2.2 The Limit of a Function, 2.3 Evaluating Limits

LIMITS

To Find a Limit: Summary

GOALS:

1. Understand that limits involve the idea of approaching a value or point.
2. Understand how the **limit** of a function at a point (approaching the point) is **different from** the **value** of the function at the point (being at the point).
3. Use tables & numerical approach to determine existence of a limit.
4. Use graphing to determine existence of a limit.
5. **Find limits analytically.**

Study 2.2 # 47-79 odd
Study 2.3 # 83 - 117 odd, limits 2.18, 2.19

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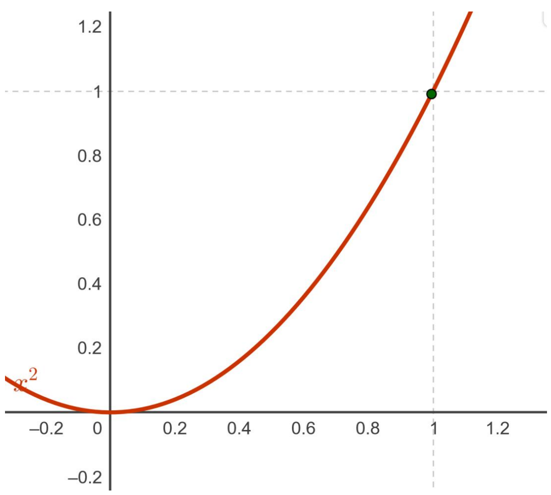
(sinx)/x*

2.2 The Limit of a Function

Given: $f(x) = x^2$

As x -> 1, what does y approach?



As x -> 1, y approaches 1. Also f(1)=1.

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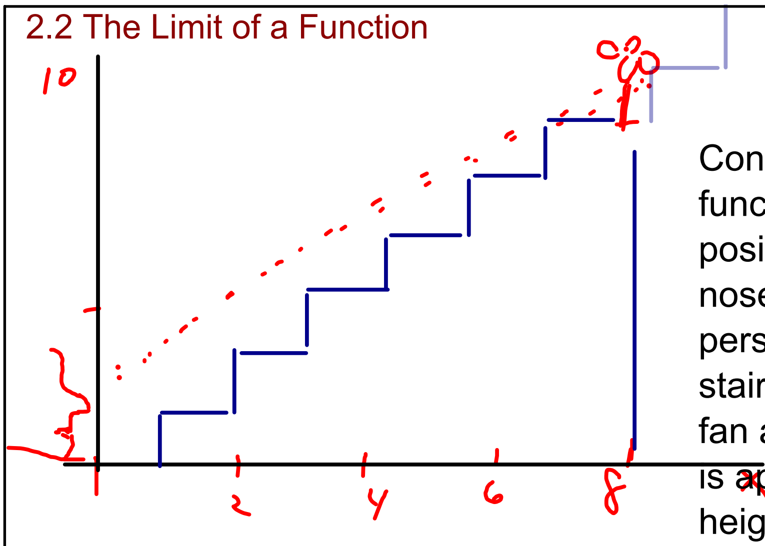
geogebra

2.2 The Limit of a Function



Consider the function that is the position of the nose. As the person walks up the stairs towards the fan at x=8, his nose is approaching a height of 10. But, does f(8) exist?

Does f(8) exist?

Does $\lim_{x \to 8} f(x)$ exist?

Or: **Is there a value that y approaches as x -> 8 ?**

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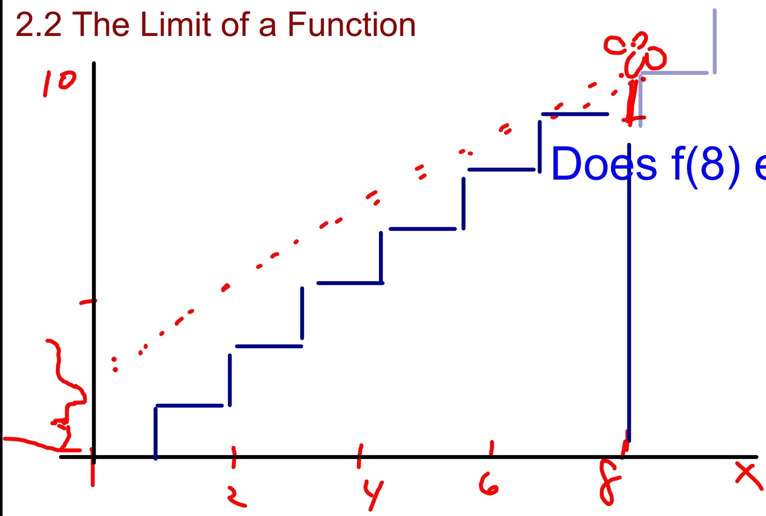
2.2 The Limit of a Function



Does f(8) exist?

