2.4 Continuity

GOAL:
1. Understand definition of continuity at a point.
2. Evaluate functions for continuity at a point, and on open and closed intervals
3. Understand the Intermediate Value Theorum (IVT)

Study 2.4, # 131-143; one of 145-149; 151, 155, 161, HW 1 HW 2

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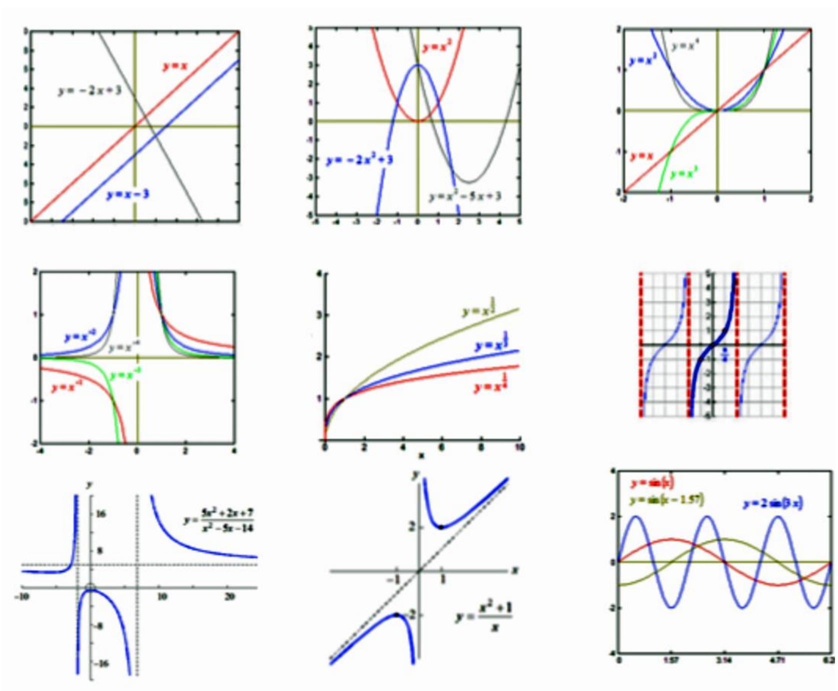
Homework

2.4 Continuity

You have seen a number of different types of graphs, some continuous for all x, some not.



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2.4 Continuity

How would you describe the graph of a curve that is continuous on an interval?

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2.4 Continuity

$$y = \frac{x^2 - 9}{x - 3}$$

Is y continuous at x = 3?
Why or why not ?

NO. y is not defined at x=3

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Homework

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2.4 Continuity

$$y = \frac{x^2 - 9}{x - 3}$$

Is y continuous at x = 3?
Why or why not ?

No. y does not exist at x=3 DNE
bec. div by 0

NO. y is not defined at x=3

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2.4 Continuity

$$y = \frac{1}{x}$$

Is y continuous at x = 0?
Why or why not ?

NO. y is not defined at x=0

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Homework

2.4 Continuity

$$y = \frac{1}{x}$$

Is y continuous at x = 0?
Why or why not ?

No. y does not exist at x=0 DNE
bec. div by 0

NO. y is not defined at x=0

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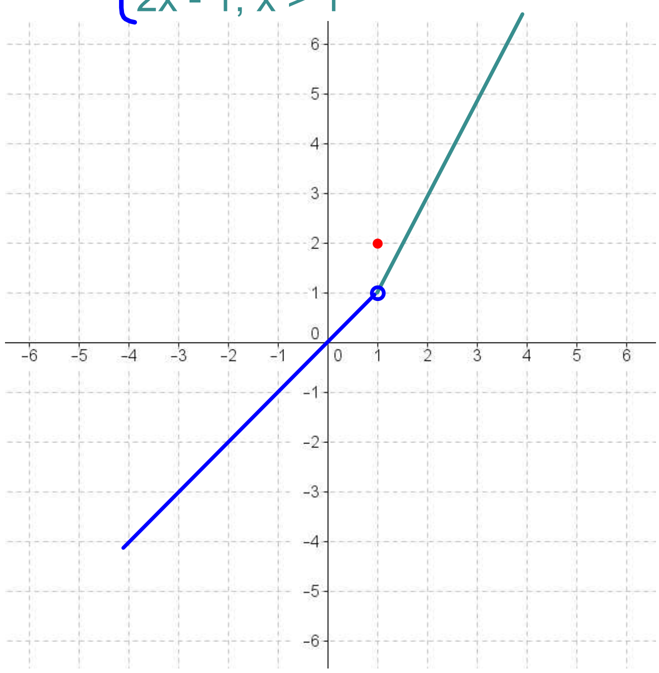
Homework

