GOALS:

- 1. For functions that are not simple, recognize those with products and/or quotients.
- 2. Learn the Product Rule and when to use it.
- 3. Learn the Quotient Rule and when to use it.

Study 3.3 # 111, 117, 139, 143-147

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derivative practice

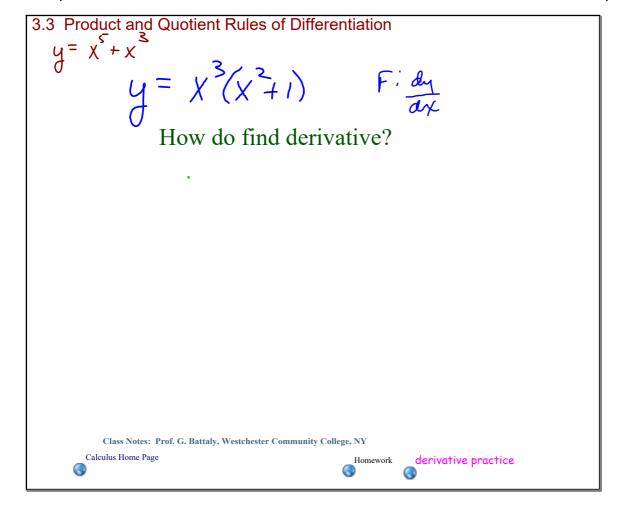
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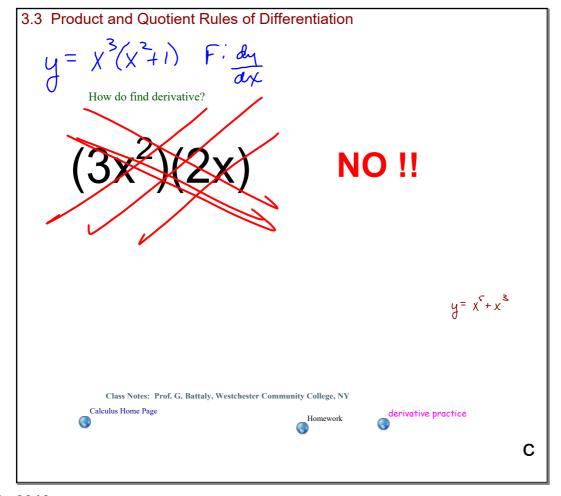
3.3 Product and Quotient Rules of Differentiation

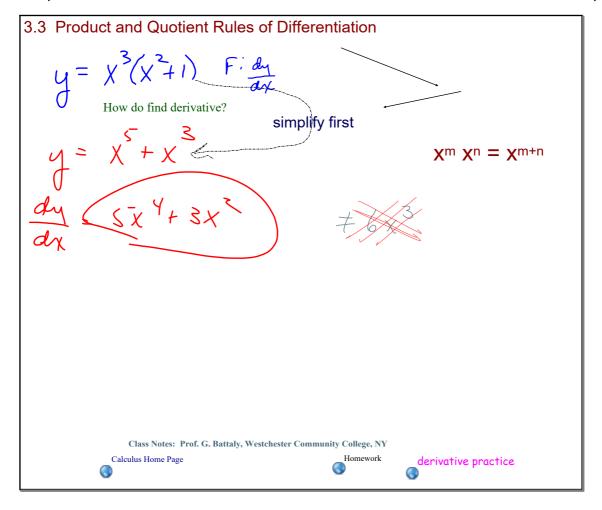
Fidy dx

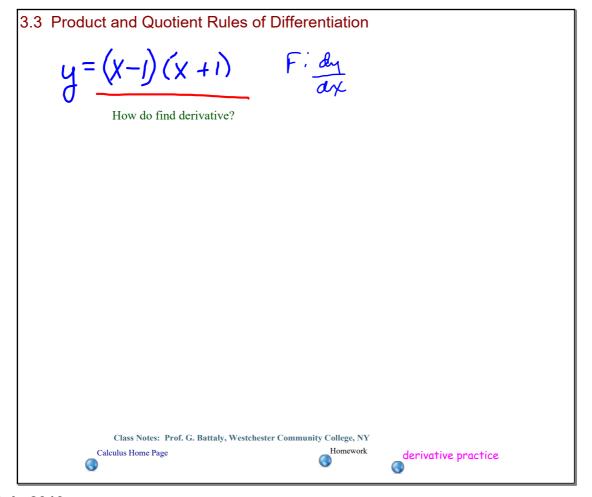
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derivative practice









