3.2 Derivative of a Function

GOALS:

1. Understand the definition of the derivative of a function (limits).
2. Apply the definition to find derivatives of specific functions.
3. Understand when a derivative does not exist.
4. Understand the relationship between continuity and the existence of a derivative at a point.

Study 3.2 #55-67, 75, 79, 85, 96

Study definitions: Derivative of a Function (3.9)

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3.2 Derivative of a Function

FROM 3.1: DERIVATIVE AT A POINT
This can be interpreted as the slope of the line tangent to the curve at a specific point (c,f(c))

$$\boxed{m_T = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \text{ at a point } (c, f(c))}$$

Now, want to generalize to find the
DERIVATIVE OF A FUNCTION
Need to consider the general point (x,f(x))

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3.2 Derivative of a Function

Derivative of a Function

$(x, f(x))$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists

$f'(x)$

$\frac{dy}{dx}$

notation

This generalizes from a specific point (c,f(c)) to any point on the curve (x,f(x))

It can be interpreted as the slope of the tangent line to the curve at any point on the curve.

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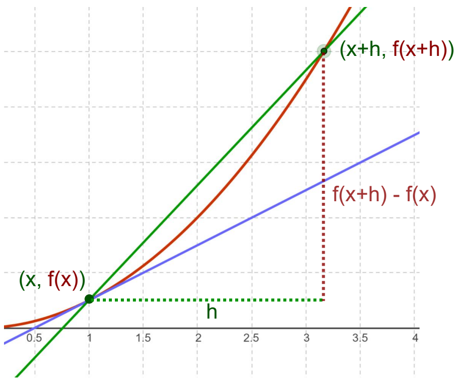
3.2 Derivative of a Function

Definition

Let f be a function. The **derivative function**, denoted by f', is the function whose domain consists of those values of x such that the following limit exists:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}. \quad (3.9)$$



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3.2 Derivative of a Function

58. $f(x) = 5x - x^2$ F: $f'(x)$

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3.2 Derivative of a Function

58. $f(x) = 5x - x^2$ F: $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{5(x+h) - (x+h)^2 - (5x - x^2)}{h}$$

$$= \lim_{h \to 0} \frac{5x + 5h - (x^2 + 2hx + h^2) - 5x + x^2}{h}$$

$$= \lim_{h \to 0} \frac{5h - x^2 - 2hx - h^2 + x^2}{h}$$

$$= \lim_{h \to 0} \frac{h(5 - 2x - h)}{h}$$

$$= \lim_{h \to 0} 5 - 2x - h = 5 - 2x$$

$$f'(x) = 5 - 2x$$

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3.2 Derivative of a Function

58. $f(x) = 5x - x^2$ F: $f'(x)$

$f'(x) = 5 - 2x$

Note: Derivative is a **function**

Compare to the derivative at a point, which is a particular numerical value.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{5(x+h) - (x+h)^2 - (5x - x^2)}{h}$$
$$= \lim_{h\to 0} \frac{5x + 5h - (x^2 + 2hx + h^2) - 5x + x^2}{h}$$
$$= \lim_{h\to 0} \frac{5h - x^2 - 2hx - h^2 + x^2}{h}$$
$$= \lim_{h\to 0} \frac{h(5 - 2x - h)}{h}$$
$$= \lim_{h\to 0} 5 - 2x - h = 5 - 2x$$

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3.2 Derivative of a Function

60. $f(x) = \sqrt{x - 6}$ $x \geq 6$ F: $f'(x)$

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3.2 Derivative of a Function

60. $f(x) = \sqrt{x-6}$ $\quad x \geq 6$ $\qquad$ F: $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h-6} - \sqrt{x-6}}{h} \cdot \frac{\sqrt{x+h-6} + \sqrt{x-6}}{\sqrt{x+h-6} + \sqrt{x-6}}$$

$$= \lim_{h \to 0} \frac{x+h-6-(x-6)}{h(\sqrt{x+h-6} + \sqrt{x-6})}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h-6} + \sqrt{x-6})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h-6} + \sqrt{x-6}} = \frac{1}{2\sqrt{x-6}}, \quad x > 6$$

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3.2 Derivative of a Function

60. $f(x) = \sqrt{x-6}$ $\quad x \geq 6$ $\qquad$ F: $f'(x)$

How is this different from **the derivative at a point** ?
(section 3.1)

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h-6} + \sqrt{x-6}} = \frac{1}{2\sqrt{x-6}}$$

$$f'(x) = \frac{1}{2\sqrt{x-6}}, \quad x > 6$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h-6} - \sqrt{x-6}}{h} \cdot \frac{\sqrt{x+h-6} + \sqrt{x-6}}{\sqrt{x+h-6} + \sqrt{x-6}}$$

$$= \lim_{h \to 0} \frac{x+h-6-(x-6)}{h(\sqrt{x+h-6} + \sqrt{x-6})}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h-6} + \sqrt{x-6})}$$

Applies to all x in the domain.