2.4 Continuity

Sketch:

$$y = \begin{cases} x & , x < 1 \\ 2 & , x = 1 \\ 2x - 1, & x > 1 \end{cases}$$

Is y continuous at x = 1?



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Homework

2.4 Continuity

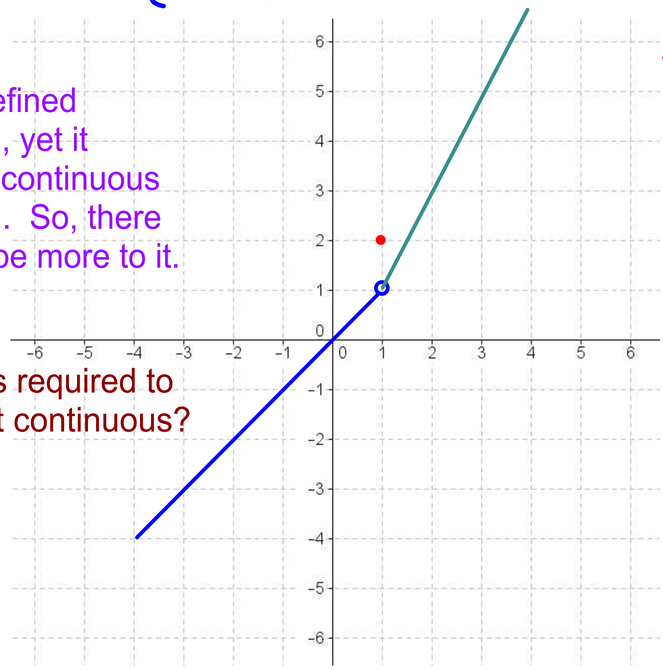
Sketch:

$$y = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$$

Is y continuous at x = 1?

NO!
Why not?

y is defined at x=1, yet it is not continuous at x=1. So, there must be more to it.



What is required to make it continuous?

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2.4 Continuity

Sketch:

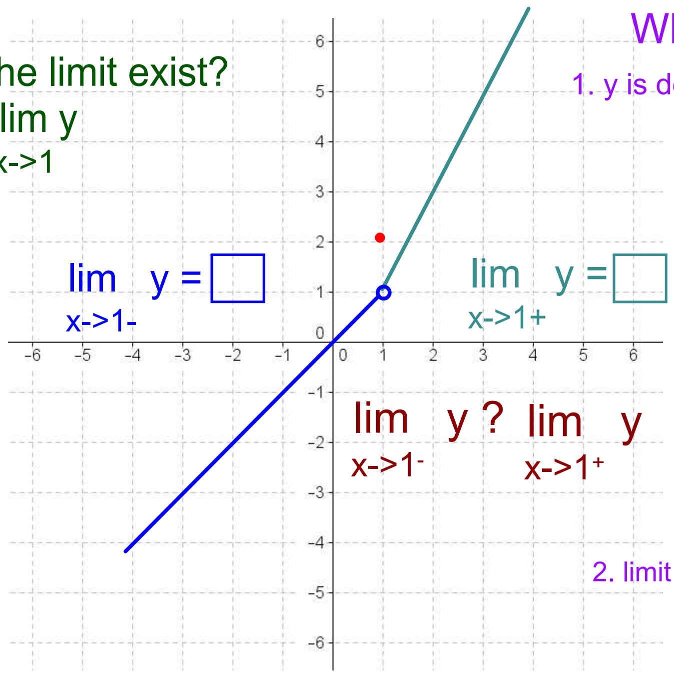
$$y = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$$

Is y continuous at x = 1?