$$y = (x-1)(x+1)$$
 Fidy

$$y = x^2 - 1$$

$$\frac{dy}{dx} = \lambda x$$

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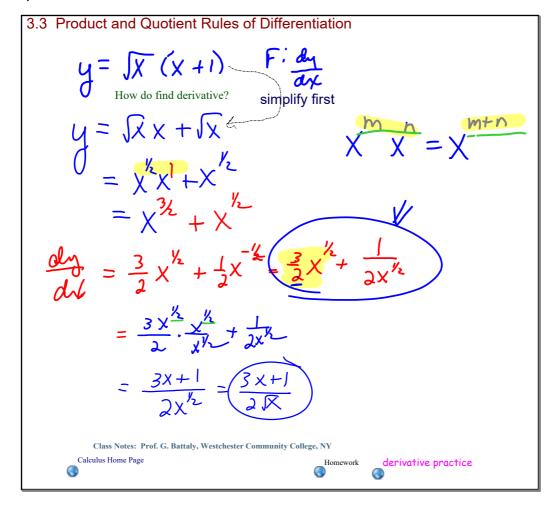
derivative practice

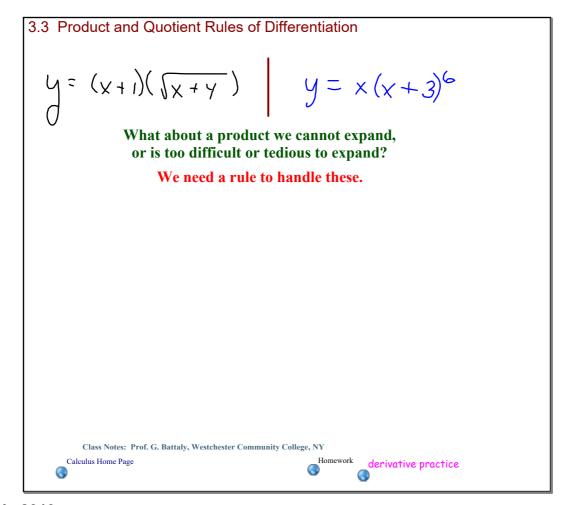
3.3 Product and Quotient Rules of Differentiation

