Ch 3.3 Basic Rules of Differentiation

GOALS:

Find the derivative of a function using basic rules:

1. Constant Rule
2. Power Rule
3. Sum & Difference Rules
4. Combination of Rules

Study 3.3

#107,109,113,115,119,121,133-139,144,149

(for sketch use calculator)

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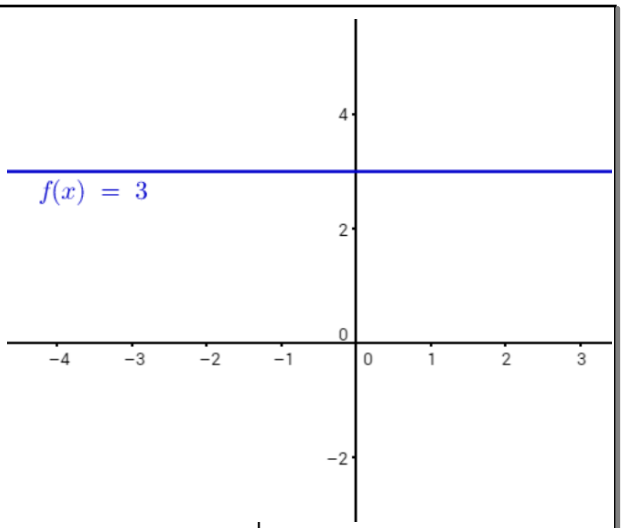
Homework Part 1

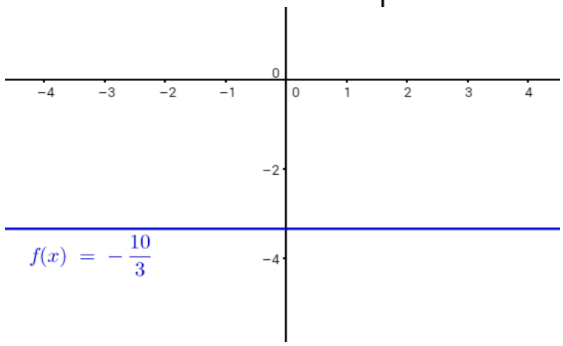
Ch 3.3 Basic Rules of Differentiation

G: f(x) = 3 F: f ' (x)



G: f(x) = 3/2 F: f ' (x) ?

Does it matter if f(x) = -10/3?
or f(x) = k?



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Homework Part 1

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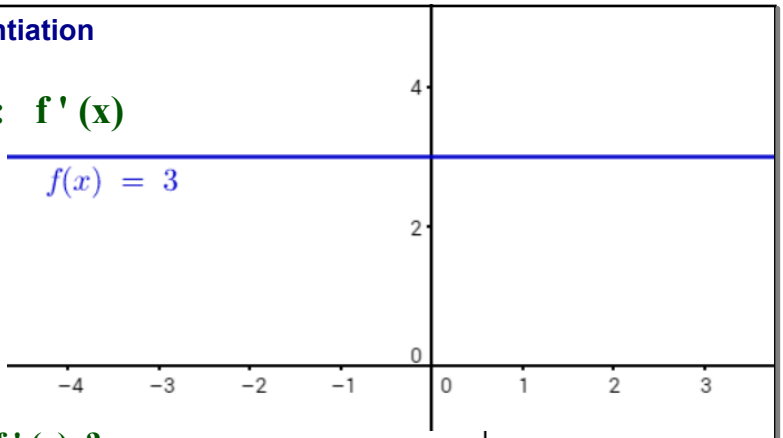
G: f(x) = 3 **F: f ' (x)**

$f'(x) = 0$



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Does it matter if f(x) = -10/3?
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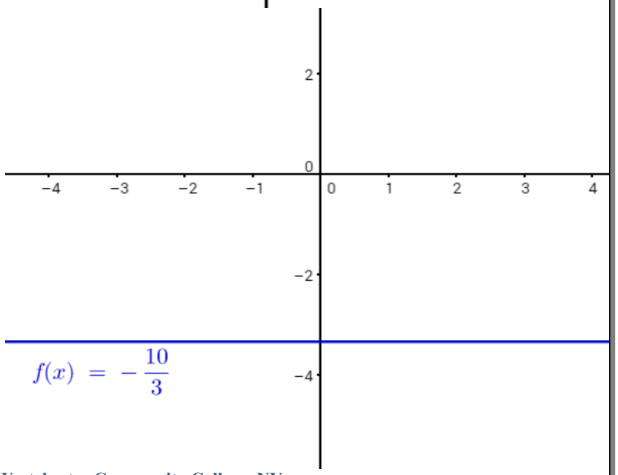

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G: f(x) = 3 - 5x **F: f ' (x)**

What if $f(x) = 3 + 2x$? $f'(x) =$ _____

What if $f(x) = 3x + 2$? $f'(x) =$ _____

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G: f(x) = 3 - 5x **F: f ' (x)**

$y = mx + b$

$f'(x) = -5$

or simply, straight line has same slope, m, for all x where y = mx + b

What if $f(x) = 3 + 2x$? $f'(x) =$ 2

What if $f(x) = 3x + 2$? $f'(x) =$ 3

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Can you see **any patterns** in finding the derivative?

