3.1 Derivative at a Point

GOALS: 1. Understand the definition of the slope of the tangent line to a curve (limits).
2. Learn the definition of the derivative at a point (limits).
3. Interpret the derivative at a point as the slope of the tangent line.

Study 3.1 #11-19, 21-29, 39-43, 47,53

Study definitions: Derivative at a Point (3.3, 3.4)

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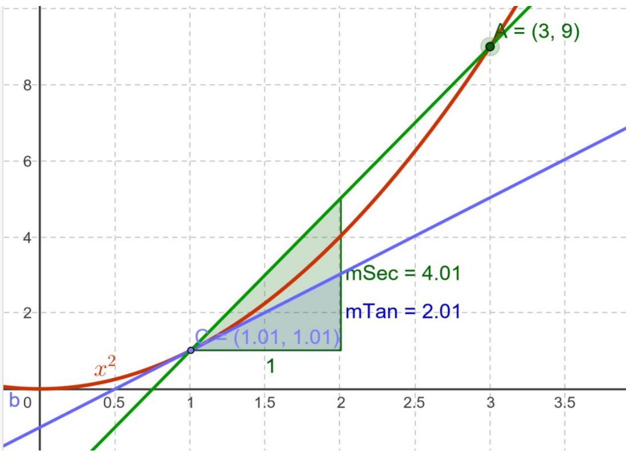
3.1 Derivative at a Point

Consider the slope of the secant line
as an approximation to the slope of the tangent line.

on-line geogebra: Slope of Tangent Line

video on disk



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3.1 Derivative at a Point

Slope of the secant line:

$$m_s = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

For $y = f(x)$, consider point $(c, f(c))$

$$m_s = \frac{f(c+\Delta x) - f(c)}{c + \Delta x - c} =$$

$$m_s = \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

In newer books, the Δx is replaced by h

Slope of the tangent line at a point (c, f(c)):

$$\boxed{m_T = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x} \text{ at a point } (c, f(c))}$$

generalize to $\boxed{\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}}$

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3.1 Derivative at a Point

Slope of the secant line:

Slope of the tangent line at a point (c, f(c)):

$$\boxed{m_T = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \text{ at a point } (c, f(c))}$$

Slope of the tangent line at a point (a, f(a)):

$$\boxed{m_T = f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

generalize to $\boxed{\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}}$

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3.1 Derivative at a Point

Slope of the secant line:

Slope of the tangent line at a point (c, f(c)):

Definition

Let $f(x)$ be a function defined in an open interval containing a. The *tangent line* to $f(x)$ at a is the line passing through the point $(a, f(a))$ having slope

$$m_{\tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad (3.3)$$

provided this limit exists.

Equivalently, we may define the tangent line to $f(x)$ at a to be the line passing through the point $(a, f(a))$ having slope

$$m_{\tan} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \quad (3.4)$$

provided this limit exists.

generalize to $\boxed{\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}}$

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3.1 Derivative at a Point

Derivative of f(x), the function: Section 3.2

f '(x) $\frac{dy}{dx}$ both are notation

Derivative of f(x) at a point (a, f(a)) Section 3.1

f '(a)

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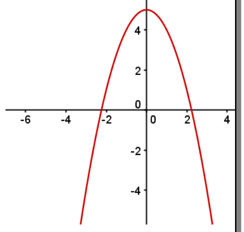
3.1 Derivative at a Point

$f(x) = 5 - x^2 \qquad F: m_T \text{ at } (2,1) \qquad 1(2)$

$f(\quad) = 5 - (\quad)^2$



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3.1 Derivative at a Point

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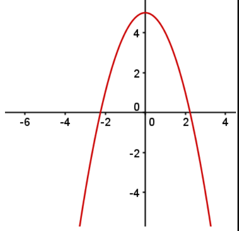

$$m_{Tan} = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$= \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2} \frac{5 - x^2 - 1}{x - 2}$$

$$= \lim_{x \to 2} \frac{-x^2 + 4}{x - 2} \to \frac{-4+4}{2-2} \to \frac{0}{0}$$ limit exists.

$$= \lim_{x \to 2} \frac{-(x^2 - 4)}{x - 2} = \lim_{x \to 2} \frac{-[(x+2)\cancel{(x-2)}]}{\cancel{(x-2)}}$$

$$m_{tan} = \lim_{x \to 2} -(x+2) = -4$$ slope at (2,1)

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3.1 Derivative at a Point

14. $f(x) = 1 - x - x^2, a = 0$

F: a) m_T at x = 0
b) eq. tan line

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3.1 Derivative at a Point

14. $f(x) = 1 - x - x^2, a = 0$

F: a) m_T at x = 0
b) eq. tan line

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{1 - x - x^2 - (1)}{x}$$

$$= \lim_{x \to 0} \frac{-x - x^2}{x} = \lim_{x \to 0} \frac{x(-1 - x)}{x} = \lim_{x \to 0} -1 - x$$

