Functions

Calculus 1

Prof. Battaly

See: www.battaly.com/calc

REVIEW TOPIC: **FUNCTIONS**

GOAL: 1. Understand functions.

2. Learn function notation and evaluation.

3. Understand composite functions.

HW: 1.1 #7-19,29-39, 43-47, 28, 32

Functions

FUNCTION

$$y = f(x)$$

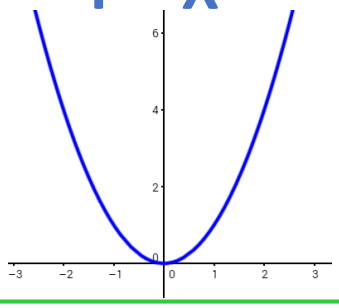
y is a function of x if:
for every value of x in the domain
there exists EXACTLY 1 value of y

Functions

Is y a function of x?

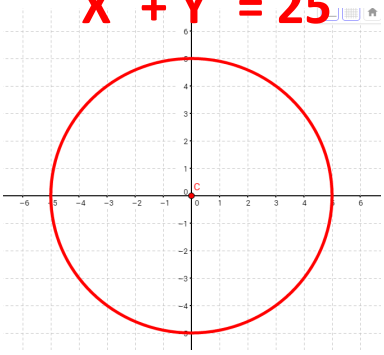
For each value of x, is there exactly 1 value for y?

$Y = X^2$



Use vertical line test.

$X^2 + Y^2 = 25$



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Calculus Home Page Homework

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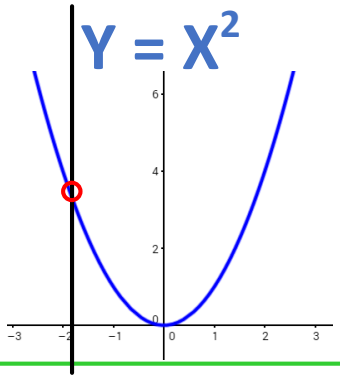
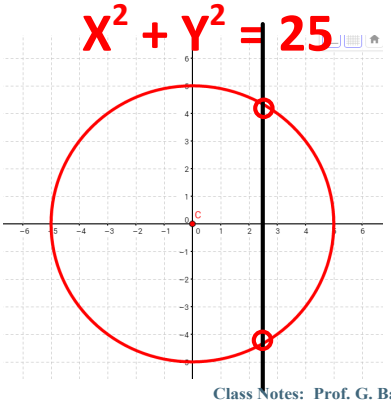


Use vertical line test.

Yes. For each value of x, there is exactly 1 value of y.

$X^2 + Y^2 = 25$



No. For many values of x, there is more than 1 value for y.

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Functions

Domain of a Function

The Domain of a function includes all the possible values of x that result in a real number for y

$y = \frac{1}{x - 1}$ D:

( ) U ( )

The Range of a function includes all the corresponding y values

R: y ?

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Functions

Domain of a Function

The Domain of a function includes all the possible values of x that result in a real number for y

$y = \frac{1}{x - 1}$ D: $x \neq 1$

$(-\infty, 1) \cup (1, \infty)$

The Range of a function includes all the corresponding y values

R: $y \neq 0$

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Calculus Home Page Homework

Functions

G: $f(x) = \sqrt{x+3}$

D: $x+3 \geq 0, \; x \geq -3$

F: $f(-2), f(6), f(-5), f(a+h), f(x+\Delta x)$

$f(\;) = \sqrt{(\;)+3}$

Place Holder:
In place of x we use ( )

$f(-2) = \sqrt{(\;)+3}$

$f(6) = \sqrt{(\;)+3}$

$f(-5) = \sqrt{(\;)+3}$

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$f(-2) = \sqrt{-2+3} = \sqrt{1} = 1$

$f(6) = \sqrt{6+3} = \sqrt{9} = 3$

$f(-5) = \sqrt{-5+3} = \sqrt{-2} \not\in$

$-5 \not\geq -3$

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$a+h \geq -3$

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$f(\quad) = \sqrt{(\quad)+3}$

Place Holder:
In place of x we use ( )

$f(x+\Delta x) = \sqrt{(x+\Delta x)+3}$

$f(x+\Delta x) = \sqrt{x+\Delta x+3}$

$x+\Delta x \geq -3$

Functions

$G: g(x) = x^2(x-4)$

$g(\ ) = (\ )^2[(\ )-4]$

a) $F: g(4) =$

b) $F: g\left(\frac{3}{2}\right) =$

Functions

$G: g(x) = x^2(x-4)$

$g(\ ) = (\ )^2[(\ )-4]$

a) $F: g(4) = (4)^2((4)-4) = 0$

b) $F: g\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2\left[\left(\frac{3}{2}\right)-4\right]$