NO!

The nose hits the fan, so it does not exist there.

Does $\lim_{x \to 8}$ (height of the nose) exist?

YES! height = 10

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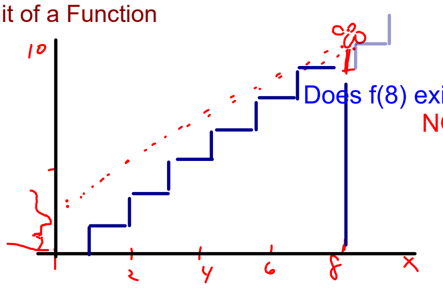
2.2 The Limit of a Function



Does f(8) exist?
NO!

Does lim (height of the nose) exist?
x -> 8

YES! height = 10

Let *f* be a function defined on an open interval containing c, (8) except possibly at c, and let L be a real number. (10)

Then $\lim_{x \to c} f(x) = L$ (c = 8, L = 10)

means that for each $\epsilon > 0 \; \exists \; \delta > 0 \; \ni$

if $0 < | x-c | < \delta$ then $| f(x)-L | < \epsilon$

x is near 8 eg: x=7.9 then δ =0.1 — y is near 10 and ε is small

Analytical definition of a limit. We are just looking at it. We will not actually apply the specifics, but will always be applying the principle.

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2.2 The Limit of a Function

$f(x) = \dfrac{x - 2}{x^2 - 4}, \quad x \neq \pm 2$

How do you Find Limits?

1. Numerically - use to show how the values are changing, or for a function that you either cannot manipulate algebraically or graph.

$$\lim_{x \to 2} \frac{x - 2}{x^2 - 4} \rightarrow \frac{2 - 2}{4 - 4} = \frac{0}{0}$$

DNE

indeterminate

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)						

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2.2 The Limit of a Function

How do you Find Limits?

1. Numerically - use only to show how the values are changing

$$\lim_{x \to 2} \frac{x-2}{x^2-4} \to \frac{2-2}{4-4} = \frac{0}{0}$$ DNE indeterminate

as x -> +2

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	0.2564	0.2506	0.2501	0.2499	0.2494	0.2439

f(x) -> 1/4

$$\lim_{x \to 2} \frac{x-2}{x^2-4} = \frac{1}{4}$$

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2.2 The Limit of a Function

How do you Find Limits?

1. Finding Limits Numerically
   Helpful
   - see where y values are going as x ->c

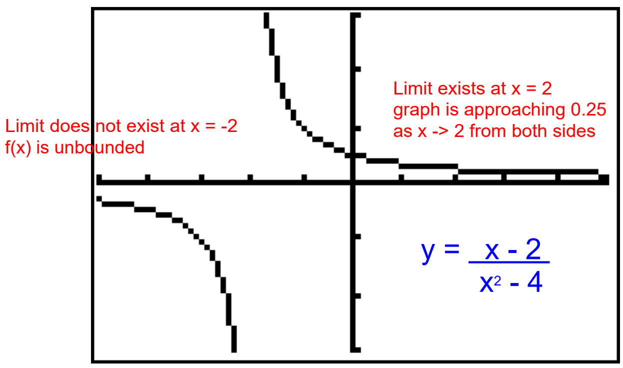
   Not so Helpful
   - tedious
   - distracts from overview of function

as x -> +2

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	0.2564	0.2506	0.2501	0.2499	0.2494	0.2439

f(x) -> 1/4

2. Graphical Approach:
   Tells us if limit exists, & suggests a value



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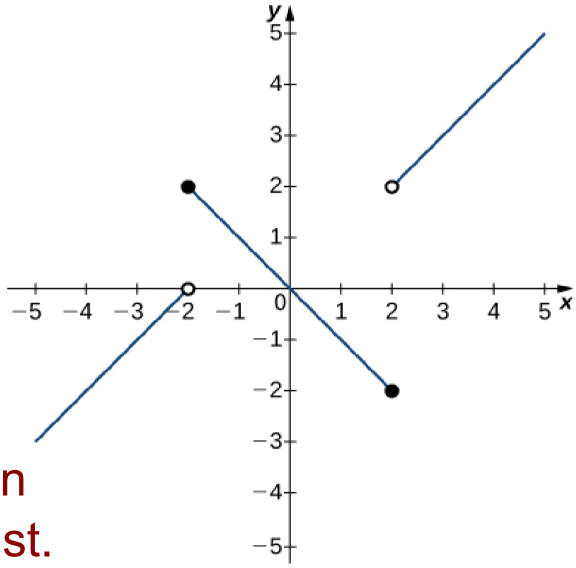
2.2 The Limit of a Function

Homework problems look like this: graphical representatons of functions, with questions about what value f approaches as x approaches a specific value.