$f(x) = 3$	$f'(x) = 0$
$f(x) = 3 - 5x$	$f'(x) = -5$
$f(x) = 3x + 2$	$f'(x) = 3$
$f(x) = x^2$	$f'(x) = 2x$
$f(x) = 4x^2$	$f'(x) = 8x$
$f(x) = x^2 + 3$	$f'(x) = 2x$
$f(x) = x^2 + 2x + 1$	$f'(x) = 2x + 2$
$f(x) = 5x - x^2$	$f'(x) = 5 - 2x$

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Can you find the derivative?

$f(x) = 3$	$f'(x) =$
$f(x) = 3 - 5x$	$f'(x) =$
$f(x) = 3x + 2$	$f'(x) =$
$f(x) = x^2$	$f'(x) =$
$f(x) = 4x^2$	$f'(x) =$
$f(x) = x^2 + 3$	$f'(x) =$
$f(x) = x^2 + 2x + 1$	$f'(x) =$
$f(x) = 5x - x^2$	$f'(x) =$

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General Formulas

1. $\frac{d}{dx}(c) = 0$
2. $\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$
3. $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$
4. $\frac{d}{dx}(x^n) = nx^{n-1}$, for real numbers n
5. $\frac{d}{dx}(cf(x)) = cf'(x)$
6. $\frac{d}{dx}(f(x) - g(x)) = f'(x) - g'(x)$
7. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$
8. $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

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Homework Part 1

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Rules of Differentiation

1. The Constant Rule $\frac{d[c]}{dx} = 0$

$f(x) = 3$

$f'(x) = 0$

2. The Power Rule $\frac{d[x^n]}{dx} = nx^{n-1}$ $\quad f(x) = x^3$

$f'(x) = 3x^2$

3. The Constant Multiple Rule $\frac{d[cf(x)]}{dx} = cf'(x)$ $\quad f(x) = 4x^2$

$f'(x) = 4(2x) = 8x$

4. The Sum & Difference Rule

$$\frac{d[f(x) \pm g(x)]}{dx} = f'(x) \pm g'(x)$$

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$f(x) = x^2 + 2x + 1$

$f'(x) = 2x + 2$

$y = 3x^4 - 2x^2 + x - 7$

$\frac{dy}{dx} = 3(4x^3) - 2(2x) + 1$

$= 12x^3 - 4x + 1$

$\frac{dy}{dx}$

$f'(x) \quad f''(x) \quad f'''(x)$

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geogebra Derivative Practice

$\frac{d[x^n]}{dx} = nx^{n-1}$

$f(x) = -2$ $\quad f'(x) =$

$f(x) = x$ $\quad f'(x) =$

$f(x) = x^8$ $\quad f'(x) =$

$f(x) = \frac{1}{x^8} = x^{-8}$ $\quad f'(x) =$

$f(x) = 2x^3 - x^2 + 3x$ $\quad f'(x) =$

$f(x) = x^{1/2}$ $\quad f'(x) =$

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Ch 3.3 Basic Rules of Differentiation

geogebra Derivative Practice

$f(x) = -2$ $\quad f'(x) = 0$

$f(x) = x$ $\quad f'(x) = 1$

$f(x) = x^8$ $\quad f'(x) = 8x^7$

$\frac{d[x^n]}{dx} = nx^{n-1}$

$f(x) = \frac{1}{x^8} = x^{-8}$ $\quad f'(x) = -8x^{-9}$

$f(x) = 2x^3 - x^2 + 3x$ $\quad f'(x) = 2(3x^2) - 2x^1 + 3(1x^0)$

$f'(x) = 6x^2 - 2x + 3$

$f(x) = x^{1/2}$ $\quad f'(x) = (1/2)x^{-1/2} = \frac{1}{2x^{1/2}}$

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$f(x) = \frac{1}{x^3}$ F: $f'(x)$

$f(x) = x^{-3}$

$f'(x) = -3x^{-3-1} = -3x^{-4} = \frac{-3}{x^4}$

$y = \frac{1}{x^2}$ F: dy/dx

$y = x^{-2}$ $\frac{dy}{dx} = -2x^{-2-1} = -2x^{-3} = \frac{-2}{x^3}$

$y = \frac{2}{x^3} = 2x^{-3}$

$\frac{dy}{dx} = 2(-3x^{-3-1}) = -6x^{-4} = \frac{-6}{x^4}$

$y = x^2\left(\frac{2}{x^2} + \frac{5}{x^3}\right) = 2 + \frac{5x^2}{x^3} = 2 + \frac{5}{x}$