NO!
Why not?

1. y is defined at x=1

Does the limit exist? $\lim_{x \to 1} y$



$\lim_{x \to 1^-} y = \square$

$\lim_{x \to 1^+} y = \square$

$\lim_{x \to 1^-} y$? $\lim_{x \to 1^+} y$

2. limit exists at x=1

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Homework

2.4 Continuity

Sketch: $y = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$

Is y continuous at x = 1?

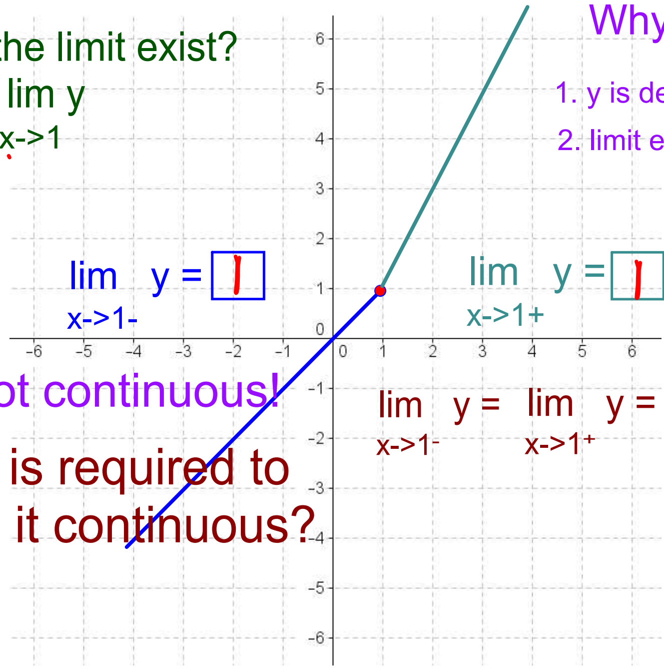
NO!

Why not?

1. y is defined at x=1
2. limit exists at x->1

Does the limit exist? $\lim_{x \to 1} y$

Yes



$\lim_{x \to 1^-} y = 1$

$\lim_{x \to 1^+} y = 1$

$\lim_{x \to 1^-} y = \lim_{x \to 1^+} y = 1$

Still not continuous!

What is required to make it continuous?

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Homework

2.4 Continuity

Sketch: $y = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$

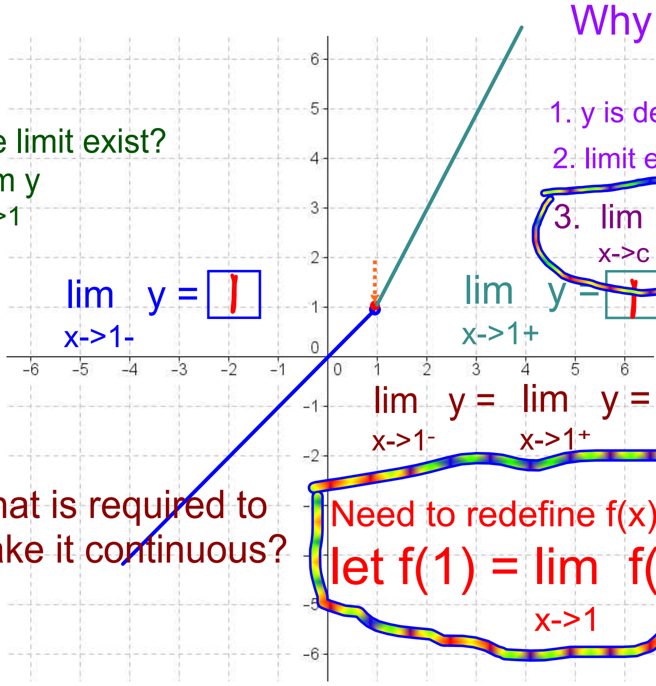
Is y continuous at x = 1?

