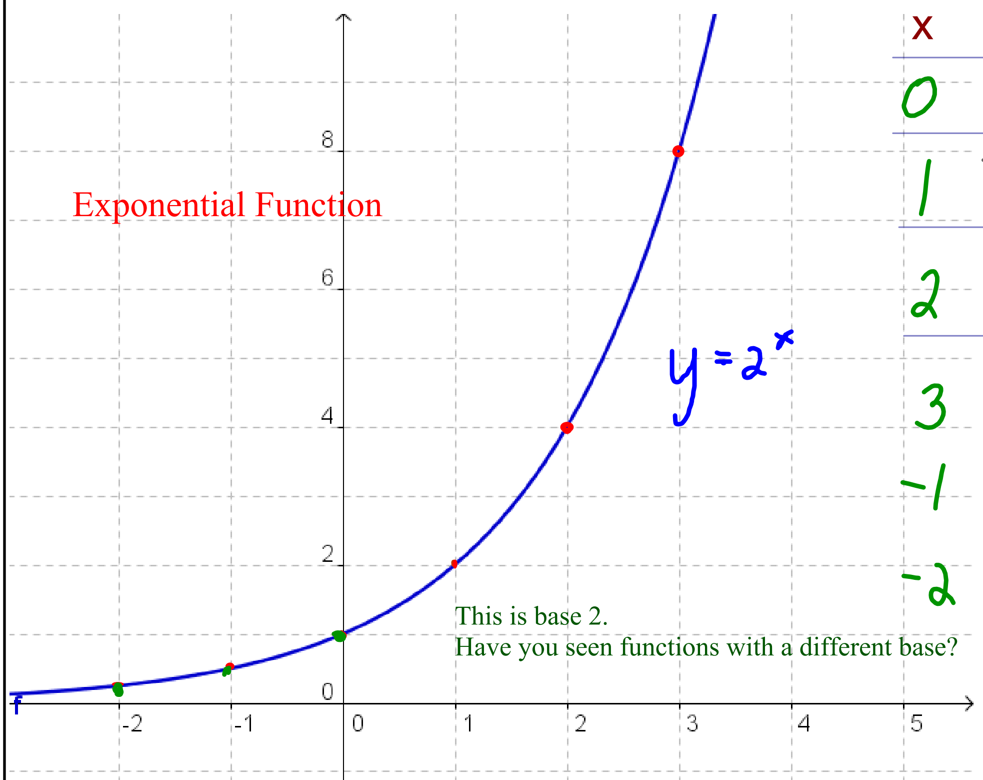

x	#	y 2^x
0	1	$2^0 = 1$
1	2	2^1
2	4	2^2
3	8	2^3
-1	$\frac{1}{2}$	2^{-1}
-2	$\frac{1}{4}$	

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3.9 Exponential & Logarithmic Functions: Derivatives

Appendix: from Pre-Calc

Laws of Exponents

$$x^m x^n = x^{m+n} \qquad \frac{x^m}{x^n} = x^{m-n}$$

$$x^{-n} = \frac{1}{x^n} \qquad (x^m)^n = x^{mn}$$

$$x^{1/n} = \sqrt[n]{x} \qquad (xy)^n = x^n y^n$$

$$x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m \qquad \sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}} \qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

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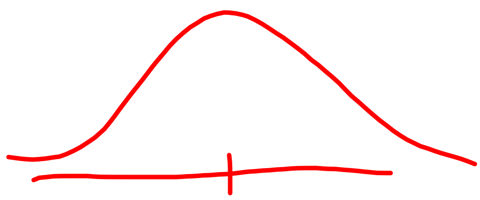
3.9 Exponential & Logarithmic Functions: Derivatives

Appendix: Exponential Function in Statistics

Standard Normal Probability Density Function

has mean of 0 and Inflection point at $\pm$ 1σ

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$$



Find the inflection points if σ=1

(See textbook to check your solution)

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3.9 Exponential & Logarithmic Functions: Derivatives

$$\ln(3e^2) = \ln 3 + \ln e^2$$
$$= \ln 3 + 2 \ln e$$
$$= \ln 3 + 2$$

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