$\frac{9}{4}\left(\frac{3}{2}-\frac{8}{2}\right)$

$\frac{9}{4}\cdot\frac{-5}{2} = -\frac{45}{8}$

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Functions

G: $g(x) = x^2(x-4)$ F: $g(t+4)$

t+4 is also a function; call it $f(t) = t+4$

Therefore g(t+4) is a composite function, a function of a function, or g(f(t))

$g(\quad) = (\quad)^2((\quad) - 4)$

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$g(\quad) = (\quad)^2((\quad) - 4)$

$g(f(t)) = (t+4)^2((t+4) - 4)$

$g(f(t)) = (t^2+8t+16)(t)$

$= t^3+8t^2+16t$ D: all real numbers

Functions

Composite Functions

G: $f(x) = \frac{3}{x}$, $g(x) = x^2 - 1$

F:

$f(g(x))$

$f \bullet g$

$g(f(x))$

$g \bullet f$

Functions

Composite Functions

G: $f(x) = \frac{3}{x}$, $g(x) = x^2 - 1$

F:

$f(g(x))$

$f \bullet g$

$g(f(x))$

$g \bullet f$

$f(\ ) = \frac{3}{(\ )}$

$f(g(x)) = \frac{3}{x^2 - 1}$

D: $x \neq \pm 1$

$g(\ ) = (\ )^2 - 1$

$g\left(\frac{3}{x}\right) = \left(\frac{3}{x}\right)^2 - 1$

$= \frac{9}{x^2} - \frac{x^2}{x^2} = \frac{9 - x^2}{x^2}$, $x \neq 0$

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Calculus Home Page Homework

$f(x)=4x^2-3x+1$

$f(\ )=4(\ )^2-3(\ )+1$

F: $f(0)$, $f(1)$, $f(3)$, $f(-x)$, $f(a)$, $f(a+h)$

$f(a+h)=4(a+h)^2-3(a+h)+1$

$=4(a^2+2ha+h^2)-3a-3h+1$

$=4a^2+8ha+4h^2-3a-3h+1$

$f(0)=0-0+1=1$

$f(3)=4(3^2)-3(3)+1$

$36-9+1=28$

$f(-x)=4(-x)^2-3(-x)+1$

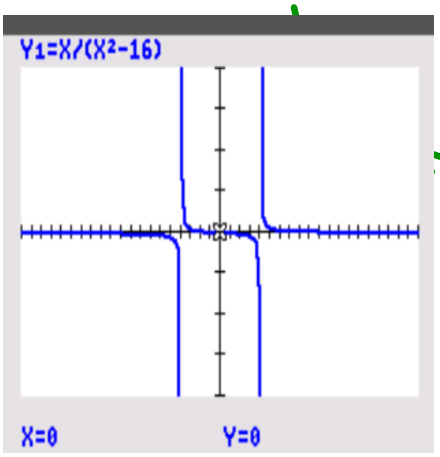
$4x^2+3x+1$

14. $f(x)=\dfrac{x}{x^2-16}$

F: D, R, zeros, inter. $(0,0)$

D: $x\neq\pm4$

R all real #'s.



44. $f(x)=2x+4$, $g(x)=x^2-2$

F: $f(g(x))$ | $g(f(x))$

$f(x^2-2)=2(x^2-2)+4$

$=2x^2-4+4$

$=2x^2$

$g(2x+4)=(2x+4)^2-2$

$=4x^2+16x+16-2$

$=4x^2+16x+14$

$y=\dfrac{1}{\frac{x^2-3x+2}{(x-2)(x-1)}}$ D: den $\neq 0$

$x\neq 2,1$

R: $y=\sqrt{x^2+2x+3}$, $\sqrt{\bigcirc}\geq 0$

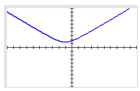
$x^2+2x+3\geq 0$ $b^2-4ac=4-4(1)(3)$

$=4-12<0$

$\therefore x^2+2x+3\neq 0$ imaginary roots

axis of symmetry $x=\frac{-b}{2a}=\frac{-2}{2}=-1$

Vertex: $(-1,\sqrt{2})$ $y|_{x=-1}=\sqrt{1-2+3}=\sqrt{2}$



$1x^2$ opens up $\therefore$ min

since $\sqrt{x^2+2x+3}$

R: $y\geq\sqrt{2}$