NO!

Why not?

1. y is defined at x=1
2. limit exists at x->1
3. $\lim_{x \to c} f(x) = f(c)$

Does the limit exist? $\lim_{x \to 1} y$



$\lim_{x \to 1^-} y = 1$

$\lim_{x \to 1^+} y = 1$

$\lim_{x \to 1^-} y = \lim_{x \to 1^+} y = 1$

What is required to make it continuous?

Need to redefine f(x) at x=1;

let $f(1) = \lim_{x \to 1} f(x) = 1$

return

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Homework

2.4 Continuity

What is needed for continuity at a point?

f(x)	c	$\lim_{x \to c} f(x)$	f(c)	Continuous at x = c?
$\frac{1}{x}$	0			
$\frac{x^2 - 9}{x - 3}$	3			
$f(x) = \begin{cases} x, & x<1 \\ 2 & x=1 \\ 2x-1, & x>1 \end{cases}$	1			
$x+3$	3			

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Homework

2.4 Continuity

What is needed for continuity at a point?

f(x)	c	$\lim_{x \to c} f(x)$	f(c)	Continuous at x = c?
$\frac{1}{x}$	0	DNE	DNE	NO
$\frac{x^2-9}{x-3}$	3	6	DNE	NO
$f(x) = \begin{cases} x, & x<1 \\ 2 & x=1 \\ 2x-1, & x>1 \end{cases}$	1	1	2	NO
$x+3$	3	6	6	YES

NEED?

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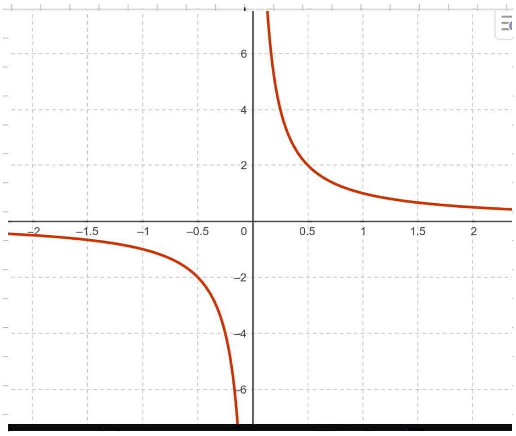
Homework

2.4 Continuity

What is needed for continuity at a point?

f(x)	c	lim f(x) x-->c	f(c)	Continuous at x = c?
$\frac{1}{X}$	0			



return

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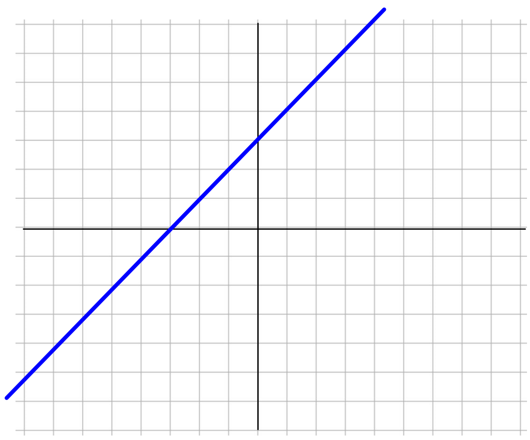
2.4 Continuity

What is needed for continuity at a point?

