

### 3.6 Chain Rule: Derivative of Composite Functions

Goal:

Find derivatives of composite functions.

Examples:

$$y = (2x^3 + 1)^{10}$$

$$g(x) = \sqrt{9 - 4x}$$

$$y = [\sin(2x^3 + 1)]^5$$

Study 3.6 # 215 - 237, 241, 243,  
253, 255 or 257, 259

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### 3.6 Chain Rule: Derivative of Composite Functions

$$y = (2x^3 + 1)^2$$

$$y = (2x^3 + 1)^{10}$$

F:  $dy/dx$

$$y = (2x^3 + 1)(2x^3 + 1)$$

$$\begin{aligned} \frac{dy}{dx} &= (2x^3 + 1)(6x^2) + (2x^3 + 1)(6x^2) \\ &= 2(6x^2)(2x^3 + 1) = \underline{12x^2(2x^3 + 1)} \end{aligned}$$

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### 3.6 Chain Rule: Derivative of Composite Functions

$$y = (2x^3 + 1)^2$$

F:  $dy/dx$ 

$$y = (2x^3 + 1)^{10}$$

$$y = (2x^3 + 1)(2x^3 + 1)$$

$$dy/dx = (2x^3 + 1)(6x^2) + (2x^3 + 1)(6x^2) \quad \text{Use Product Rule}$$

$$dy/dx = 2(2x^3 + 1)(6x^2)$$

$$dy/dx = 12x^2(2x^3 + 1)$$

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### 3.6 Chain Rule: Derivative of Composite Functions

$$y = (2x^3 + 1)^{10}$$

$$y = (2x^3 + 1)^2$$

F:  $dy/dx$ 

Can we use the Power Rule to get  
the correct answer?

$$\frac{dy}{dx} \neq 2(2x^3 + 1)'$$

?

$$dy/dx = 12x^2(2x^3 + 1)$$

$$6x^2 \rightarrow$$

from previous

**NO! Not alone.**

NEED FACTOR:  $6x^2 = \frac{d[2x^3+1]}{dx}$

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### 3.6 Chain Rule: Derivative of Composite Functions

$$y = (2x^3 + 1)^{10}$$

What makes this a composite function?

$$y = (2x^3 + 1)^2$$

let  $u = 2x^3 + 1$ , then  $y = u^2$

$u$  is a function of  $x$  and  
 $y$  is a function of  $u$

OR: let  $f(x) = 2x^3 + 1$ , and  $g(x) = x^2$

From Chapter 1

then  $y = g(f(x))$

$$g(f(x)) = (2x^3 + 1)^2$$

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### 3.6 Chain Rule: Derivative of Composite Functions

$$y = (2x^3 + 1)^{10}$$

$$y = (2x^3 + 1)^2$$

F:  $dy/dx$

Find  $dy/dx$  again, from the beginning to the end.

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### 3.6 Chain Rule: Derivative of Composite Functions

$$y = (2x^3 + 1)^{10} \quad F: dy/dx$$

For composite functions where  $y = f(u)$  and  $u = f(x)$ ,

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

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### 3.6 Chain Rule: Derivative of Composite Functions

$$y = (2x^3 + 1)^{10}$$

F:  $dy/dx$

$$y = u^{10}$$

$$\frac{dy}{du} = 10u^9$$

$$u = 2x^3 + 1$$

$$\frac{du}{dx} = 6x^2$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= 10u^9 \cdot 6x^2 \\ &= 10(2x^3 + 1)^9 \cdot 6x^2 \\ &= 60x^2(2x^3 + 1)^9 \end{aligned}$$

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### 3.6 Chain Rule: Derivative of Composite Functions

$$y = (2x^3 + 1)^{10}$$

F:  $dy/dx$

Directly

$$dy/dx = 10(2x^3 + 1)^9 (6x^2)$$

$$dy/dx = 60x^2(2x^3 + 1)^9$$

Using u substitution for inner function

$$y = u^{10} \quad u = 2x^3 + 1$$

$$\frac{dy}{du} = 10u^9 \quad \frac{du}{dx} = 6x^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 10u^9 (6x^2)$$

$$= 10(2x^3 + 1)^9 (6x^2)$$

$$= 60x^2(2x^3 + 1)^9$$

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### 3.6 Chain Rule: Derivative of Composite Functions