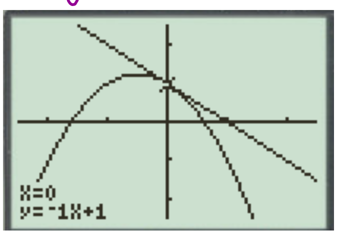
$$= -1$$

$$y - y_1 = m_T(x - x_1)$$

$$y - 1 = -1(x - 0)$$

$$y - 1 = -x$$

$$y = -x + 1$$



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3.1 Derivative at a Point

28. $f(x) = \dfrac{1}{x-3}$, $a = -1$ F: f '(-1)

3.1 Derivative at a Point

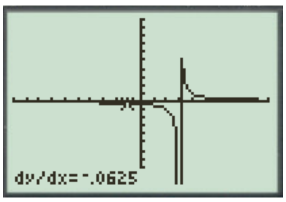
28. $f(x) = \dfrac{1}{x-3}$, $a = -1$ F: f '(-1)

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(-1) = \lim_{x \to -1} \frac{f(x) - f(-1)}{x - (-1)} = \lim_{x \to -1} \frac{\frac{1}{x-3} - \frac{1}{-1-3}}{x+1}$$

$$= \lim_{x \to -1} \frac{\frac{1}{x-3} + \frac{1}{4}}{x+1} \cdot \frac{4(x-3)}{4(x-3)}$$

$$= \lim_{x \to -1} \frac{4 + x - 3}{4(x+1)(x-3)} = \lim_{x \to -1} \frac{x+1}{4(x+1)(x-3)} = \lim_{x \to -1} \frac{1}{4(x-3)} = -\frac{1}{16}$$



3.1 Derivative at a Point

A position function represents the position of a particle traveling along a horizontal line.

G: $s(t)=2t^3-3t^2-12t+8$

F: the initial velocity

Velocity is the instantaneous rate of change of distance with respect to time.

$$v(c)=s'(c)=\lim_{t\to c}\frac{s(t)-s(c)}{t-c}$$

So, looking for the slope of the tangent line to s(t) when t = 0.

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3.1 Derivative at a Point

A position function represents the position of a particle traveling along a horizontal line at time *t*.

G: $s(t)=2t^3-3t^2-12t+8$

F: the initial velocity

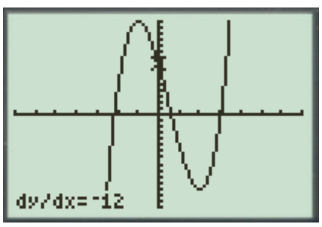
Velocity is the instantaneous rate of change of distance with respect to time.

$$v(c)=s'(c)=\lim_{t\to c}\frac{s(t)-s(c)}{t-c}$$

Initial velocity is the velocity at t=0.

$$v(0)=s'(0)=\lim_{t\to 0}\frac{s(t)-s(0)}{t-0}=\lim_{t\to 0}\frac{(2t^3-3t^2-12t+8)-(8)}{t}$$

$$s'(0)=\lim_{t\to 0}\frac{2t^3-3t^2-12t}{t}=\lim_{t\to 0}\frac{t(2t^2-3t-12)}{t}=\lim_{t\to 0}2t^2-3t-12=-12$$



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3.1 Derivative at a Point

HW PROBLEM:

157. A potato is launched vertically upward with an initial velocity of 100 ft/s from a potato gun at the top of an 85-foot-tall building. The distance in feet that the potato travels from the ground after t seconds is given by $s(t) = -16t^2 + 100t + 85$.

F: velocity at t = 1 sec

3.1 Derivative at a Point

HW PROBLEM:

157. A potato is launched vertically upward with an initial velocity of 100 ft/s from a potato gun at the top of an 85-foot-tall building. The distance in feet that the potato travels from the ground after t seconds is given by $s(t) = -16t^2 + 100t + 85$.

F: velocity at t = 1 sec

$$v(1)=s'(1)=\lim_{t\to 1}\frac{s(t)-s(1)}{t-1}=\lim_{t\to 1}\frac{(-16t^2+100t+85)-(-16+100+85)}{t-1}$$

$$=\lim_{t\to 1}\frac{-16t^2+100t+85-84-85}{t-1}=\lim_{t\to 1}\frac{-16t^2+100t-84}{t-1}\to\frac{0}{0}$$

$$=\lim_{t\to 1}\frac{-4(4t^2-25t+21)}{t-1}=-4\lim_{t\to 1}\frac{(t-1)(4t-21)}{t-1}=-4\lim_{t\to 1}4t-21=-4(-17)=68$$

at t = 1 sec, v(1)= 68 ft/sec

3.1 Derivative at a Point

If a function is differentiable, is it continuous?
If a function is continuous, is it differentiable?

f is differentiable at pt. ? f continuous at pt.