f(x)	c	lim f(x) x-->c	f(c)	Continuous at x = c?
x+3	0			



return

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Homework

2.4 Continuity

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} \longrightarrow$$

return

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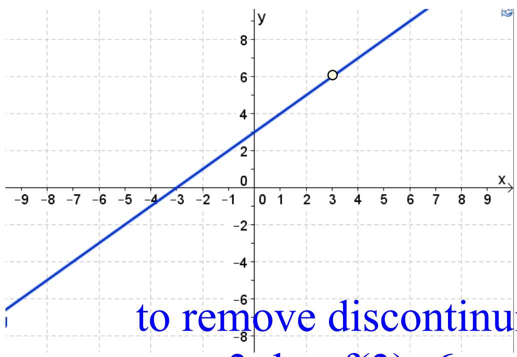
Homework

2.4 Continuity

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} \longrightarrow \frac{0}{0} \quad \text{lim. exists.}$$

$$\frac{(x+3)(x-3)}{(x-3)} = \lim_{x \to 3} x+3 = 6$$

$$\frac{x^2-9}{x-3} = x+3, \quad x \neq 3$$



to remove discontinuity at x=3, let: f(3)=6

return

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2.4 Continuity

What is needed for continuity at a point?

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Homework

2.4 Continuity

What is needed for continuity at a point?

Continuity at a Point

A function f is continuous at c if:

1. f(c) is defined.
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

Required to show continuity.

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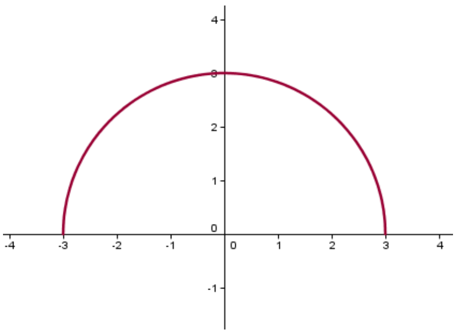
Homework

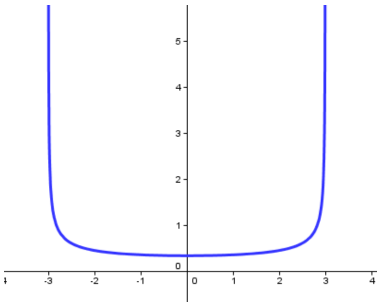
2.4 Continuity

What about continuity on an interval?

[a, b] or (a, b)

$f(x) = \sqrt{(9-x^2)}$ $g(x) = \dfrac{1}{\sqrt{(9-x^2)}}$





continuous on [-3, 3] continuous on (-3, 3)

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2.4 Continuity

Continuity on an interval

Consider $f(x) = \sqrt{(9-x^2)}$ $g(x) = \dfrac{1}{\sqrt{(9-x^2)}}$

I. A function is continuous on an **open** interval (a,b) if it is continuous at each point in (a,b).

(a) b

II. A function is continuous on an **closed** interval [a,b] if it is continuous at each point in (a,b), and

[a] b

$\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$

concerned with inside the interval and the ends

III. Functions continuous on entire real number line are everywhere continuous.

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2.4 Continuity

G: $g(x) = \dfrac{1}{x^2 - 4}$ F: Is g continuous on [-1,2]?

G: $g(x) = \dfrac{1}{x^2 - 4}$ F: Is g continuous on (-1,2)?

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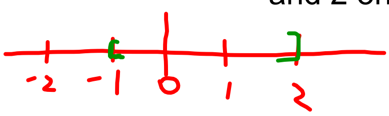
2.4 Continuity

G: $g(x) = \dfrac{1}{x^2 - 4}$ F: Is g continuous on [-1,2]?

$x \neq \pm 2$

NO at $x=2$, g DNE

and 2 on [-1,2]



G: $g(x) = \dfrac{1}{x^2 - 4}$ F: Is g continuous on (-1,2)?

$x \neq 2$

Yes.

bec. $2 \notin (-1,2)$

g differentiable and continuous for all points other than x = -2,+2

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2.4 Continuity

G: $f(x) = \dfrac{x^2-1}{x+1}$ F: Is f continuous for all x?

G: $g(x) = \dfrac{x^2-1}{x^2+1}$ F: Is g continuous for all x?