60. $\lim_{x \to -2^+} f(x)$

62. $\lim_{x \to 2^-} f(x)$

64. $\lim_{x \to 2} f(x)$



Need to know when a limit does not exist.

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2.2 The Limit of a Function

When does a Limit NOT exist?

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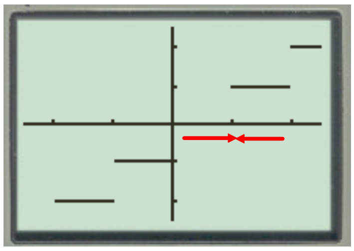
2.2 The Limit of a Function

When does a Limit NOT exist?

y = [x]



1. $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$

$\lim_{x \to 1^-} [x] \neq \lim_{x \to 1^+} [x]$

$0 \neq 1$

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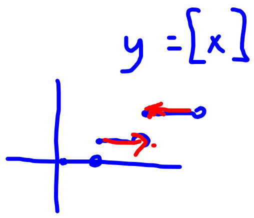
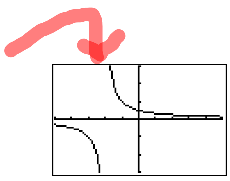
2.2 The Limit of a Function

When does a Limit NOT exist?



1. $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$

2. $\lim_{x \to c^-} f(x)$ unbounded



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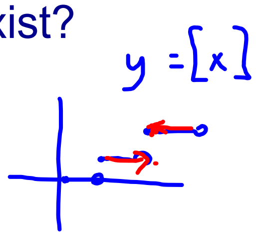
2.2 The Limit of a Function

When does a Limit NOT exist?



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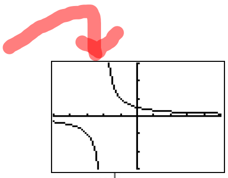
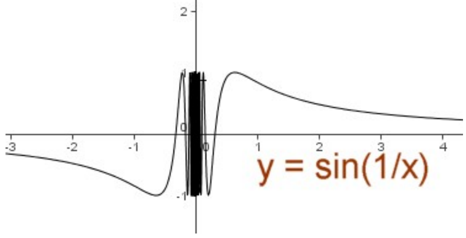


3. f(x) oscillating
   $\lim_{x \to c^-} f(x)$ DNE



on calc:
-.05<x<.05
-1 <y< 1

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2.2 The Limit of a Function

When does a Limit NOT exist?

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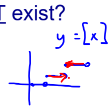
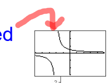
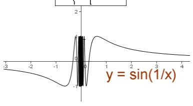


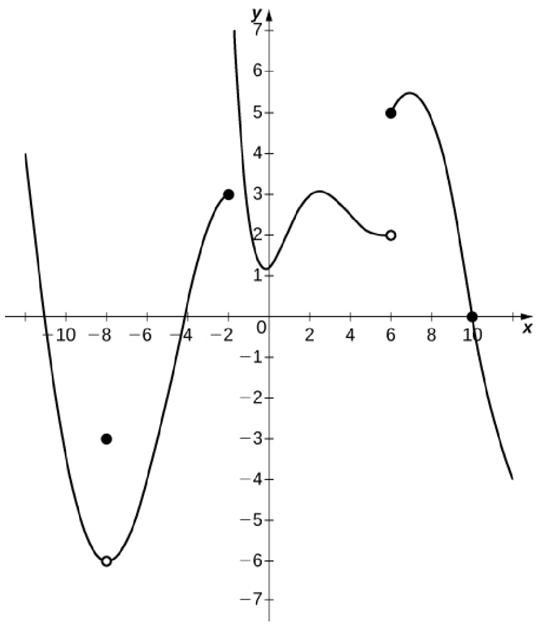




True or False?

46. $\lim_{x \to 10} f(x) = 0$

48. $\lim_{x \to -8} f(x) = f(-8)$



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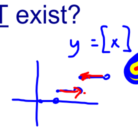
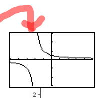
2.2 The Limit of a Function

When does a Limit NOT exist?