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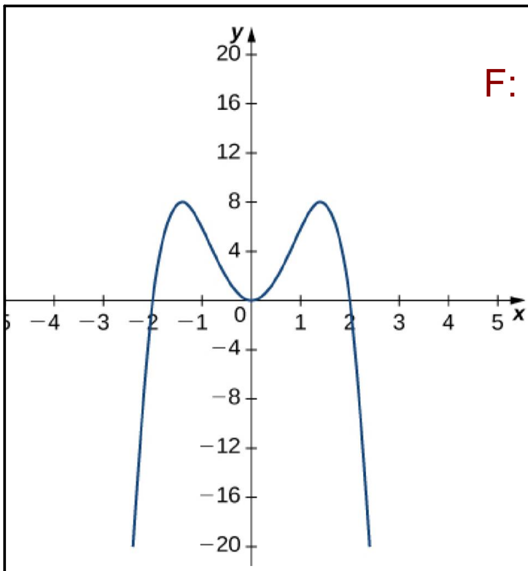

F: sketch the graph of *f '(x)*

f'(x)

slope of tangent line

1. remove secant line
2. expand axes
3 change to $y=-x^4 + 4x^2$

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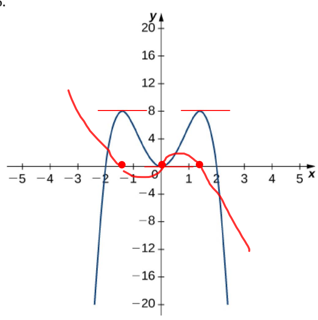
3.2 Derivative of a Function

66.



F: sketch the graph of *f '(x)*

Think about the slope of the tangent line.

Start with max & mins. -> slope=0
so f '(x)=0 at those values of x.
Next, where is the slope positive?
Where is the slope negative?
Can we expect f '(x) to be continuous?

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3.2 Derivative of a Function

76. $f(x) = \begin{cases} -x^2 + 2, & x \le 1 \\ x, & x > 1 \end{cases}$

F: a) sketch the graph
b) Show that not differentiable at x=1

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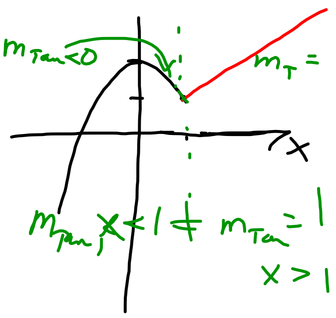
3.2 Derivative of a Function

76. $f(x) = \begin{cases} -x^2 + 2, & x \le 1 \\ x, & x > 1 \end{cases}$

F: a) sketch the graph
b) Show that not differentiable at x=1

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$



$m_T = f'(x) = 1$

$\therefore f(x)$ not diff. at $x=1$

m_{Tan}, $x<1$ $\neq$ $m_{Tan} = 1$, $x > 1$

Slope from the left of x=1 is neg. Slope from right =1. Slopes not equal. Therefore derivative DNE.

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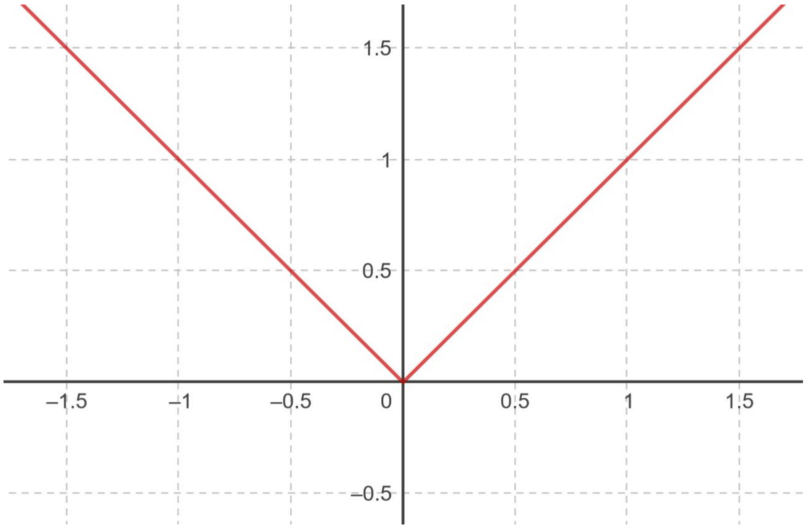
$y = |x|$

$$y = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



3.2 Derivative of a Function

If a function is differentiable, is it continuous?

