3.4 Derivative as a Rate of Change

GOALS:

1. Recognize the derivative as a rate of change of one variable with respect to another.
2. Recognize velocity as the instantaneous rate of change of position with respect to time:

$$v(t) = s'(t)$$

3. Recognize acceleration as the instantaneous rate of change of velocity with respect to time:

$$a(t) = v'(t) = s''(t)$$

4. Consider rates of change of populations.

Study 3.4, p 267-270 motion, population;
151 -157, 165 a, b,c

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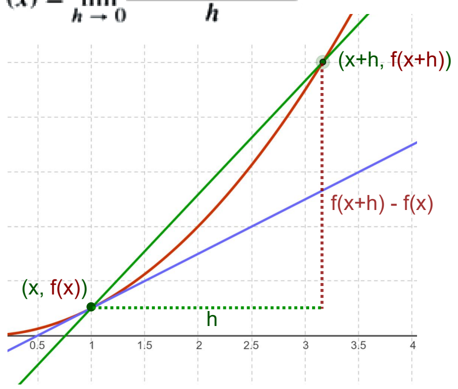
3.2 Derivative of a Function

Definition

Let f be a function. The **derivative function**, denoted by f', is the function whose domain consists of those values of x such that the following limit exists:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad (3.9)$$



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3.4 Derivative as a Rate of Change

1. Recognize the derivative as a rate of change of one variable with respect to another.

Average Rate of Change on Interval	vs	Instantaneous Rate of Change
Slope of Secant Line		Slope of Tangent Line
eg: $\frac{\text{total distance}}{\text{total time}}$		eg: velocity at specific time
average population over last 10 years		rate of population growth at a particular time:

These are examples. There are many more.

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Homework Part 1

3.4 Derivative as a Rate of Change

2. Recognize velocity as the instantaneous rate of change of position with respect to time: v(t) = s '(t)
3. Recognize acceleration as the instantaneous rate of change of velocity with respect to time: a(t) = v '(t) = s "(t)

Motion: Position, Velocity, Acceleration

Definition

Let $s(t)$ be a function giving the position of an object at time t.

The velocity of the object at time t is given by $v(t) = s'(t)$.

The speed of the object at time t is given by $|v(t)|$.

The acceleration of the object at t is given by $a(t) = v'(t) = s''(t)$.

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3.4 Derivative as a Rate of Change

2. Recognize velocity as the instantaneous rate of change of position with respect to time: v(t) = s '(t)

Motion: Position, Velocity, Acceleration

156. The position of a hummingbird flying along a straight line in t seconds is given by $s(t) = 3t^3 - 7t$ meters.

a. Determine the velocity of the bird at $t = 1$ sec.
b. Determine the acceleration of the bird at $t = 1$ sec.
c. Determine the acceleration of the bird when the velocity equals 0.

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3.4 Derivative as a Rate of Change

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a. Determine the velocity of the bird at $t = 1$ sec.
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c. Determine the acceleration of the bird when the velocity equals 0.

$s(t)= 3t^3 - 7t$

a) $v(t)=s'(t)= 9t^2 - 7$

$v(1)= 9-7 = 2$ m/s

b) $a(t)=v'(t)=s''(t)= 18t$

$a(1)= 18$ m/s/s

c) $v(t)=s'(t)= 9t^2 - 7 = 0$

$9t^2 = 7$, $t^2 = 7/9$ $t = \pm\sqrt{7/9} = +\sqrt{7}/3$ sec

$a(\sqrt{7}/3) = 18\sqrt{7}/3 = 6\sqrt{7}$ m/s/s

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3.4 Derivative as a Rate of Change

Motion: Speeding Up (SU) and Slowing Down (SD)

I am taking a trip to Montauk. What happens to my motion as I back out of my driveway to begin the trip?

	v(t)	a(t)	motion
Car parked			
Start backing up			
Beginning to stop on road			
Stop on road			
Drive to Highway			
Getting ready to turn			

0 0

0 0

− −

− −

+ +

+ +

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3.4 Derivative as a Rate of Change

Motion: Speeding Up (SU) and Slowing Down (SD)

I am taking a trip to Montauk. What happens to my motion as I back out of my driveway to begin the trip?

	v(t)	a(t)	motion
Car parked	0	0	
Start backing up	−	−	
Beginning to stop on road	−	+	
Stop on road	0	0	
Drive to Highway	+	+	
Getting ready to turn	+	−	

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3.4 Derivative as a Rate of Change

Motion: Speeding Up (SU) and Slowing Down (SD)

I am taking a trip to Montauk. What happens to my motion as I back out of my driveway to begin the trip?

