3.5 Derivatives of Trig Functions

GOALS:

1. Recognize the derivatives of:
   f(x)= sin(x), g(x)= cos(x), h(x)= tan(x)
2. Recognize the derivatives of reciprocal functions:
   f(x)= csc(x), g(x)= sec(x), h(x)= cot(x)
3. Find the derivatives of all 6 functions.
4. Notice that the derivatives of all the cofunctions are negative. cosine(x) cosecant(x) cotangent(c)

Study 3.5 # 175-197, 201, 203

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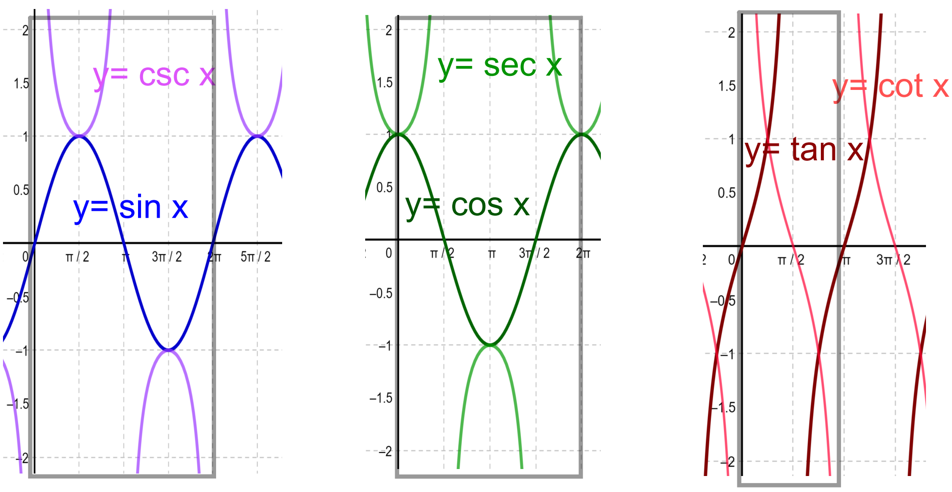
Homework

Calc Web Pages

3.5 Derivatives of Trig Functions

Reference Graphs of Trig Functions



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Homework

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3.5 Derivatives of Trig Functions

Investigate: $\frac{d[\sin(x)]}{dx}$

1. Go to: https://www.geogebra.org/classic
   This opens an online software called geogebra.

2. Click upper right bars and select the folder option.

3. In the window for the file name, type:
http://www.battaly.com/calc/geogebra/trig/derivative_sinx.ggb

4. Click the X under the previous bars to clear the graphing window.

5. Then click the circles to the left of f: y= sin(x) and A=(c,sin(c))

6. Click and drag either the point A or the c bar and watch the point move along the curve of y=sin(x)

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Homework

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3.5 Derivatives of Trig Functions

Investigate: $\frac{d[\sin(x)]}{dx}$

7. Click the circle to the left of T, the tangent line at point A.

8. Notice the slope of the tangent line. What is its value? What is it, in words? How does it change as the A is moved? When is it positive? negative? zero?

9. Find point B on the left side, and click the circle to the left of B. B has the same x-coordinate as A, but its y-coordinate is the slope of the tangent line T or the derivative of y=sin(x) at that x value.

10. Move point A to see what happens to point B.

10. Return to point B on the left. Right click on it and Turn ON SHOW TRACE. Then move point A again.

11. What do you see? What does the resulting curve represent?

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3.5 Derivatives of Trig Functions

Investigate: $\frac{d[\sin(x)]}{dx}$

Just finished a very visual representation of the derivative of sin(x)
How do we do that analytically?

Definition

Let f be a function. The **derivative function**, denoted by f', is the function whose domain consists of those values of x such that the following limit exists:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad (3.9)$$

$f(x) = \sin x$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$

$= \lim_{h \to 0} \frac{\sin x \cosh h + \cos x \sin h - \sin x}{h}$

$= \lim_{h \to 0} \frac{\sin x(\cos h - 1)}{h} + \frac{\cos x \sin h}{h}$

$\lim_{h \to 0} \sin x \frac{(\cos h - 1)}{h} + \lim_{h \to 0} \cos x \frac{\sin h}{h}$

$(\sin x)(0) + (\cos x)(1)$

$f'(x) = \cos x$

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3.5 Derivatives of Trig Functions