## Chain Rule

If:

1)  $y = f(u)$  is a differentiable function of  $u$ , and

2)  $u = g(x)$  is a differentiable function of  $x$ ,

then

$y = f(g(x))$  is a differentiable function of  $x$ , and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

or

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

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## 3.6 Chain Rule: Derivative of Composite Functions

228.  $y = (3x^2 + 3x - 1)^4$  F:  $dy/dx$

$$\frac{dy}{dx} = \frac{4(3x^2 + 3x - 1)^3 (6x + 3)}{1} = 12(2x + 1)(3x^2 + 3x - 1)^3$$

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## 3.6 Chain Rule: Derivative of Composite Functions

228.  $y = (3x^2 + 3x - 1)^4$

F:  $dy/dx$ 

Directly

$$dy/dx = 4(3x^2 + 3x - 1)^3 (6x + 3)$$

$$dy/dx = 4(6x + 3)(3x^2 + 3x - 1)^3$$

Using u substitution for inner function

$$y = u^4 \quad u = 3x^2 + 3x - 1$$

$$\frac{dy}{du} = 4u^3 \quad \frac{du}{dx} = 6x + 3$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 4u^3 (6x + 3)$$

$$= 4(3x^2 + 3x - 1)^3 (6x + 3)$$

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## 3.6 Chain Rule: Derivative of Composite Functions

222.  $y = \sin^5(x)$  F:  $dy/dx$

Rewrite:  $y = [\sin x]^5$

## 3.6 Chain Rule: Derivative of Composite Functions

222.  $y = \sin^5(x)$  F:  $dy/dx$

Rewrite:  $y = [\sin x]^5$

Directly

$$dy/dx = 5(\sin x)^4 (\cos x)$$

Using u substitution for inner function

$$y = u^5 \quad u = \sin x$$

$$\frac{dy}{du} = 5u^4 \quad \frac{du}{dx} = \cos x$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 5u^4 (\cos x)$$

$$= 5(\sin x)^4 (\cos x)$$

### 3.6 Chain Rule: Derivative of Composite Functions

$$y = \sin x^5$$

F:  $dy/dx$

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### 3.6 Chain Rule: Derivative of Composite Functions

$$y = \sin x^5$$

F:  $dy/dx$

$$\begin{aligned}\frac{dy}{dx} &= (\cos x^5)(5x^4) \\ &= 5x^4 \cos x^5\end{aligned}$$

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## 3.6 Chain Rule: Derivative of Composite Functions

$$y = (\sin x^2)^3 \quad F: dy/dx$$

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## 3.6 Chain Rule: Derivative of Composite Functions

$$y = (\sin x^2)^3$$

$$y = (\sin x^2)^3 \quad F: dy/dx$$

$$y = (\quad)^3$$

Outer function is cube of something:  $(\square)^3$

$$y = (\sin \quad)^3$$

Inner function is sine of something:  $\sin \square$

$$y = (\sin x^2)^3$$

Innermost function is  
square of something:  $(\square)^2$

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## 3.6 Chain Rule: Derivative of Composite Functions

$$y = (\sin x^2)^3 \quad F: dy/dx$$

$$y = f(u) = u^3$$

$$u = g(v) = \sin v$$

$$v = h(x) = x^2$$

$$\text{then } y = f(g(h(x)))$$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

expanded chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$$

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## 3.6 Chain Rule: Derivative of Composite Functions

