3.8 Implicit Differentiation

Goal: to find the derivative of a variable that is not expressed explicitly in terms of other variables.

Examples: Find *dy/dx*

$xy = 1$

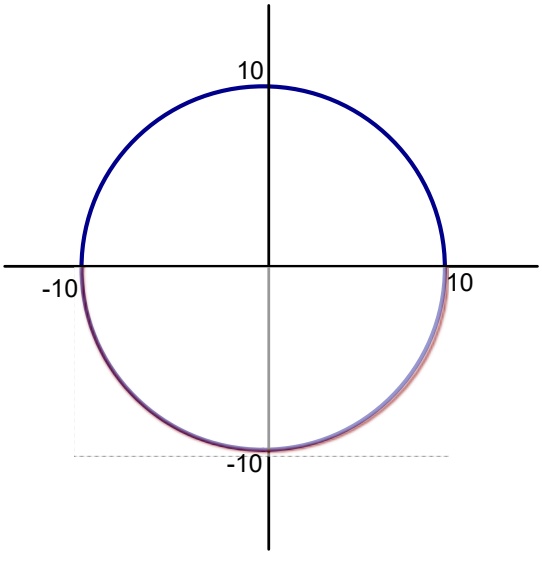
$x^2 + y^2 = 25$

$\sin y = x$

Study 3.8, # 301,303, 307-311, 315-319,325,305

3.8 Implicit Differentiation

G: $x^2 + y^2 = 100$ F: dy/dx



If we solve explicitly for y in terms of x, we have 2 functions, the upper semicircle, a pos. radical, and the lower semicircle, a neg. radical.

3.8 Implicit Differentiation

G: $x^2 + y^2 = 100$ F: dy/dx

If we solve explicitly for x,
we have 2 functions,
the upper semicircle, + radical,
and the lower semicircle, - radical.

$y^2 = 100 - x^2$

$y = \pm\sqrt{100 - x^2} = \pm(100 - x^2)^{1/2}$

for $y > 0$: $\frac{dy}{dx} = \frac{1}{2}(100 - x^2)^{-1/2}(-2x)$

$\frac{dy}{dx} = \frac{-x}{\sqrt{100 - x^2}} = \frac{-x}{y}$

for $y < 0$, $\frac{dy}{dx} = \frac{x}{\sqrt{100 - x^2}} = \frac{x}{-y}$

$\frac{dy}{dx} = \frac{-x}{y}$

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3.8 Implicit Differentiation

G: $x^2 + y^2 = 100$ F: dy/dx

derivative with **respect to x**

Since $y = \pm\sqrt{100 - x^2}$

treat y^2 as a composite function

Instead of solving explicitly,
use the **Chain Rule** on y^2
to solve implicitly.

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3.8 Implicit Differentiation

G: $x^2 + y^2 = 100$ F: dy/dx

derivative with **respect to x**

$2x + 2y\frac{dy}{dx} = 0$

Since $y = \pm\sqrt{100 - x^2}$

treat y^2 as a composite function

$2y\frac{dy}{dx} = -2x$

Instead of solving explicitly, use the **Chain Rule** on y^2 to solve implicitly.

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

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3.8 Implicit Differentiation

G: $x^2 - y^2 = 16$ F: dy/dx

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3.8 Implicit Differentiation

G: $x^2 - y^2 = 16$ F: dy/dx

$2x - 2y\frac{dy}{dx} = 0$

$-2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{-2y} = \frac{x}{y}$

$\frac{dy}{dx} = \frac{x}{y}$

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3.8 Implicit Differentiation

G: $x^3 + y^3 = 8$ F: dy/dx

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3.8 Implicit Differentiation

G: $x^3 + y^3 = 8$ F: dy/dx

$$x^3+y^3=8$$

$$3x^2+3y^2\frac{dy}{dx}=0$$

$$3y^2\frac{dy}{dx}=-3x^2$$

$$\frac{dy}{dx}=-\frac{x^2}{y^2}$$

$$\frac{dy}{dx}=\frac{-x^2}{y^2}$$

3.8 Implicit Differentiation

G: $2 \sin x \cos y = 1$ F: dy/dx

3.8 Implicit Differentiation

G: $2 \sin x \cos y = 1$ F: dy/dx

$$(2\sin x)\left(-\sin y \frac{dy}{dx}\right) + \cos y\,(2\cos x) = 0$$

$$-2\sin x \sin y \frac{dy}{dx} = -2\cos x \cos y$$

$$\frac{dy}{dx} = \frac{\cos x \cos y}{\sin x \sin y}$$

3.8 Implicit Differentiation

G: $x^2 + y^2 - 4x + 6y + 9 = 0$ F: dy/dx

3.8 Implicit Differentiation

G: $x^2 + y^2 - 4x + 6y + 9 = 0$ F: dy/dx

$$2x + 2y\frac{dy}{dx} - 4 + 6\frac{dy}{dx} + 0 = 0$$

$$2y\frac{dy}{dx} + 6\frac{dy}{dx} = 4 - 2x$$

$$(2y+6)\frac{dy}{dx} = 4 - 2x$$

$$\frac{dy}{dx} = \frac{4-2x}{2y+6} = \frac{2-x}{y+3}$$

3.8 Implicit Differentiation

alternative approach (not recommended, depends on ability to factor to $(a+b)^2$)