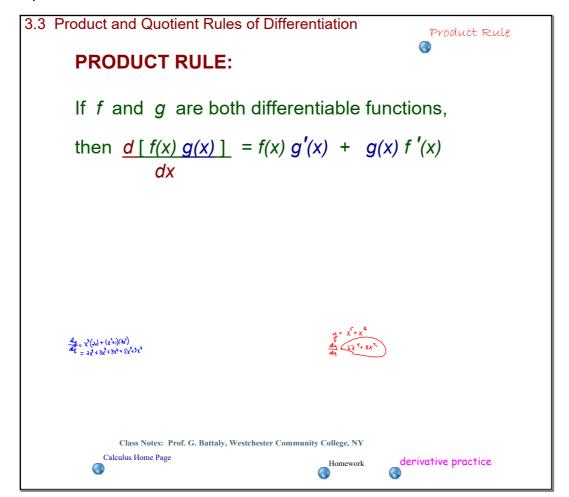
$$y = \int X (x + 1)$$
 Findy

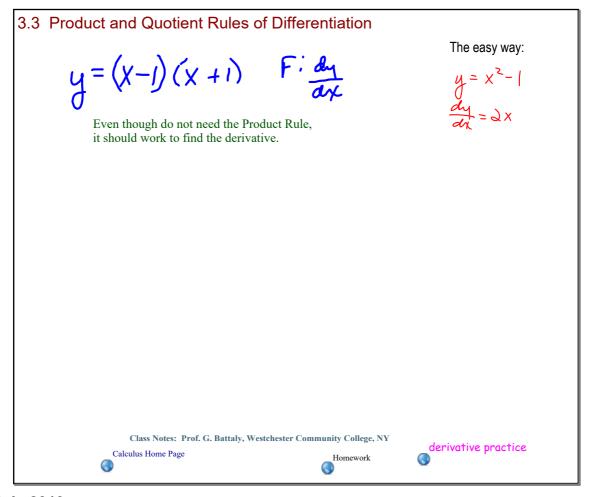
How do find derivative?

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y = (x-1)(x+1) Figure

The easy way:

$$y = x^2 - 1$$

$$dy = 2x$$

Even though do not need the Product Rule, it should work to find the derivative.

$$\frac{dy}{dx} = (x - 1) (1) + (x + 1) (1)$$

$$\frac{dy}{dx} = x - 1 + x + 1 = 2x$$

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derivative practice

3.3 Product and Quotient Rules of Differentiation

 $y = \chi^{3} \left(\chi^{2} + 1\right) \qquad \text{Fidular}$ $\frac{d \left[f(x) g(x) \right]}{d(x)} \cdot = f(x) g'(x) + g(x) f'(x)$

Product Rule

 $= 5X_4 + 3X_4 + 3X_5 = 2X_4 + 3X_5$ = $X_3(5x) + (X_7 + 1)(3x_7)$

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derivative practice 0

3.3 Product and Quotient Rules of Differentiation

$$y = \chi^{3}(\chi^{2}+1) \qquad F : d_{1}$$

$$\frac{d[f(x)g(x)]}{dx} = f(x)g'(x) + g(x)f'(x)$$

$$\frac{d}{dx} = \chi^{3}(2x) + (\chi^{3}+1)(3x^{3})$$

$$= 2\chi^{4} + 3\chi^{4} + 3\chi^{2}$$

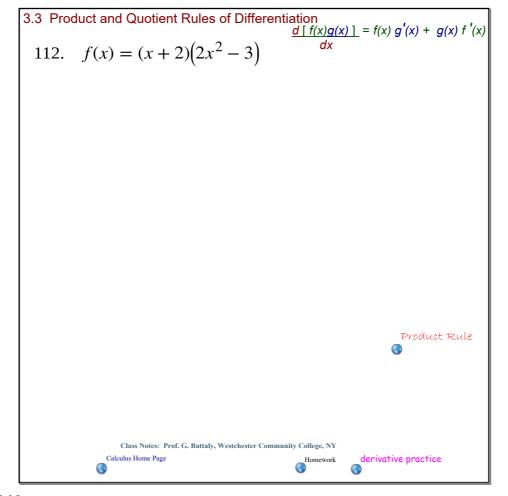
$$= 5\chi^{4} + 3\chi^{2}$$
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derivative practice



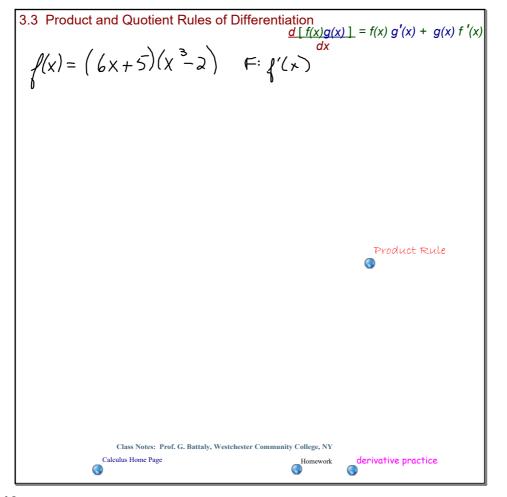
3.3 Product and Quotient Rules of Differentiation
$$\frac{d [f(x)g(x)]}{dx} = f(x) g'(x) + g(x) f'(x)$$
112. $f(x) = (x+2)(2x^2-3)$