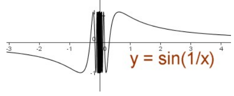


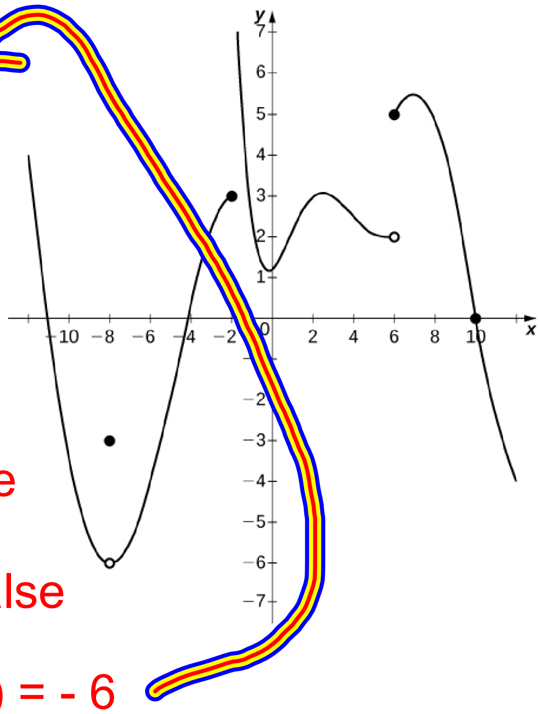
1. $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$

2. $\lim_{x \to c^-} f(x)$ unbounded



3. f(x) oscillating
   $\lim_{x \to c^-} f(x)$ DNE





True or False?

46. $\lim_{x \to 10} f(x) = 0$ True

48. $\lim_{x \to -8} f(x) = f(-8)$ False

$f(-8) = -3 \neq \lim_{x \to -8} f(x) = -6$

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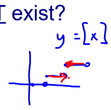
2.2 The Limit of a Function

When does a Limit NOT exist?



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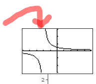
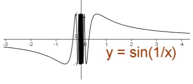
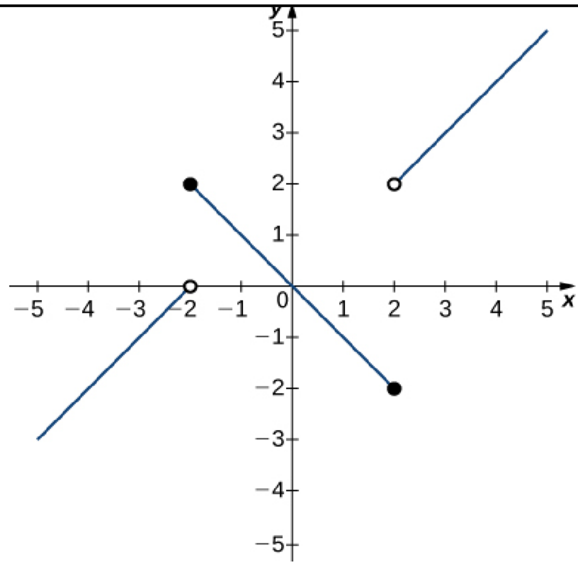


3. f(x) oscillating
   $\lim_{x \to c^-} f(x)$ DNE





60. $\lim_{x \to -2^+} f(x)$

62. $\lim_{x \to 2^-} f(x)$

64. $\lim_{x \to 2} f(x)$

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2.2 The Limit of a Function

When does a Limit NOT exist?

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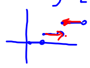
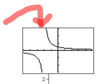
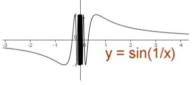
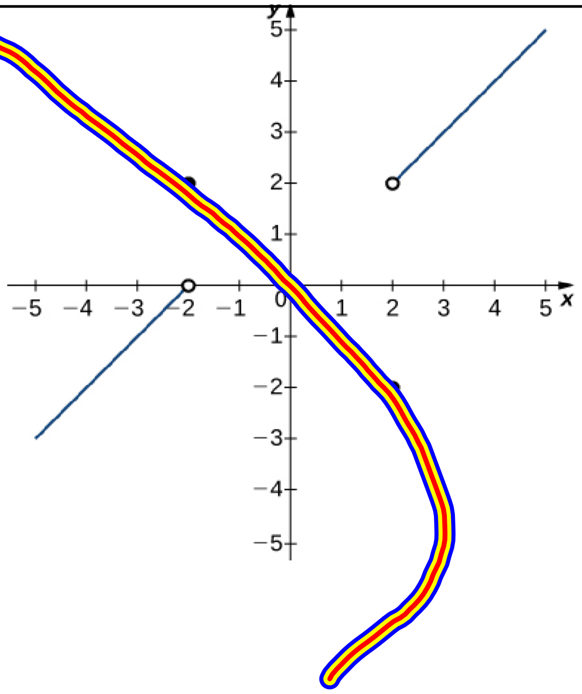