$$y = (\sin x^2)^3 \quad F: dy/dx$$

$$y = f(u) = u^3$$

$$u = g(v) = \sin v$$

$$v = h(x) = x^2$$

$$\text{then } y = f(g(h(x)))$$

$$\begin{aligned} \frac{dy}{dx} &= 3(\sin x^2)^2 \cdot (\cos x^2) (2x) \\ &= 6x (\cos x^2) (\sin x^2)^2 \end{aligned}$$

$$\text{NOTE: } \sin^2 x \neq \sin x^2$$

$$\sin^2 x = (\sin x)^2$$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

expanded chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$$

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## 3.6 Chain Rule: Derivative of Composite Functions

$$y = \sin x^2$$

$$\frac{dy}{dx} = +(\cos x^2)(2x)$$

$$= +2x \cos x^2$$

$$\sin^2 x \neq \sin x^2$$

$$(\sin x)^2 \neq \sin x^2$$

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## 3.6 Chain Rule: Derivative of Composite Functions

co-functions

$$\frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

$$\frac{d(\tan x)}{dx} = \sec^2 x$$

$$\frac{d(\cot x)}{dx} = -\csc^2 x$$

$$\frac{d(\sec x)}{dx} = \sec x \tan x$$

$$\frac{d(\csc x)}{dx} = -\csc x \cot x$$

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## 3.6 Chain Rule: Derivative of Composite Functions

co-functions

$$\frac{d(\sin u)}{dx} = \cos u \frac{du}{dx} \qquad \frac{d(\cos u)}{dx} = -\sin u \frac{du}{dx}$$


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$$\frac{d(\tan u)}{dx} = \sec^2 u \frac{du}{dx} \qquad \frac{d(\cot u)}{dx} = -\csc^2 u \frac{du}{dx}$$


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$$\frac{d(\sec u)}{dx} = \sec u \tan u \frac{du}{dx} \qquad \frac{d(\csc u)}{dx} = -\csc u \cot u \frac{du}{dx}$$

$$\sin^2(3x^2 - 4x + 1) + \cos^2(3x^2 - 4x + 1) = 1$$

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$$y = \cos \sqrt{t} \quad F: \frac{dy}{dt}$$

$$\frac{dy}{dt} = -(\sin \sqrt{t}) \left( \frac{1}{2} t^{-1/2} \right)$$

$$\sqrt{t} = t^{1/2}$$

$$\frac{d(t^{1/2})}{dt} = \frac{1}{2} t^{-1/2}$$

$$= -\frac{\sin \sqrt{t}}{2\sqrt{t}}$$

**3.6 Chain Rule: Derivative of Composite Functions**

226.  $y = \cot^2 x$

232.  $y = \frac{1}{\sin^2(x)}$

234.  $y = x^2 \cos^4 x$

224.  $y = \tan(\sec x)$

242. **[T]** Find the equation of the tangent line to  $y = \left(3x + \frac{1}{x}\right)^2$  at the point  $(1, 16)$ . Use a calculator to graph the function and the tangent line together.

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**3.6 Chain Rule: Derivative of Composite Functions**

226.  $y = \cot^2 x$

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## 3.6 Chain Rule: Derivative of Composite Functions

226.  $y = \cot^2 x$

$$\begin{aligned}\frac{dy}{dx} &= 2 \cot x (-\csc^2 x) \\ &= -2 \cot x \csc^2 x\end{aligned}$$

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## 3.6 Chain Rule: Derivative of Composite Functions

232.  $y = \frac{1}{\sin^2(x)}$

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## 3.6 Chain Rule: Derivative of Composite Functions

$$232. \quad y = \frac{1}{\sin^2(x)} = \csc^2 x$$

$$y = (\sin x)^{-2}$$

$$\frac{dy}{dx} = -2(\sin x)^{-3} \cos x$$

$$= -\frac{2 \cos x}{\sin^3 x}$$

$$\frac{dy}{dx} = 2 \csc x (-\csc x \cot x)$$

$$= -2 \csc^2 x \cot x$$

$$= -2 \frac{1}{\sin^2 x} \cdot \frac{\cos x}{\sin x}$$

$$= -\frac{2 \cos x}{\sin^3 x}$$

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## 3.6 Chain Rule: Derivative of Composite Functions