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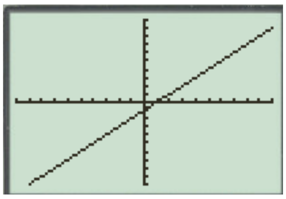
2.4 Continuity

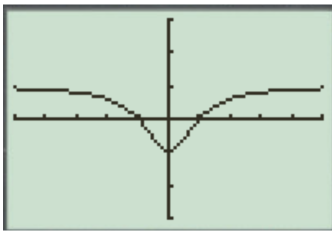
G: $f(x) = \dfrac{x^2-1}{x+1}$ F: Is f continuous for all x?



No. At x=-1, f DNE
because division by 0.

Discontinuity not visible on graph

G: $g(x) = \dfrac{x^2-1}{x^2+1}$ F: Is g continuous for all x?



Yes.
No values of x cause division by 0
No radicals that restrict domain

No discontinuity is visible on graph

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Homework

2.4 Continuity

G: $f(x) = \frac{x - 1}{x^2-3x-10}$

F: a) all x where f not continuous
b) any removable? how?

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2.4 Continuity

G: $f(x) = \frac{x - 1}{x^2-3x-10}$

F: a) all x where f not continuous
b) any removable? how?

a) Factor den: $x^2-3x-10 = (x-5)(x+2)$

at x=5, -2 div by 0
f DNE, and f is discontinuous

what 2 numbers mult to -10 and add to -3? -5 and +2

b) $\lim_{x \to 5} \frac{x - 1}{x^2-3x-10} \to \frac{4}{0}$

as x->5 f is unbounded and NOT removable

$\lim_{x \to -2} \frac{x - 1}{x^2-3x-10} \to \frac{-3}{0}$

as x->-2 f is unbounded and NOT removable

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2.4 Continuity

G: $f(x) = \dfrac{x + 2}{x^2-3x-10}$

F: a) all x where f not continuous
b) any removable? how?

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Homework

2.4 Continuity

G: $f(x) = \dfrac{x + 2}{x^2-3x-10}$

F: a) all x where f not continuous
b) any removable? how?

a) Factor den: $x^2-3x-10 = (x-5)(x+2)$

at x=5, -2 div by 0

f DNE, and f is discontinuous

what 2 numbers mult to -10 and add to -3? -5 and +2

b) $f(x) = \dfrac{x + 2}{x^2-3x-10} = \dfrac{x + 2}{(x-5)(x+2)} = \dfrac{1}{x-5}$, $x \neq 5,-2$

$\lim_{x \to 5} f(x) = \lim \dfrac{1}{x-5} \to \dfrac{4}{0}$

as x->5 f is unbounded and NOT removable

$\lim_{x \to -2} f(x) = \lim \dfrac{1}{x-5} \to \dfrac{1}{-7}$

as x->-2, f -> -1/7 and **IS removable bec limit exists**; let f(-2)=-1/7

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2.4 Continuity

let $f(x) = \begin{cases} \dfrac{x+2}{x^2-3x-10}, & x \neq -2 \\ \dfrac{-1}{7}, & x = -2 \end{cases}$

Discontinuity at x = -2 is removable.

Discontinuity at x = 5 is NOT removable.
No limit as x ->5.

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Homework

141. G: $g(u) = \begin{cases} \dfrac{6u^2+u-2}{2u-1} & \text{if } u \neq \frac{1}{2} \\ \dfrac{7}{2}, & u = \frac{1}{2} \end{cases}$

F: Is $g(u)$ continuous at $u = \frac{1}{2}$?