$y = 2 + 5x^{-1}$

$\frac{dy}{dx} = 0 + 5(-x^{-1-1}) = -5x^{-2} = \frac{-5}{x^2}$

$y = x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$

$= \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

$\frac{d[x^n]}{dx} = nx^{n-1}$

$y = x^{\frac{1}{3}}$ F: $\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}}$

$= \frac{1}{3x^{\frac{2}{3}}}$

$-\frac{2}{3} - 1$

$-\frac{2}{3} - \frac{3}{3} = -\frac{5}{3}$

F: $\frac{d^2y}{dx^2} = \frac{1}{3}\left(-\frac{2}{3}x^{-\frac{2}{3}-1}\right)$

$= -\frac{2}{9}x^{-\frac{5}{3}} = \frac{-2}{9x^{5/3}}$

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$\frac{d[x^n]}{dx} = nx^{n-1}$

$$y = \frac{2}{3x^2}$$

$$y = \frac{\pi}{(3x)^2}$$

Ch 3.3 Basic Rules of Differentiation

$\frac{d[x^n]}{dx} = nx^{n-1}$

$$y = \frac{2}{3x^2} = \frac{2}{3}x^{-2}$$

$$\frac{dy}{dx} = \frac{2}{3}[-2x^{-2-1}] = \frac{2}{3}[-2x^{-3}] = \frac{-4}{3x^3}$$

$$y = \frac{\pi}{(3x)^2} = \frac{\pi}{9x^2} = \frac{\pi}{9}x^{-2}$$

$$\frac{dy}{dx} = \frac{\pi}{9}[-2x^{-3}] = \frac{-2\pi}{9x^3}$$

Ch 3.3 Basic Rules of Differentiation

$\frac{d[x^n]}{dx} = nx^{n-1}$

108. $f(x) = 4x^2 - 7x$

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110. $f(x) = x^4 + \frac{2}{x} = x^4 + 2x^{-1}$

Ch 3.3 Basic Rules of Differentiation

$\frac{d[x^n]}{dx} = nx^{n-1}$

108. $f(x) = 4x^2 - 7x$

$f'(x) = 8x - 7$

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110. $f(x) = x^4 + \frac{2}{x}$

$f(x) = x^4 + 2x^{-1}$

$f'(x) = 4x^3 + 2(-x^{-2}) = 4x^3 - \frac{2}{x^2}$

Ch 3.3 Basic Rules of Differentiation

$\frac{d[x^n]}{dx} = nx^{n-1}$

For the following exercises, find the equation of the tangent line $T(x)$ to the graph of the given function at the indicated point. Use a graphing calculator to graph the function and the tangent line.

118. **[T]** $y = 3x^2 + 4x + 1$ at $(0, 1)$

Ch 3.3 Basic Rules of Differentiation

For the following exercises, find the equation of the tangent line $T(x)$ to the graph of the given function at the indicated point. Use a graphing calculator to graph the function and the tangent line.

$\frac{d[x^n]}{dx} = nx^{n-1}$

$y - y_1 = m_T(x - x_1)$

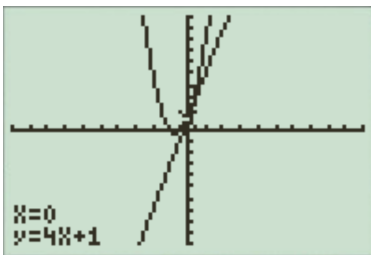
118. **[T]** $y = 3x^2 + 4x + 1$ at $(0, 1)$

$m_T = f'(x) = 6x + 4$

$m_T = f'(0) = 6(0) + 4 = 4$

$y - 1 = 4(x - 0) = 4x$

$y = 4x + 1$

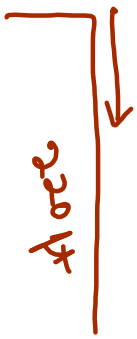


Ch 3.3 Basic Rules of Differentiation

G: $s(t) = -16t^2 + v_0t + s_0$

G: A ball is thrown straight down from the top of a 220 ft. building with an initial velocity of -22 ft/sec.

F: What is its velocity after 3 sec? after falling 200 ft?



Ch 3.3 Basic Rules of Differentiation

G: $s(t) = -16t^2 + v_0t + s_0$

G: A ball is thrown straight down from the top of a 220 ft. building with an initial velocity of -22 ft/sec.

F: What is its velocity after 3 sec? after falling 20 ft?

$s(t) = -16t^2 + v_0t + s_0 = -16t^2 - 22t + 220$

$v(t) = s'(t) = -16(2t) - 22 + 0$
$= -32t - 22$

$v(t) = -32t - 22$

$v(3) = -32(3) - 22 = -118$ ft/sec

need t when s=200 ft.