$$234. \quad y = x^2 \cos^4 x$$

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## 3.6 Chain Rule: Derivative of Composite Functions

$$234. \quad y = (x^2)(\cos^4 x)$$

$$\frac{dy}{dx} = x^2 \frac{d(\cos^4 x)}{dx} + \cos^4 x \frac{d(x^2)}{dx}$$

$$= x^2 4 \cos^3 x (-\sin x) + 2x \cos^4 x$$

$$= -4x^2 \sin x \cos^3 x + 2x \cos^4 x$$

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## 3.6 Chain Rule: Derivative of Composite Functions

$$224. \quad y = \tan(\sec x)$$

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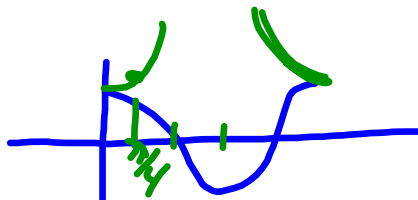
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## 3.6 Chain Rule: Derivative of Composite Functions

224.  $y = \tan(\sec x)$

$$\frac{dy}{dx} = \sec^2(\sec x) \sec x \tan x$$



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## 3.6 Chain Rule: Derivative of Composite Functions

242. [T] Find the equation of the tangent line to  $y = \left(3x + \frac{1}{x}\right)^2$  at the point  $(1, 16)$ . Use a calculator to graph the function and the tangent line together.

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## 3.6 Chain Rule: Derivative of Composite Functions

242. [T] Find the equation of the tangent line to  $y = \left(3x + \frac{1}{x}\right)^2$  at the point  $(1, 16)$ . Use a calculator to graph the function and the tangent line together.

$$y = \left(3x + \frac{1}{x}\right)^2$$

$$y = \left(3x + x^{-1}\right)^2$$

$$\frac{dy}{dx} = 2\left(3x + x^{-1}\right)' \left(3 - x^{-2}\right)$$

$$\frac{dy}{dx} = 2\left(3x + \frac{1}{x}\right) \left(3 - \frac{1}{x^2}\right)$$

$$\left. \frac{dy}{dx} \right|_{(1,16)} = 2(3+1)(3-1)$$

$$m_T = 2(4)(2) = 16$$

F: eq. tan. line.  
at  $(1, 16)$

$$y - y_1 = m_T(x - x_1)$$

$$y - 16 = m_T(x - 1)$$

$$y - 16 = 16(x - 1)$$

$$y - 16 = 16x - 16$$

$$y = 16x$$

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## 3.6 Chain Rule: Derivative of Composite Functions

Follow-up

236.  $y = \sqrt{6 + \sec \pi x^2}$

Tr y  $y = \sqrt{6 + \sin x}$

$$y = \sqrt{10 + \sec x}$$

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## 3.6 Chain Rule: Derivative of Composite Functions

$$G: y = \sqrt[3]{6x^2+1} \quad F: \frac{dy}{dx}$$

$$= (6x^2+1)^{1/3}$$

$$u = 6x^2 + 1$$

$$\frac{dy}{dx} = \frac{1}{3}(6x^2+1)^{-2/3} (12x)$$

$$= \frac{4x}{(6x^2+1)^{2/3}}$$

$$\frac{4x}{\sqrt[3]{(6x^2+1)^2}}$$

This is the beginning of extra problems that were not done in class.