① $g(\frac{1}{2})$ [illegible]

141. G: $g(u) = \begin{cases} \dfrac{6u^2+u-2}{2u-1} & \text{if } u \neq \frac{1}{2} \\ \frac{7}{2}, & u = \frac{1}{2} \end{cases}$

F: Is $g(u)$ continuous at $u = \frac{1}{2}$?

disc. $2u-1=0$, $u=\frac{1}{2}$

G: $g\left(\frac{1}{2}\right) = \frac{7}{2}$ ✓

Def. continuous. ① $g\left(\frac{1}{2}\right)$ exist ✓

② $\lim_{u \to \frac{1}{2}} g(u) = \frac{7}{2}$?

$$\lim_{u\to\frac{1}{2}} g(u) = \lim_{u\to\frac{1}{2}} \frac{6u^2+u-2}{2u-1} = \lim_{u\to\frac{1}{2}} 3u+2 = 3\cdot\frac{1}{2}+2 = \frac{3}{2}+\frac{4}{2} = \frac{7}{2} \checkmark$$

$$\frac{(2u-1)(3u+2)}{(2u-1)}$$

$\therefore \lim_{u\to\frac{1}{2}} g(u) = \frac{7}{2} = g\left(\frac{1}{2}\right)$

g is cont. at $u = \frac{1}{2}$

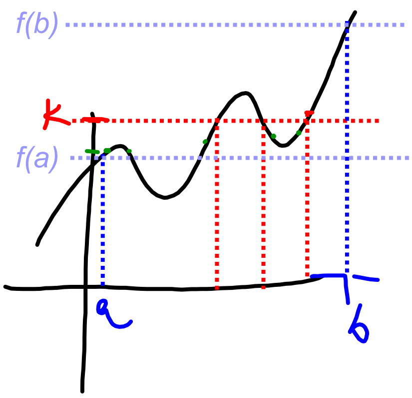
2.4 Continuity

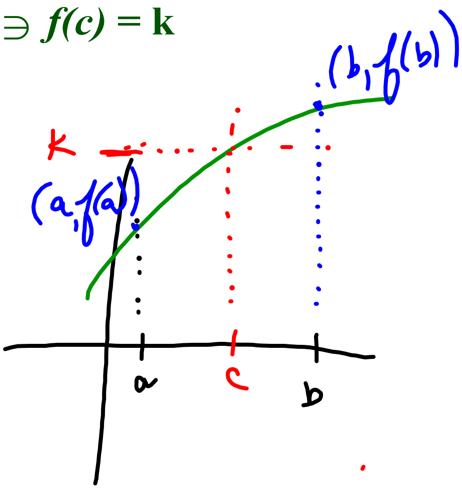
Intermediate Value Theorum (IVT)

If:

1. ***f*** **is continuous on closed interval [a,b]**
2. $f(a) \neq f(b)$
3. **k is any # between** $f(a)$ **and** $f(b)$

then $\exists$ **at least one c** $\in$ **[a,b]** $\ni f(c) = k$





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2.4 Continuity

G: $f(x) = x^2 - 6x + 8$

F: a) Verify IVT on [0,3]

b) c so f(c)=0

Note that k=0, the y value

Homework

2.4 Continuity

G: $f(x) = x^2 - 6x + 8$

F: a) Verify IVT on [0,3]

b) c so f(c)=0

Note that k=0, the y value

Intermediate Value Theorum (IVT)

1. ***f*** **is continuous on [a,b]**

YES. Polynomial

2. $f(a) \neq f(b)$

$f(0) = 0 - 0 + 8 = 8$

$f(3) = 9 - 18 + 8 = -1$

YES. $8 \neq -1$

3. $f(a) < k < f(b)$ **or** $f(b) < k < f(a)$

YES. $-1 < 0 < 8$

then $\exists$ **at least one c** $\in$ **[a,b]** $\ni f(c) = k$

So, guaranteed there is a c on [0, 3] where f(c) = 0

$f(c) = c^2 - 6c + 8 = 0$

$c^2 - 6c + 8 = (c-4)(c-2)0$

c = 2, ~~4~~

c=2 [0,3]

results in f(2)=0

c=4 not on [0,3]

Homework

2.4 Continuity

G: $f(x)= x^3-x^2+x-2$

F: a) Verify IVT on [0,3]
b) c, so $f(c)=4$

Homework

2.4 Continuity

G: $f(x)= x^3-x^2+x-2$

F: a) Verify IVT on [0,3]
b) c, so $f(c)=4$

Intermediate Value Theorum (IVT)