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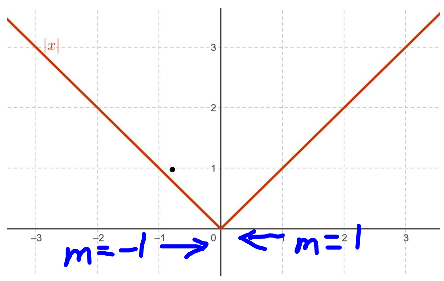
3.1 Derivative at a Point

f is differentiable at pt. $\underset{\Longrightarrow}{\not\Longleftarrow}$ f continuous at pt.

continuous

$y = |x|$



$m = -1 \rightarrow$ $\leftarrow m = 1$ not diff. at $x = 0$

$y' = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ DNE

bec. $\lim_{x \to 0^-} \cdots \neq \lim_{x \to 0^+}$

NO SHARP TURNS

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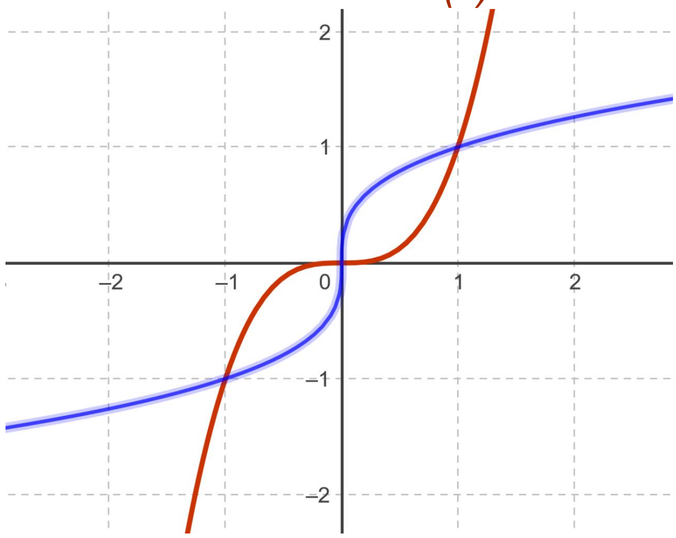
3.1 Derivative at a Point

$f(x) = x^3$



$f(x) = x^{1/3}$

$f(x) = x^3$ diff and cont

$f(x) = x^{1/3}$ not diff at x=0, yet cont

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3.1 Derivative at a Point

G: $f(x) = x^2 + 3$

$f(\ ) = (\ )^2 + 3$

F: eq tang. at $(1, 4)$

$y - y_1 = m_T(x - x_1)$

$y - \ \ = m_T(x - \ \ )$

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3.1 Derivative at a Point

p.104 #25 G: $f(x) = x^2 + 3$

$f(\ ) = (\ )^2 + 3$

F: eq. tang. at $(1, 4)$

$y - y_1 = m_T(x - x_1)$

$y - 4 = m_T(x - 1)$

$$\boxed{m_T = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

$$= \lim_{\Delta x \to 0} \frac{[(x+\Delta x)^2 + 3] - [x^2 + 3]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + (\Delta x)^2 + \cancel{3} - \cancel{x^2} - \cancel{3}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x + \Delta x)}{\cancel{\Delta x}} = \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

at $(1, 4)$ $m_T = f'(1) = 2(1) = 2 = m_T$

$y - 4 = 2(x - 1)$

$y - 4 = 2x - 2$

$y = 2x + 2$

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3.1 Derivative at a Point

12. $f(x) = \frac{x}{5} + 6,\ a = -1$

F: a) m_T at $x = -1$

b) eq. tan line

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3.1 Derivative at a Point $-\frac{1}{5}+6=$

12. $f(x) = \frac{x}{5} + 6,\ a = -1$

F: a) m_T at $x = -1$
b) eq. tan line

Can use definition formulas, but do not need to because this is a straight line, and already know the slope and already given the equation.

$$f'(x) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$f'(-1) = \lim_{x \to -1} \frac{f(x) - f(-1)}{x - (-1)}$$

$$= \lim_{x \to -1} \frac{\frac{x}{5} + 6 - \left(-\frac{1}{5} + 6\right)}{x + 1}$$

$$\lim_{x \to -1} \frac{\frac{x}{5} + 6 + \frac{1}{5} - 6}{x + 1}$$

$$\lim_{x \to -1} \frac{\frac{x}{5} + \frac{1}{5}}{x + 1} \cdot \frac{5}{5} = \lim_{x \to -1} \frac{x + 1}{5(x + 1)} = \lim_{x \to -1} \frac{1}{5} = \frac{1}{5}$$

m_T at $x = -1$

$$y - y_1 = m_T(x - x_1)$$

$$y - \frac{29}{5} = \frac{1}{5}(x - (-1))$$

$$y - \frac{29}{5} = \frac{1}{5}x + \frac{1}{5}$$

$$y = \frac{1}{5}x + \frac{30}{5}$$

$$y = \frac{1}{5}x + 6$$

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slopeTangentVS.wmv