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## 3.6 Chain Rule: Derivative of Composite Functions

$$g(x) = \sqrt{9-4x} \quad F: g'(x)$$

$$y = (9-4x)^{1/2}$$

$$u = 9-4x$$

$$\frac{du}{dx} = -4$$

$$y = u^{1/2}$$

$$\frac{dy}{du} = \frac{1}{2} u^{-1/2} = \frac{1}{2\sqrt{u}}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{2\sqrt{9-4x}} \cdot (-4) = -\frac{2}{\sqrt{9-4x}}$$

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## 3.6 Chain Rule: Derivative of Composite Functions

$$h(t) = \left( \frac{t^2}{t^3+2} \right)^2 \quad F: h'(t)$$

$$u = \frac{t^2}{t^3+2}$$

$$h = u^2$$

$$\frac{dh}{dt} = 2u \cdot \frac{du}{dt}$$

$$= 2 \cdot \frac{t^2}{(t^3+2)} \cdot \frac{t(4-t^3)}{(t^3+2)^2}$$

$$= \frac{2t^3(4-t^3)}{(t^3+2)^3}$$

$$\frac{du}{dt} = \frac{(t^3+2)(2t) - t^2(3t^2)}{(t^3+2)^2}$$

$$= \frac{2t^4 + 4t - 3t^4}{(t^3+2)^2}$$

$$= \frac{4t - t^4}{(t^3+2)^2}$$

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## 3.6 Chain Rule: Derivative of Composite Functions

$$h(t) = \left( \frac{t^2}{t^3+2} \right)^2 \quad F: h'(t)$$

one option:  $\left( \frac{a}{b} \right)^2 = \frac{a^2}{b^2}$

$$= \frac{t^4}{(t^3+2)^2} = \frac{t^4}{t^6+4t^3+4}$$

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then use Quotient Rule

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## 3.6 Chain Rule: Derivative of Composite Functions

$$G: f(x) = \frac{1}{(x^2 - 3x)^2}$$

$$= (x^2 - 3x)^{-2}$$

$$f'(x) = -2(x^2 - 3x)^{-3}(2x - 3)$$

$$= \frac{-2(2x - 3)}{(x^2 - 3x)^3}$$

$$f'(4) = \frac{-2(5)}{(16 - 12)^3} = \frac{-10}{64} = \left(-\frac{5}{32}\right)$$

$$F: f'(x) \text{ at } (4, \frac{1}{16})$$

or  $f'(4)$

$$u = x^2 - 3x$$

$$\frac{du}{dx} = 2x - 3$$

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## 2.4 Chain Rule: Derivative of Composite Functions

$$y = \sqrt{\frac{2x}{x+1}} = \left(\frac{2x}{x+1}\right)^{1/2}$$

$$y = u^{1/2}$$

$$\frac{dy}{du} = \frac{1}{2} u^{-1/2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{2\sqrt{u}} \cdot \frac{2}{(x+1)^2}$$

$$= \frac{1}{\sqrt{\frac{2x}{x+1}}} \cdot \frac{1}{(x+1)^2} = \sqrt{\frac{x+1}{2x}} \cdot \frac{1}{(x+1)^2} = \frac{(x+1)^{1/2}}{\sqrt{2x}(x+1)^2}$$

$$= \frac{1}{\sqrt{2x}(x+1)^{3/2}}$$

$$u = \frac{2x}{x+1}$$

$$\frac{du}{dx} = \frac{(x+1)2 - 2x(1)}{(x+1)^2}$$

$$= \frac{2x + 2 - 2x}{(x+1)^2}$$

$$= \frac{2}{(x+1)^2}$$

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## 2.4 Chain Rule: Derivative of Composite Functions

$$g(t) = \sqrt{\sqrt{t+1} + 1} = \left( \underline{\underline{(\underline{t+1}^{\frac{1}{2}} + 1)}}^{\frac{1}{2}} \right)$$

$$g'(t) = \frac{1}{2} \left[ (\underline{t+1}^{\frac{1}{2}} + 1) \right]^{-\frac{1}{2}} \underline{\underline{\frac{d}{dt}(\underline{t+1}^{\frac{1}{2}} + 1)}}$$

$$= \frac{1}{2} \left[ (\underline{t+1}^{\frac{1}{2}} + 1) \right]^{-\frac{1}{2}} \cdot \left[ \frac{1}{2} (\underline{t+1})^{-\frac{1}{2}} \underline{\underline{\frac{d(t+1)}{dt}}} \right]$$