If a function is continuous, is it differentiable?

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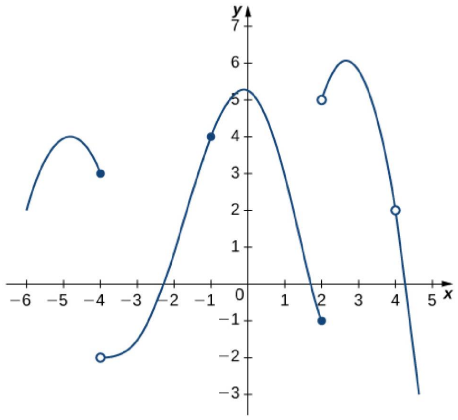
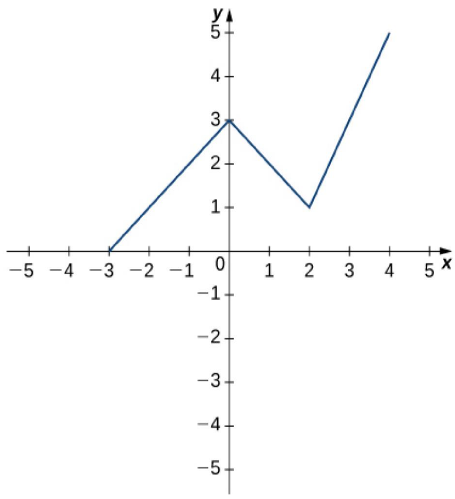
3.2 Derivative of a Function

If a function is differentiable, is it continuous?
If a function is continuous, is it differentiable?

78.



80. Use the graph to evaluate a. $f'(-0.5)$, b. $f'(0)$, c. $f'(1)$, d. $f'(2)$, and e. $f'(3)$, if it exists.



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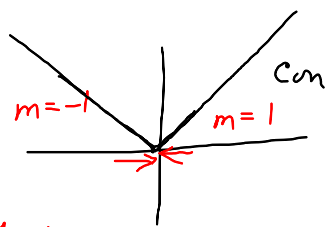
3.2 Derivative of a Function

f is differentiable at a point $\Rightarrow$ **continuous at the point**

f is differentiable at a point $\not\Leftarrow$ **continuous at the point**

$y = |x|$



continuous

not diff. at x=0

$y' = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$ DNE

No sharp turns!

bec. $\lim_{x\to 0^-} \cdots \neq \lim_{x \to 0^+}$

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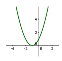
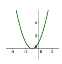
3.2 Derivative of a Function

G: $f(x) = x^2+2x+1$ F: eq of tan line at $(-3,4)$



3.2 Derivative of a Function

G: $f(x) = x^2+2x+1$ F: eq of tan line at $(-3,4)$ $y-y_1 = m_T(x-x_1)$



$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f(x+h) = (x+h)^2 + 2(x+h) + 1$$
$$= x^2+2hx+h^2+2x+2h+1$$

$$= \lim_{h\to 0} \frac{x^2+2hx+h^2+2x+2h+1-(x^2+2x+1)}{h}$$

$$= \lim_{h\to 0} \frac{2hx+h^2+2h}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h+2)}{\cancel{h}}$$

$$\lim_{h\to 0} 2x+h+2 = 2x+2$$

$$y-y_1 = m_T(x-x_1)$$

m_T at $(-3,4)$
$$= 2(-3)+2$$
$$= -6+2 = -4$$

$$y-4 = -4(x-(-3))$$
$$y-4 = -4(x+3) = -4x-12$$
$$y = -4x-8$$

3.2 Derivative of a Function

G: $f(x) = x^2+3$ F: eq tang. at $(1,4)$

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3.2 Derivative of a Function

G: $f(x) = x^2+3$ F: eq tang. at $(1,4)$

$f(1) = (1)^2 + 3$

$$\boxed{m_T = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

$$= \lim_{\Delta x \to 0} \frac{[(x+\Delta x)^2+3] - [x^2+3]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + (\Delta x)^2 + \cancel{3} - \cancel{x^2} - \cancel{3}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x+\Delta x)}{\cancel{\Delta x}} = \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

$y - y_1 = m_T(x - x_1)$

$y - 4 = m_T(x-1)$

$y - 4 = 2(x-1)$

$y - 4 = 2x - 2$

$y = 2x + 2$

at $(1,4)$ $m_T = f'(1) = 2(1) = 2 = m_T$

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Attachments

slopeTangentVS.wmv