$$f'(x) = (x+2) d(2x-3) + (2x-3) d(x+2)$$

$$= (x+2) (4x) + (2x^2-3) (1)$$

$$= 4x^2 + 8x + 2x^2 - 3$$

$$= (6x^2 + 8x - 3)$$
expanding:
$$f(x) = 2x^3 + 4x^2 - 3x - 6$$
Product Rule



3.3 Product and Quotient Rules of Differentiation
$$\frac{d[f(x)g(x)]}{dx} = f(x) g'(x) + g(x) f'(x)$$

$$f(x) = (6x+5)(x^3-2) \iff f'(x)$$

$$= (6x+5)(x^3-2) + (x^3-2) d(6x+5)$$

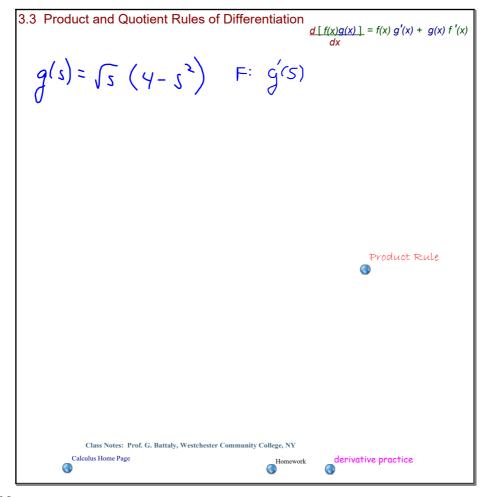
$$= (6x+5)(3x^2) + (x^3-2)(6)$$

$$= (6x+5)(3x+6)(3x+6)$$

$$= (6x+6)(3x+6)(3x+6)$$

$$= (6x+6)(3x+6)$$

$$= (6x+6)($$



3.3 Product and Quotient Rules of Differentiation

$$\frac{d}{d} (x) = \sqrt{3} \left(\frac{1}{4} - \frac{3}{4} \right) = f(x) g'(x) + g(x) f'(x)$$

$$= \sqrt{3} \left(\frac{1}{4} - \frac{3}{4} \right) = f(x) g'(x) + g(x) f'(x)$$

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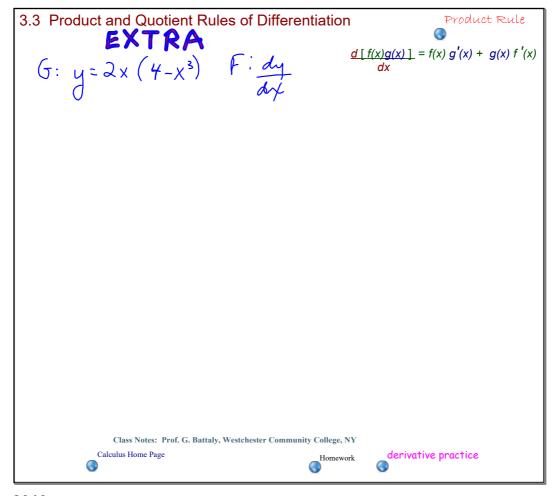
$$= \sqrt{3} \left(\frac{1}{4} - \frac{3}{4} \right) = f(x) g'(x) + g(x) f'(x)$$

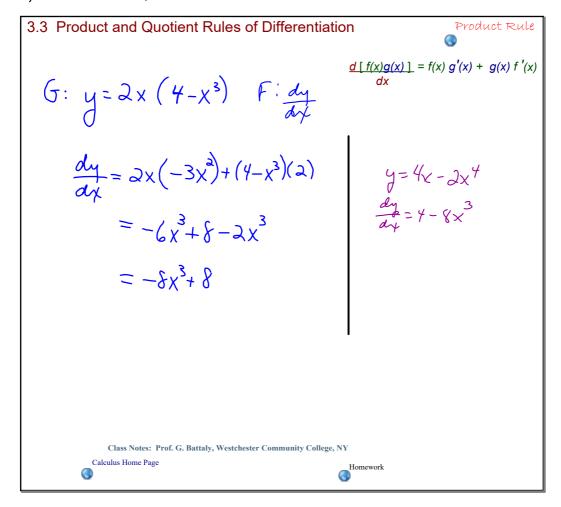
$$= \sqrt{3} \left(\frac{1}{4} - \frac{3}{4} \right)$$