$$\frac{d(t+1)}{dt} = 1$$

$$= \frac{1}{2} \left[ (\underline{t+1}^{\frac{1}{2}} + 1) \right]^{-\frac{1}{2}} \cdot \left[ \frac{1}{2} (\underline{t+1})^{-\frac{1}{2}} (1) \right]$$

$$= \frac{1}{2 \sqrt{\underline{t+1} + 1}} \cdot \frac{1}{2 \sqrt{\underline{t+1}}}$$

$$= \frac{1}{4 \sqrt{\underline{t+1}} \sqrt{\underline{t+1} + 1}}$$

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## 2.4 Chain Rule: Derivative of Composite Functions

$$y = (\underline{9x+2})^{\frac{2}{3}}$$

$$F: \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2}{3} (\underline{9x+2})^{-\frac{1}{3}} (9)$$

$$u = 9x+2$$

$$\frac{du}{dx} = 9$$

$$= \frac{6}{(\underline{9x+2})^{\frac{1}{3}}}$$

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G:  $y = 2 \tan^3 x$

$$\frac{dy}{dx} = 2 \cdot 3 (\tan x)^2 \sec^2 x$$

$$= 6 \tan^2 x \sec^2 x$$

$$\left. \frac{dy}{dx} \right|_{(\frac{\pi}{4}, 2)} = 6 \left( \tan \frac{\pi}{4} \right)^2 \left( \sec \frac{\pi}{4} \right)^2$$


$$= 6 \cdot 1 \cdot \left( \frac{1}{\cos \frac{\pi}{4}} \right)^2 = 6 \cdot 1 \cdot (\sqrt{2})^2 = 12$$

F: eq. of tang at  $(\frac{\pi}{4}, 2)$

$$y - y_1 = m_T (x - x_1)$$

$$y - 2 = m_T (x - \frac{\pi}{4})$$

$u = \tan x$   
 $\frac{du}{dx} = \sec^2 x$



$$y - 2 = 12(x - \frac{\pi}{4})$$

$$y = 12x - 3\pi + 2$$

## 2.4 Chain Rule: Derivative of Composite Functions

$y = (9x+2)^{2/3}$

F:  $\frac{dy}{dx}$

$y = u^{2/3}$

$u = 9x+2$

$$\frac{dy}{du} = \frac{2}{3} u^{-1/3}$$

$$\frac{du}{dx} = 9$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{2}{3} u^{-1/3} \cdot 9 = \frac{6}{u^{1/3}} = \frac{6}{(9x+2)^{1/3}}$$

## 2.4 Chain Rule: Derivative of Composite Functions

$$y = \sqrt{5-3x}$$

F:  $dy/dx$ 

$$y = \sqrt{5-3x} = (5-3x)^{1/2}$$

$$u = 5-3x$$

$$\frac{du}{dx} = -3$$

$$y = u^{1/2}$$

$$\frac{dy}{du} = \frac{1}{2} u^{-1/2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2} u^{-1/2} (-3) = \frac{-3}{2u^{1/2}}$$

$$= \frac{-3}{2\sqrt{5-3x}}$$

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## 2.4 Chain Rule: Derivative of Composite Functions

$$y = \sin \underline{\pi x}$$

F:  $dy/dx$ 

$$u = \pi x$$

$$\frac{du}{dx} = \pi$$

$$\frac{dy}{dx} = (\cos \pi x) \pi$$

$$= \pi \cos \pi x$$

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## 2.4 Chain Rule: Derivative of Composite Functions

$$y = \sin \frac{3\pi}{2}x$$

$$\frac{dy}{dx} = \frac{3\pi}{2} \cos \frac{3\pi}{2}x$$

$$F: dy/dx$$

$$u = \frac{3\pi}{2}x$$

$$\frac{du}{dx} = \frac{3\pi}{2}$$

$$x \frac{dy}{dx} \sin \frac{3\pi}{2} = \frac{3\pi}{2}$$

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