1. ***f*** **is continuous on [a,b]**

YES. Polynomial

2. $f(a) \neq f(b)$

$f(0) = 0-0+0-2 = -2$

$f(3) = 27-9+3-2 = 19$

YES. $-2 \neq 19$

3. $f(a) < k < f(b)$ **or** $f(b) < k , f(a)$

YES. $-2 < 4 < 19$

then $\exists$ **at least one c** $\in$ **[a,b]** $\ni f(c) = k$

So, guaranteed there is at least one c on [0, 3] where f(c) = 4

$f(c)= c^3-c^2+c-2=4$

$f(c)= c^3-c^2+c-6=0$

want factor of 6 on (0,3)

2	1	-1	1	-6
		2	2	6
	1	1	3	0

$c^3-c^2+c-6=(c-2)(c^2+c+3)=0$

$c=2 \quad [0,3]$

$b^2-4ac=1-12=-11< 0$
so no more real roots

results in $f(2)=4$

Homework

2.4 Continuity

HOMEWORK problem

G: $f(x)= x^2-4x+3$

F: Use IVT to explain why f(x) has a zero on [2,4]

extra at link

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Homework

2.4 Continuity

HOMEWORK problem

G: $f(x)= x^2-4x+3$

F: Use IVT to explain why f(x) has a zero on [2,4]

y = 0

1. f is polynomial, continuous everywhere
2. f(2) = 4-8+3 = -1 not equal to f(4)= 16-16+3 = 3
   and -1 < 0 < 3
3. So, IVT guarantees there is a c on [2,4]
   where f(c) = 0

extra at link

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Homework

2.4 Continuity

HOMEWORK problem

G: $f(x)= x^2+x-1$

F: a) Verify IVT on [0,5]
b) c so f(c) = 11

#91 at link

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Homework

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146. $f(\theta)=\begin{cases}\sin\theta, & 0\le\theta<\frac{\pi}{2}\\ \cos(\theta+k), & \frac{\pi}{2}\le\theta\le\pi\end{cases}$

F: k so F(θ) is continuous over the interval, [0,π]

sin θ and cos (θ+ k) are both continuous everywhere, so only possible discontinuity is at $\theta = \pi/2$

$f(\pi/2) = \cos(\pi/2+k)$

$\lim_{\theta \to \pi/2} f(\theta) = ?$

---

146. $f(\theta)=\begin{cases}\sin\theta, & 0\le\theta<\frac{\pi}{2}\\ \cos(\theta+k), & \frac{\pi}{2}\le\theta\le\pi\end{cases}$

only θ value at $\theta = \pi/2$

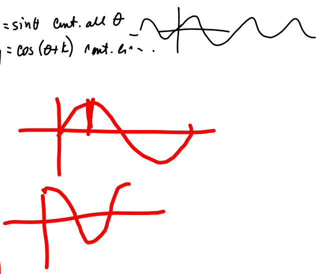
$\lim_{x\to a} f(x) = f(a)$

$f(\frac{\pi}{2}) = \cos(\frac{\pi}{2}+k)$

need $\lim_{x\to\frac{\pi}{2}} f(x) = \cos(\frac{\pi}{2}+k)$

$\lim_{\theta\to\frac{\pi}{2}^-} \sin\theta = \lim_{\theta\to\frac{\pi}{2}^+} (\cos\theta+k)$

$\sin\frac{\pi}{2} = \cos(\frac{\pi}{2}+k) = f(\frac{\pi}{2})$



$1 = \cos(\frac{\pi}{2}+k)$

let $\theta = 0$

$\frac{\pi}{2}+k=0$

$k=-\frac{\pi}{2}$

Continuous: $\lim_{\theta\to\frac{\pi}{2}} f(\theta) = f(\frac{\pi}{2})$