Investigate: $\frac{d[\sin(x)]}{dx}$

p.773

$$\sin 2x = 2 \sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$
$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$
$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

sin x csc x = 1

cos x sec x = 1

tan x cot x = 1

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3.5 Derivatives of Trig Functions

Derivatives of Trig Functions

Co-functions: derivatives are neg

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

3.5 Derivatives of Trig Functions

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178. $y = x - x^3 \sin x$

180. $y = \sin x \tan x$

182. $y = \dfrac{\tan x}{1 - \sec x}$

188. **[T]** $f(x) = \sec x,\ x = \frac{\pi}{4}$

F: tangent line

202. After a diver jumps off a diving board, the edge of the board oscillates with position given by $s(t) = -5\cos t$ cm at t seconds after the jump.

a. Sketch one period of the position function for $t \geq 0$.
b. Find the velocity function.
c. Sketch one period of the velocity function for $t \geq 0$.
d. Determine the times when the velocity is 0 over one period.
e. Find the acceleration function.
f. Sketch one period of the acceleration function for $t \geq 0$.

198. Find all x values on the graph of $f(x) = x - 2\cos x$ for $0 < x < 2\pi$ where the tangent line has slope 2.

3.5 Derivatives of Trig Functions

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Homework

3.5 Derivatives of Trig Functions

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$\frac{d}{dx}(\sec x) = \sec x \tan x$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$

F: $\frac{dy}{dx} = 1 - \left[x^3(\cos x) + (\sin x)(3x^2)\right]$

$= 1 - x^3 \cos x - 3x^2 \sin x$

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Homework

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180. $y = \sin x \tan x$

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$\sin^2 x + \cos^2 x = 1$

3.5 Derivatives of Trig Functions

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$\frac{d}{dx}(\sec x) = \sec x \tan x$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$

180. $y = \sin x \tan x$

$$\frac{dy}{dx} = \sin x \sec^2 x + \tan x(\cos x)$$
$$= \sin x \sec^2 x + \frac{\sin x}{\cos x} \cdot \cos x$$
$$= \sin x \sec^2 x + \sin x$$
$$= \sin x(\sec^2 x + 1)$$

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$\sin^2 x + \cos^2 x = 1$

3.5 Derivatives of Trig Functions

182. $y = \dfrac{\tan x}{1 - \sec x}$

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$$\frac{dy}{dx} = \frac{(1-\sec x)\sec^2 x - \tan x(-\sec x \tan x)}{(1-\sec x)^2}$$

$$= \frac{\sec^2 x - \sec^3 x + \sec x \tan^2 x}{(1-\sec x)^2}$$

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[T] $f(x) = \sec x,\ x = \dfrac{\pi}{4}$

F: eq of tangent line at x = $\pi/4$

$\frac{d}{dx}(\sin x) = \cos x$ $\quad$ $\frac{d}{dx}(\cos x) = -\sin x$

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$\frac{d}{dx}(\sec x) = \sec x \tan x$ $\quad$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$

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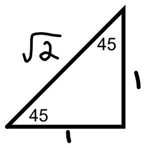
[T] $f(x) = \sec x,\ x = \dfrac{\pi}{4}$

$\frac{d}{dx}(\tan x) = \sec^2 x$ $\quad$ $\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$ $\quad$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$

F: eq of tangent line at x = $\pi/4$

need (x_1, y_1) and m_T



$(x_1,y_1) = (\pi/4, \sqrt{2})$; need m_T

$y - y_1 = m_T(x - x_1)$

$m_T = f'(x) = \sec x \tan x$

$m_T \Big|_{x=\pi/4} = f'\left(\frac{\pi}{4}\right) = \sec\frac{\pi}{4} \cdot \tan\frac{\pi}{4} = \sqrt{2} \cdot 1 = \sqrt{2}$

$y - \sqrt{2} = \sqrt{2}\left(x - \frac{\pi}{4}\right) = \sqrt{2}\,x - \frac{\pi\sqrt{2}}{4}$