G: $x^2 + y^2 - 4x + 6y + 9 = 0$ F: dy/dx

$$x^2 + y^2 - 4x + 6y + 9 = 0$$

$$y^2 + 6y + 9 = -x^2 + 4x$$

$$(y+3)^2 = -x^2 + 4x$$

$$y + 3 = \pm\sqrt{-x^2 + 4x}$$

$$y = -3 \pm \sqrt{-x^2 + 4x}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(x+b)^2 = x^2 + 2bx + b^2$$

3.8 Implicit Differentiation

Skip to even HW problems

G: $x^2 - y^3 = 0$ F: $\left.\frac{dy}{dx}\right|_{(1,1)}$

3.8 Implicit Differentiation

G: $x^2 - y^3 = 0$ F: $\left.\frac{dy}{dx}\right|_{(1,1)}$

$2x - 3y^2 \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{-3y^2} = \frac{2x}{3y^2}$

at $(1,1)$ $\frac{dy}{dx} = \frac{2}{3}$

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3.8 Implicit Differentiation

G: $x^2 + y^2 = 36$

F: eq. tang line and normal line at (6,0) and at $(5, \sqrt{11})$

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3.8 Implicit Differentiation

G: $x^2 + y^2 = 36$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$

$\left.\frac{dy}{dx}\right|_{(6,0)} = \frac{-6}{0}$ DNE

$\left.\frac{dy}{dx}\right|_{(5,\sqrt{11})} = \frac{-5}{\sqrt{11}} = m_T$

$m_N = \frac{\sqrt{11}}{5}$

F: eq. tang line and normal line at (6,0) and at $(5, \sqrt{11})$

$y - y_1 = m_T(x - x_1)$

$y - 0 = m_T(x - 6)$

$y = m_T(x - 6)$

$x = 6$ tang.

$y = 0$ normal

$y - \sqrt{11} = m_T(x - 5)$

tangent

$y - \sqrt{11} = \frac{-5}{\sqrt{11}}(x - 5)$

$y = \frac{-5}{\sqrt{11}}x + \frac{25}{\sqrt{11}} + \sqrt{11}$

normal

$y - \sqrt{11} = \frac{\sqrt{11}}{5}(x - 5)$

$y = \frac{\sqrt{11}}{5}x - \sqrt{11} + \sqrt{11}$

$y = \frac{\sqrt{11}}{5}x$

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3.8 Implicit Differentiation

300. $x^2 - y^2 = 4$

302. $x^2 y = y - 7$

304. $xy - \cos(xy) = 1$

306. $-xy - 2 = \frac{x}{2}$

310. **[T]** $x^4 y - xy^3 = -2, \ (-1, -1)$ tangent line at (-1,-1)

318. Find all points on the graph of $y^3 - 27y = x^2 - 90$ at which the tangent line is vertical.

3.8 Implicit Differentiation

300. $x^2 - y^2 = 4$

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3.8 Implicit Differentiation

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3.8 Implicit Differentiation

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3.8 Implicit Differentiation

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3.8 Implicit Differentiation

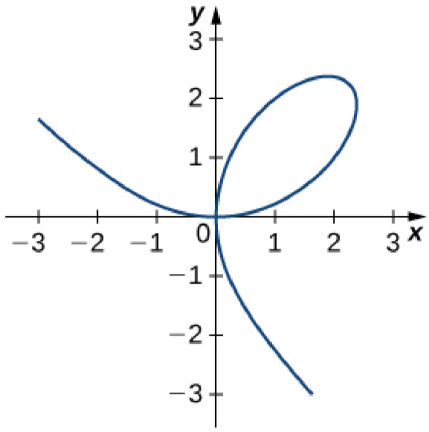
310. **[T]** $x^4 y - xy^3 = -2, (-1, -1)$ tangent line at (-1,-1)

3.8 Implicit Differentiation

318. Find all points on the graph of $y^3 - 27y = x^2 - 90$ at which the tangent line is vertical.

3.8 Implicit Differentiation

316. **[T]** The graph of a folium of Descartes with equation $2x^3 + 2y^3 - 9xy = 0$ is given in the following graph.



a. Find the equation of the tangent line at the point $(2, 1)$. Graph the tangent line along with the folium.

b. Find the equation of the normal line to the tangent line in a. at the point $(2, 1)$.