2. $\lim_{x \to c^-} f(x)$ unbounded



3. f(x) oscillating
   $\lim_{x \to c^-} f(x)$ DNE





60. $\lim_{x \to -2^+} f(x)$ 2

62. $\lim_{x \to 2^-} f(x)$ 0

64. $\lim_{x \to 2} f(x)$ DNE because 2 ≠ 0

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2.3 Evaluating Limits

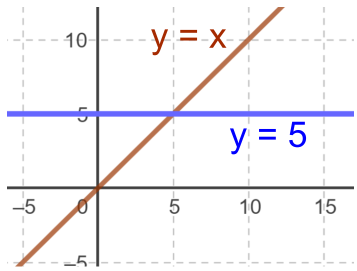
Find Limits Analytically

Theorem 2.4: Basic Limit Results

For any real number a and any constant c,

i. $\lim_{x \to a} x = a$

ii. $\lim_{x \to a} c = c$



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2.3 Evaluating Limits

Find Limits Analytically

Theorem 2.5: Limit Laws

Let $f(x)$ and $g(x)$ be defined for all $x \neq a$ over some open interval containing a. Assume that L and M are real numbers such that $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$. Let c be a constant. Then, each of the following statements holds:

Sum law for limits: $\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = L + M$

Difference law for limits: $\lim_{x\to a}(f(x)-g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = L - M$

Constant multiple law for limits: $\lim_{x\to a} cf(x) = c \cdot \lim_{x\to a} f(x) = cL$

Product law for limits: $\lim_{x\to a}(f(x)\cdot g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) = L \cdot M$

Quotient law for limits: $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{L}{M}$ for $M \neq 0$

Power law for limits: $\lim_{x\to a}(f(x))^n = \left(\lim_{x\to a} f(x)\right)^n = L^n$ for every positive integer n.

Root law for limits: $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)} = \sqrt[n]{L}$ for all L if n is odd and for $L \geq 0$ if n is even.

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2.3 Evaluating Limits

Find Limits Analytically

$$f(x) = \frac{x - 1}{x^2 - 1}, \quad x \neq \pm 1$$

f(x) does not exist at +1 or -1, but what is the value of *f(x)* when x is near +1? near -1?

Investigate limit of f(x) as $x \Rightarrow +1$ and as $x \Rightarrow -1$

$x \Rightarrow +1$

Subst: $f(1) = \frac{1 - 1}{1 - 1} = \frac{0}{0}$

Indeterminate Form
1. hole in graph at x=1
2. limit as x-> 1 exists

$x \Rightarrow -1$

$f(-1) \Rightarrow \frac{-2}{0}$

Unbounded

DNE

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2.3 Evaluating Limits

Find Limits Analytically (Algebraically)

$$f(x) = \frac{x - 1}{x^2 - 1}, \quad x \neq \pm 1$$

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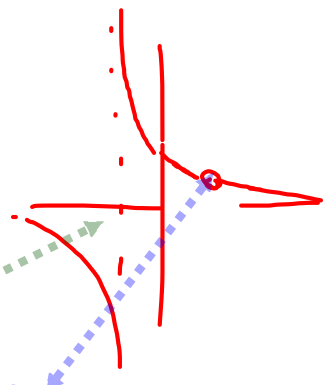
2.3 Evaluating Limits

Find Limits Analytically

$$f(x) = \frac{x - 1}{x^2 - 1}, \quad x \neq \pm 1$$



Subst: $f(1) = \frac{1 - 1}{1 - 1} = \frac{0}{0}$

$$f(x) = \frac{x - 1}{(x+1)(x-1)} = \frac{1}{x+1} \cdot \frac{x - 1}{x - 1}$$

as x->-1
f(x) unbounded
VA: vertical asymptote

as x->+1
f(x) has single undefined point
(hole)

as x->-1
limit DNE

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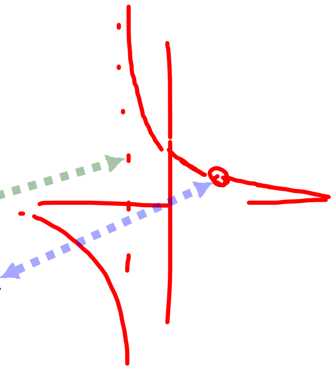
2.3 Evaluating Limits