$y = \sqrt{2}\,x - \frac{\pi\sqrt{2}}{4} + \sqrt{2}$

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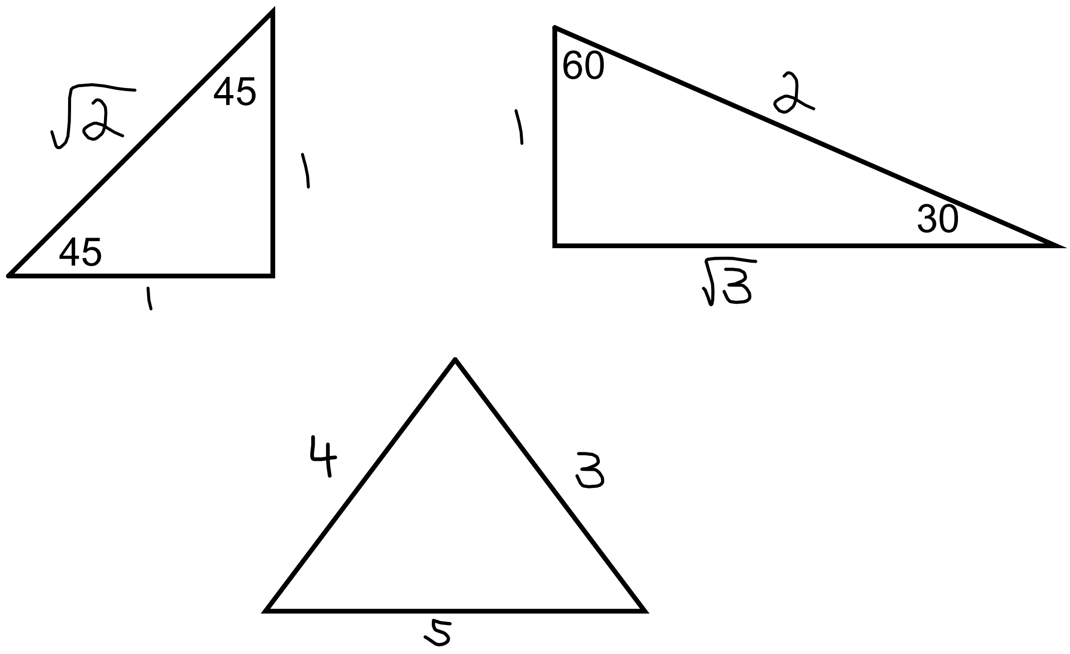

3.5 Derivatives of Trig Functions

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Derivative Practice

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Calculus Home Page

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a. Sketch one period of the position function for $t \geq 0$.

v(t)=s '(t)=-5(-sint)=5sint

b. Find the velocity function.

c. Sketch one period of the velocity function for $t \geq 0$.

d. Determine the times when the velocity is 0 over one period. v(t)=0 at t=0,π, 2π

e. Find the acceleration function.

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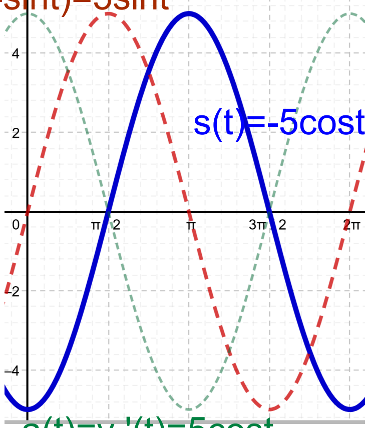

a(t)=v '(t)=5cost

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Homework

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Homework

3.5 Derivatives of Trig Functions

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Derivative Practice

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198. Find all x values on the graph of $f(x) = x - 2\cos x$ for $0 < x < 2\pi$ where the tangent line has slope 2. $\longrightarrow m_T = 2$

$f(x) = x - 2\cos x$

$m_T = f'(x) = 1 - 2(-\sin x)$

$= 1 + 2\sin x$

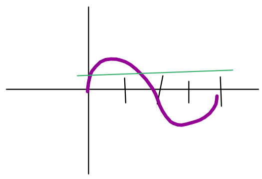
$1 + 2\sin x = 2$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = 30°, 150°$

$= \frac{\pi}{6}, \frac{5\pi}{6}$



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HW: 183 answer can look different and still be correct.

$$= \frac{2\csc^2 x}{(1+\cot x)^2} = \frac{2\cdot\frac{1}{\sin^2 x}}{\left(1+\frac{\cos x}{\sin x}\right)^2} = \frac{2\cdot\frac{1}{\sin^2 x}}{\left(\frac{\sin x}{\sin x}+\frac{\cos x}{\sin x}\right)^2}$$

$$= \frac{\frac{2}{\sin^2 x}}{\frac{(\sin x+\cos x)^2}{\sin^2 x}} = \frac{2}{(\sin x+\cos x)^2}$$

$$= \frac{2}{\sin^2 x+2\sin x\cos x+\cos^2 x} =$$

$$= \frac{2}{1+2\sin x\cos x}$$