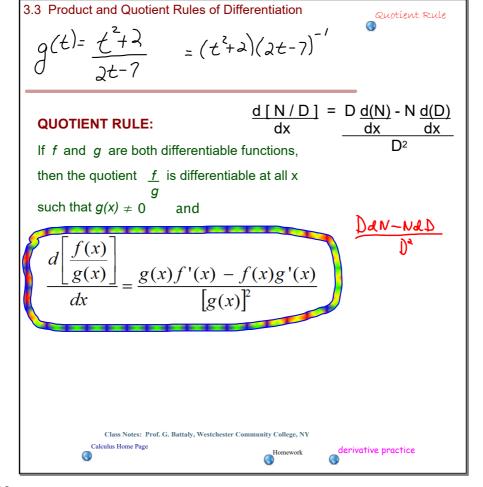
$$= -\sqrt{3} \left(\frac{1}{4} - \frac{3}{4} \right) = f(x) g'(x) + g(x) f'(x)$$

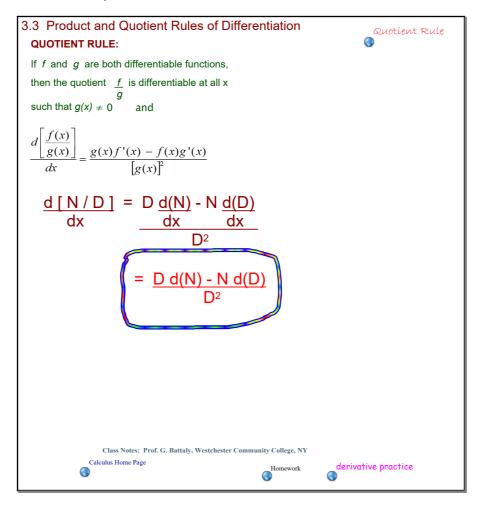
$$= \sqrt{3} \left(\frac{1}{4} - \frac{3}{4} \right)$$

$$= -\sqrt{3} \left(\frac{1}{4} - \frac{3}{4} \right)$$









$$\frac{d \left[\frac{f(x)}{g(x)} \right]}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x) \right]^2}$$

$$= \frac{D \, dN - N \, dD}{D^2}$$
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$$\int_{D}^{A} (x) = \frac{x}{x+1}$$

$$\int_{D}^{A} \frac{d \left[\frac{f(x)}{g(x)} \right]}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^{2}}$$

$$= \frac{D dN - N dD}{D^{2}}$$

$$\int_{D}^{A} (x) = \frac{(x+1)(1) - x(1)}{(x+1)^{2}}$$

$$= \frac{x+1-x}{(x+1)^{2}}$$

using definition of derivative: shown to appreciate the rule as being less complex than the definition to find the derivative

$$\frac{1}{(x)} = \frac{x}{x+1}$$

$$\frac{1}{(x+1)} = \frac{x+1}{(x+1)} = \frac{x+1}{(x+1)}$$

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$$g(t) = \frac{t^2 + 2}{2t - 7}$$

Quotient Rule
$$\frac{d\left[\frac{f(x)}{g(x)}\right]}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

$$= \frac{D dN - N dD}{D^2}$$

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3.3 Product and Quotient Rules of Differentiation
$$h(t) = \frac{t^2 + \lambda}{\lambda t - 7}$$

$$d \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

$$h'(t) = \frac{(\lambda t - 7)\lambda(t^2 + \lambda) - (t^2 + \lambda)\lambda(\lambda t - 7)}{(\lambda t - 7)\lambda} = \frac{D dN - N dD}{(\lambda t - 7)\lambda}$$

$$= \frac{(\lambda t - 7)(\lambda t) - (t^2 + \lambda)(\lambda)}{(\lambda t - 7)\lambda}$$

$$= \frac{(\lambda t - 7)(\lambda t) - (t^2 + \lambda)(\lambda)}{(\lambda t - 7)\lambda}$$

$$= \frac{(\lambda t - 7)(\lambda t) - (t^2 + \lambda)(\lambda t)}{(\lambda t - 7)\lambda}$$