$f(x) = \frac{x-1}{x^2-1}, \quad x \neq \pm 1$

Subst: $f(1) = \frac{1-1}{1-1} = \frac{0}{0}$

$f(x) = \frac{x-1}{(x+1)(x-1)} = \frac{1}{x+1} \cdot \frac{x-1}{x-1}$



$f(x) = \frac{x-1}{x^2-1} \stackrel{?}{=} \frac{1}{x+1}$

$x \neq \pm 1 \qquad x \neq -1$

Not quite equal; different domains.

Right one includes (1,1/2)

$\lim_{x \to 1} \frac{x-1}{x^2-1} = \lim_{x \to 1} \frac{1}{x+1} = \frac{1}{2}$

Yet, as x->1 limit exists

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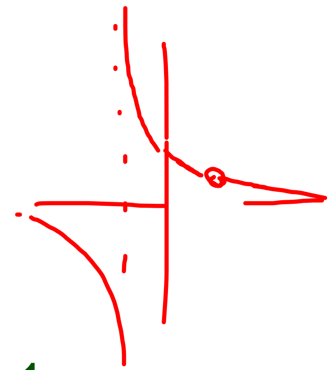
2.3 Evaluating Limits

$f(x) = \frac{x-1}{x^2-1}, \quad x \neq \pm 1$

$f(x) = \frac{x-1}{x^2-1} \stackrel{?}{=} \frac{1}{x+1}$

$x \neq \pm 1 \qquad x \neq -1$



$\lim_{x \to 1} \frac{x-1}{x^2-1} = \lim_{x \to 1} \frac{1}{x+1} = \frac{1}{2}$

The function on the left, f(x), is not defined at x=1 or x=-1.
The function on the right, 1/(x+1), is not defined at x= -1, but it is defined at x=1, at the point (1, 0.5).
So, these functions are equal, except for the point (1,0.5).
That means that as x -> 1, the y values of both are equal, and they are approaching the same limit, 0.5.

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2.3 Evaluating Limits

Summary: To Find a Limit

1. Substitute x = c

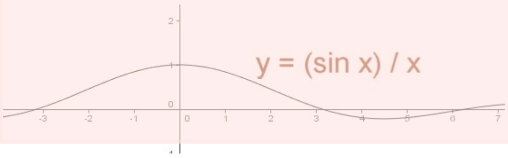
a) If finite number, L, then the limit is L.

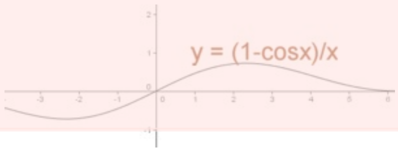
b) If results in form, k / 0 , then f is unbounded and the limit DNE

2. If indeterminate, use algebra to find a function that is equivalent at all but the undefined point, and substitute again. $\frac{0}{0}$ $\frac{\infty}{\infty}$

3. If still indeterminate, consider special limits:

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \lim_{x\to 0}\frac{1-\cos x}{x}=0$$





(sinx)/x*

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2.3 Evaluating Limits

$$\lim_{x\to 1} (-x^2 + 1)$$

$$\lim_{x\to 3} \frac{3 - x}{x^2-9}$$

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2.3 Evaluating Limits

$$\lim_{x \to 1} (-x^2 + 1) = -1 + 1 = 0$$

$$\lim_{x \to 3} \frac{3 - x}{x^2 - 9} \to \frac{0}{0}$$ limit exists

$$\lim_{x \to 3} \frac{-(x - 3)}{(x+3)(x-3)} = \lim_{x \to 3} \frac{-1}{x+3} = \frac{-1}{6}$$

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2.3 Evaluating Limits

$$\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$$

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2.3 Evaluating Limits

$$\lim_{x\to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x} \rightarrow \frac{\sqrt{2}-\sqrt{2}}{0} = \frac{0}{0}$$ ∴ limit exist

$$\frac{\sqrt{2+x} - \sqrt{2}}{x} \cdot \frac{\sqrt{2+x} + \sqrt{2}}{\sqrt{2+x} + \sqrt{2}} \qquad x\neq 0,\ x>-2$$

$$\frac{2+x-2}{x(\sqrt{2+x}+\sqrt{2})} = \frac{x}{x(\sqrt{2+x}+\sqrt{2})} = \frac{1}{\sqrt{2+x}+\sqrt{2}}$$

$$\lim_{x\to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x} = \lim_{x\to 0} \frac{1}{\sqrt{2+x}+\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