$s(t) = -16t^2 - 22t + 220 = 200$

$-16t^2 - 22t + 20 = 0$

$8t^2 + 11t - 10 = 0$

Consider - or +

80 = (2)(40)	38or42
(4)(20)	18or24
(8)(10)	2 or 18
(16)(5)	11or21

(-5) (16) = -80

(-5) + (16) = 11

$8t^2 + 16t - 5t - 10 = 0$

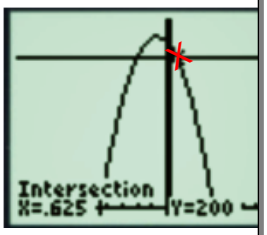
$8t(t + 2) - 5(t + 2) = 0$

$(8t - 5)(t + 2) = 0$

$t = 5/8, -2$ no meaning

$v(t) = -32t - 22$

$v(5/8) = -32(5/8) - 22$
$= -4(5) - 22$
$= -42$ ft/sec



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Ch 3.3 Basic Rules of Differentiation

142. Determine all points on the graph of $f(x) = x^3 + x^2 - x - 1$ for which the slope of the tangent line is

a. horizontal

b. -1.

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Ch 3.3 Basic Rules of Differentiation

142. Determine all points on the graph of $f(x) = x^3 + x^2 - x - 1$ for which the slope of the tangent line is

a. horizontal

b. -1.

$m_T = f'(x) = 3x^2 + 2x - 1$

horizontal means $m_T = 0$

$(3x-1)(x+1) = 0$

$3x-1=0 \mid x+1=0$

$x=1/3 \mid x=-1$

$(1/3, -32/27) \mid (-1,0)$

$\frac{1}{27} + \frac{1}{9} - \frac{1}{3} - 1 \mid -1 + 1 + 1 - 1$

$\frac{1+3-9-27}{27} \mid -1 + 1 + 1 - 1 = 0$

$m_T = -1$

$3x^2+2x-1 = -1$

$3x^2+2x = 0$

$x(3x+2) = 0$

$x=0 \mid 3x+2=0$

$x=0 \mid x=-2/3$

$(0,-1) \mid (-2/3, -5/27)$

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Homework Part 1

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geogebra Derivative Practice

$y = 3x^3 - 6$ Find slope of graph at $(2, 18)$

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Homework Part 1

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Ch 3.3 Basic Rules of Differentiation

geogebra Derivative Practice

$y = 3x^3 - 6$ F: m of graph at $(2, 18)$

$\frac{dy}{dx} = 3(3x^2) + 0 = 9x^2$

$\left.\frac{dy}{dx}\right|_{(2,18)} = 9(2)^2 = 9(4) = 36$

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G: $y = x^3 - 3x$ F: a) eq tang. line at $(2, 2)$

Homework Part 1

Ch 3.3 Basic Rules of Differentiation

G: $y = x^3 - 3x$

F: a) eq tang. line at $(2,2)$

$m_T = \frac{dy}{dx} = 3x^2 - 3$

$\left.\frac{dy}{dx}\right|_{(2,2)} = 3(2^2) - 3 = 12 - 3 = 9$

$y = m_T x + b$

$y - y_1 = m_T(x - x_1)$

$y - 2 = m_T(x - 2)$

$y - 2 = 9(x - 2)$

$y - 2 = 9x - 18$

$y = 9x - 16$

Homework Part 1

Ch 3.3 Basic Rules of Differentiation

Average Rate of Change on Interval	vs	Instantaneous Rate of Change
Slope of Secant Line		Slope of Tangent Line
eg: $\frac{\text{total distance}}{\text{total time}}$		eg: velocity at specific time
speed, average velocity		instantaneous velocity

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Ch 3.3 Basic Rules of Differentiation

Average Rate of Change on Interval

G: $f(x) = x^2 - 7$
F: average rate of change, [3,3.1]

$f(3) = 9 - 7 = 2$

Instantaneous Rate of Change

[3, 3.1]

G: $f(x) = x^2 - 7$
F: Instantaneous rates of change at endpoints.

Ch 3.3 Basic Rules of Differentiation

Average Rate of Change on Interval

G: $f(x) = x^2 - 7$
F: average rate of change, [3,3.1]

$$Ave = \frac{f(3.1) - f(3)}{3.1 - 3}$$

$$= \frac{(3.1^2 - 7) - (3^2 - 7)}{0.1}$$

$$= \frac{9.61 - 7 - 9 + 7}{0.1} = \frac{0.61}{0.1} = 6.1$$

Instantaneous Rate of Change

G: $f(x) = x^2 - 7$
F: Instantaneous rates of change at endpoints.

$f'(x) = 2x$

$f'(3) = 6$

$f'(3.1) = 6.2$

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Homework Part 1