	$v(t)$	$a(t)$	motion
Car parked	0	0	**none**
Start backing up	−	−	SU
Beginning to stop on road	−	+	SD
Stop on road	0	0	**none**
Drive to Highway	+	+	SU
Getting ready to turn	+	−	SD

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3.4 Derivative as a Rate of Change

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Car parked	0	0	**none**
Start backing up	−	−	SU
Beginning to stop on road	−	+	SD
Stop on road	0	0	**none**
Drive to Highway	+	+	SU
Getting ready to turn	+	−	SD

Speeds Up (SU) when ***v*** and ***a*** are the ***same sign***.

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3.4 Derivative as a Rate of Change

Motion: Speeding Up (SU) and Slowing Down (SD)

I am taking a trip to Montauk. What happens to my motion as I back out of my driveway to begin the trip?

	$v(t)$	$a(t)$	motion
Car parked	0	0	**none**
Start backing up	−	−	SU
Beginning to stop on road	−	+	SD
Stop on road	0	0	**none**
Drive to Highway	+	+	SU
Getting ready to turn	+	−	SD

Slows Down (SD) when ***v*** and ***a*** have ***opposite signs***.

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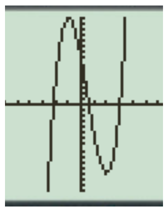
3.4 Derivative as a Rate of Change

2. Recognize velocity as the instantaneous rate of change of position with respect to time: v(t) = s '(t)

Motion: Position, Velocity, Acceleration



For the following exercises, the given functions represent the position of a particle traveling along a horizontal line.

a. Find the velocity and acceleration functions.

b. Determine the time intervals when the object is slowing down or speeding up.

150. $s(t) = 2t^3 - 3t^2 - 12t + 8$

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3.4 Derivative as a Rate of Change

Motion: Position, Velocity, Acceleration

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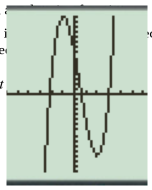

a) $v(t) = s'(t) = 6t^2 - 6t - 12$

$a(t) = v'(t) = s''(t) = 12t - 6$

b) slowing down when v and a have opposite signs
speeding up wheen v and a have the same signs
∴ need to know the intervals where each is + and where each is -

$v(t) = 6(t^2 - t - 2) = 6(t-2)(t+1)$

∴ v changes sign at $t = -1, 2$

$a(t) = 6(2t-1)$ ∴ a changes sign at $t = \frac{1}{2}$

	-1	1/2	2	
v(t)	+	−	−	+
a(t)	−	−	+	+
s(t)	slow	faster	slower	faster

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3.4 Derivative as a Rate of Change

Motion: Position, Velocity, Acceleration

150. $s(t) = 2t^3 - 3t^2 - 12t + 8$

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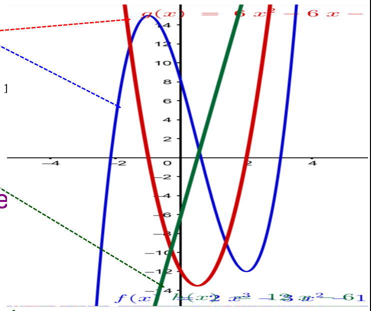

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$a(t) = 6(2t-1)$ ∴ a changes sign at $t = \frac{1}{2}$

v(t)	+	−	−	+
a(t)	−	−	+	+
s(t)	slow	fast	slow	fast
	-1	1/2	2	

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3.4 Derivative as a Rate of Change

164. A small town in Ohio commissioned an actuarial firm to conduct a study that modeled the rate of change of the town's population. The study found that the town's population (measured in thousands of people) can be modeled by the function $P(t) = -\frac{1}{3}t^3 + 64t + 3000$, where t is measured in years.

a. Find the rate of change function $P'(t)$ of the population function.

b. Find $P'(1)$, $P'(2)$, $P'(3)$, and $P'(4)$. Interpret what the results mean for the town.

c. Find $P''(1)$, $P''(2)$, $P''(3)$, and $P''(4)$. Interpret what the results mean for the town's population.

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c. Find $P''(1)$, $P''(2)$, $P''(3)$, and $P''(4)$. Interpret what the results mean for the town's population.

$P(t) = -\frac{1}{3}t^3 + 64t + 3000$

a) $P'(t) = -t^2 + 64$

b) $P'(1) = -1 + 64 = 63$/yr

$P'(2) = -4 + 64 = 60$/yr

$P'(3) = -9 + 64 = 55$/yr

$P'(4) = -16 + 64 = 48$/yr

For the next 4 years, the number of people added per year is declining.

c) $P''(t) = -2t$

$P''(1) = -2(1) = -2$/yr/yr

$P''(2) = -2(2) = -4$/yr/yr

$P''(3) = -2(3) = -6$/yr/yr

$P''(4) = -2(4) = -8$/yr/yr

The rate of population change is declining at the rate of 2000 $\frac{\text{people/yr}}{\text{yr}}$

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