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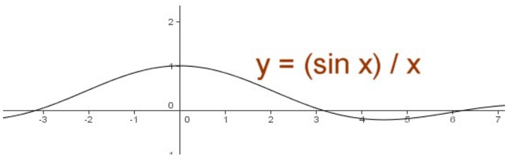
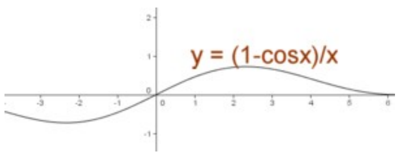
2.3 Evaluating Limits

Summary: To Find a Limit

1. **Substitute x = c**
   a) If finite number, L, then the limit is L.
   b) If results in form, k / 0 , then f is unbounded and the limit DNE
2. **If indeterminate, use algebra to find a function that is equivalent at all but the undefined point, and substitute again.** $\frac{0}{0}$ $\frac{\infty}{\infty}$
3. **If still indeterminate, consider special limits:**

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x\to 0} \frac{1-\cos x}{x} = 0$$





(sinx)/x*

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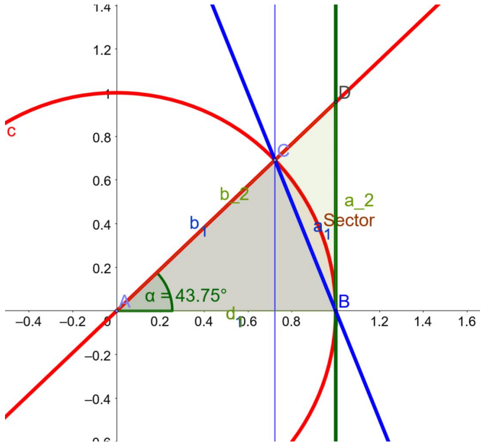
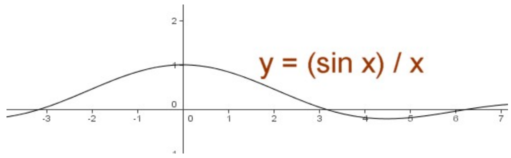
2.3 Evaluating Limits

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} \to \frac{0}{0}$ limit exists





(sinx)/x*

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2.3 Evaluating Limits

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sin θ < θ < tan θ

$\frac{\sin\theta}{\sin\theta} < \frac{\theta}{\sin\theta} < \frac{\tan\theta}{\sin\theta}$

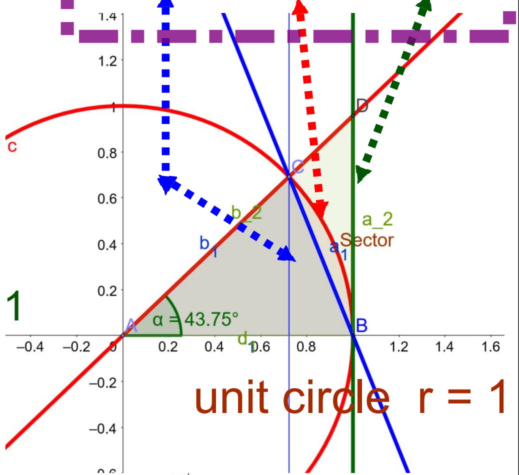
$1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$

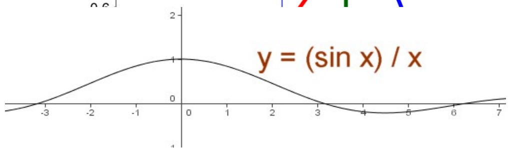
$1 > \frac{\sin\theta}{\theta} > \cos\theta$

$\lim_{\theta\to 0} 1 = 1$ $\lim_{\theta\to 0} \cos\theta = 1$

$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$

by Squeeze Theorum





(sinx)/x*

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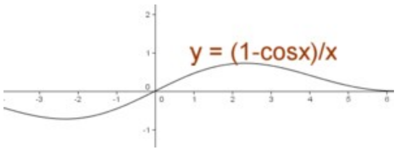
2.3 Evaluating Limits