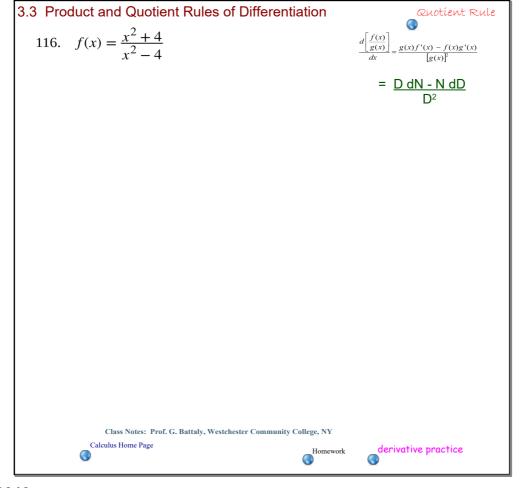
$$= \frac{(\lambda t - 7)(\lambda t) - (t^2 + \lambda)(\lambda t)}{(\lambda t - 7)\lambda}$$
no common factors; cannot reduce to lowest terms (simplify)

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using definition of derivative: shown to appreciate the rule as being less complex than the definition to find the derivative

$$\frac{1'(x) = \frac{1}{2} \cdot \frac{1}{2$$



3.3 Product and Quotient Rules of Differentiation

$$116. \quad f(x) = \frac{x^2 + 4}{x^2 - 4}$$

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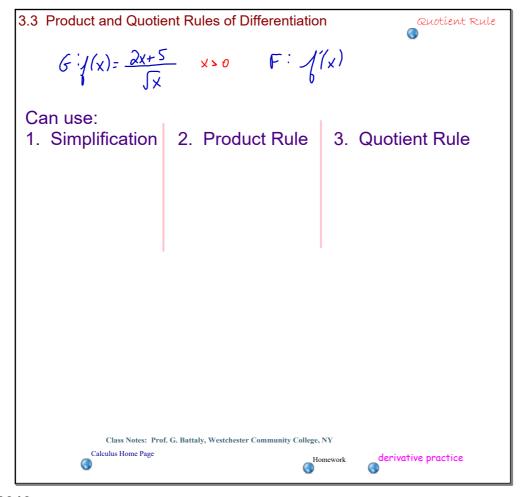
$$116. \quad f(x) = \frac{x^2 + 4}{x^2 - 4}$$

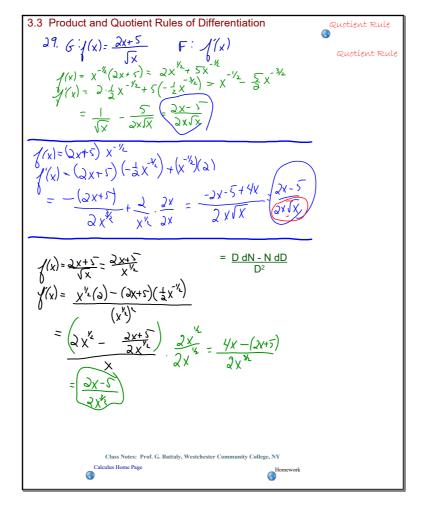
$$116. \quad f(x) = \frac{x^2 + 4}{x^2 - 4}$$

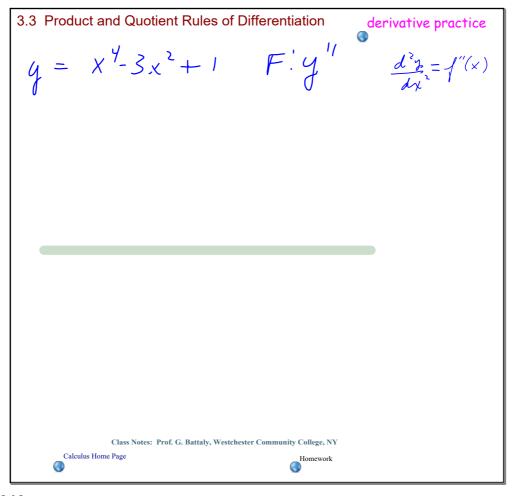
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$$116. \quad f(x) =$$