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

$\frac{0}{0}$ limit exists



(sinx)/x*

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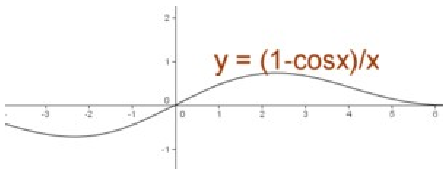

$$\lim_{x \to 0} \frac{1 - \cos x}{x}$$

$$\frac{1 - \cos x}{x} \cdot \frac{1 + \cos x}{1 + \cos x}$$

$$\frac{1 - \cos^2 x}{x(1 + \cos x)} = \frac{\sin^2 x}{x(1 + \cos x)} = \frac{\sin x}{x} \cdot \frac{\sin x}{1 + \cos x}$$

$$\lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{1 + \cos x} = 1 \cdot \frac{0}{2} = 0$$

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2.3 Evaluating Limits

$$\lim_{x\to 2} \frac{x^3 - 8}{x - 2} \rightarrow \frac{0}{0} \quad \therefore \text{limit exists}$$

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2.3 Evaluating Limits

$$\lim_{x\to 2} \frac{x^3 - 8}{x - 2} \rightarrow \frac{0}{0} \quad \therefore \text{limit exists}$$

synthetic division to factor x^3-8

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & |\,0 \end{array}$$

$$\frac{(x-2)(1x^2+2x+4)}{(x-2)}$$

$$\lim_{x\to 2} \frac{x^3 - 8}{x - 2} = \lim_{x\to 2} x^2+2x+4 = 12$$

synthetic division

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2.3 Evaluating Limits

G: $g(x) = \frac{x^2 - x}{x}$ F: $\lim_{x \to 0} g(x)$

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2.3 Evaluating Limits

G: $g(x) = \frac{x^2 - x}{x}$ F: $\lim_{x \to 0} g(x)$

$$g(x) = \frac{x(x-1)}{x} = x - 1, \; x \neq 0$$

$$\lim_{x \to 0} \frac{x^2 - x}{x} = \lim_{x \to 0} x - 1 = -1$$

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2.3 Evaluating Limits

$$\lim_{x\to 4} \frac{\sqrt{x+5} - 3}{x - 4} \rightarrow \frac{3-3}{0} \quad \frac{0}{0}$$

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2.3 Evaluating Limits

$$\lim_{x\to 4} \frac{\sqrt{x+5} - 3}{x - 4} \rightarrow \frac{3-3}{0} \quad \frac{0}{0}$$

$$\frac{\sqrt{x+5} - 3}{x - 4} \cdot \frac{\sqrt{x+5} + 3}{\sqrt{x+5} + 3}$$

$(a+b)(a-b)$

$a^2 - b^2$

$$= \frac{x+5-9}{(x-4)(\sqrt{x+5}+3)} = \frac{x-4}{(x-4)(\sqrt{x+5}+3)} = \frac{1}{(\sqrt{x+5}+3)}$$

$$\lim_{x\to 4} \frac{\sqrt{x+5} - 3}{x - 4} = \lim_{x\to 4} \frac{1}{(\sqrt{x+5}+3)} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

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2.3 Evaluating Limits

$$\lim_{x \to 0} \frac{\tan x}{\tan 2x} \to \frac{0}{0}$$

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2.3 Evaluating Limits

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$$\lim_{x \to 0} \frac{\tan x}{\tan 2x} \to \frac{0}{0}$$

$$\frac{\tan x}{\tan 2x} = \frac{\sin x / \cos x}{\sin 2x / \cos 2x}$$

$$\frac{\sin x}{\cos x} \cdot \frac{\cos 2x}{\sin 2x} = \frac{\sin x}{\cos x} \cdot \frac{\cos 2x}{2 \sin x \cos x} = \frac{\cos 2x}{2\cos^2 x}$$

$$\lim_{x \to 0} \frac{\tan x}{\tan 2x} = \lim_{x \to 0} \frac{\cos 2x}{2\cos^2 x} = \frac{1}{2}$$

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2.3 Evaluating Limits

$$\lim_{\Delta x \to 0} \frac{2(x+\Delta x) - 2x}{\Delta x}$$

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2.3 Evaluating Limits

$$\lim_{\Delta x \to 0} \frac{2(x+\Delta x) - 2x}{\Delta x} \to \frac{2(x+0) - 2x}{0}$$

$$\frac{2x-2x}{0} \quad \frac{0}{0}$$

$\therefore$ limit <u>exists</u>

$$= \lim_{\Delta x \to 0} \frac{2x + 2\Delta x - 2x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2\Delta x}{\Delta x} = \lim_{\Delta x \to 0} 2$$

$$= \boxed{2}$$

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