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3.3 Product and Quotient Rules of Differentiation
$$q = \chi^{4} - 3\chi^{2} + 1 \quad F \cdot y^{11} \qquad \frac{d^{3}\chi}{dx} = f''(x)$$

$$\frac{d^{3}\chi}{dx} = 4\chi^{3} - 6\chi$$

$$\frac{d^{3}\chi}{dx} = 12\chi^{2} - 6$$

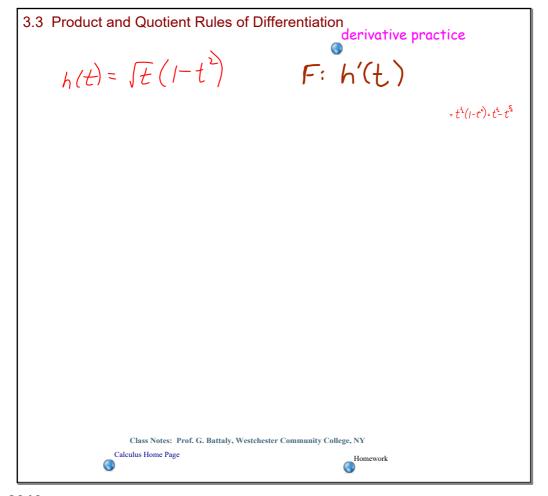
$$\frac{d^{3}\chi}{dx} = 24\chi$$

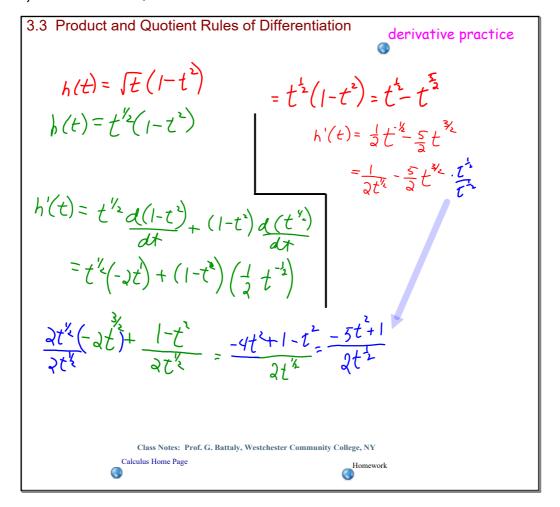
$$\frac{d^{3}\chi}{dx} = 24\chi$$
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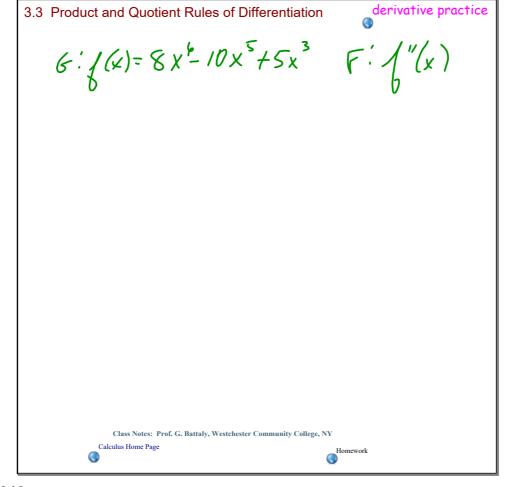
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$$G: f(x) = 8x^{4} - 10x^{5} + 5x^{3}$$

$$f'(x) = 48x^{5} - 50x^{4} + 15x^{2}$$

$$f''(x) = 240x^{4} - 200x^{3} + 30x$$

G: $\int (x) = x \left(1 - \frac{4}{x+3}\right)$ F: $\int (x)$

derivative practice

Quotient Rule

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