11.2 Series

Goals:

1. Infinite series: A **series** is an infinite **summation**.
2. Consider **partial sums** of a series. Then consider the sequence of partial sums.
3. If the sequence of partial sums converges, then the **series converges**.
4. If the sequence of partial sums diverges, then the **series diverges**.
4. For **telescoping series**, intermediate terms 'drop out' - opposites add to 0.
5. Each term in a **geometric series** is a fixed multiple of the previous term.
6. **Repeating decimals** result from a geometric series.

Study 11.2 # 1, 3, 5, 9, 15, 21-25, 29-33, 41-45, 49, 57

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11.2 Series

1. Infinite series: A **series** is an infinite **summation**.

Sequence: $a_n = \frac{3n}{n+1}$

$$\left\{ \frac{3}{2}, 2, \frac{9}{4}, \frac{12}{5}, \frac{15}{6}, \frac{18}{7}, \ldots.. \right\}$$

Series:

$$\sum_{n=1}^{\infty} \frac{3n}{n+1} = \frac{3}{2} + 2 + \frac{9}{4} + \frac{12}{5} + \frac{15}{6} + \frac{18}{7} + \ldots..$$

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$\sum_{n=1}^{\infty}$

11.2 Series

Consider **partial sums** of a series.
Then consider the **sequence of partial sums**.

Sequence: $a_n = \frac{3n}{n+1}$ $\left\{\frac{3}{2}, 2, \frac{9}{4}, \frac{12}{5}, \frac{15}{6}, \frac{18}{7}, ...\right\}$

Series: $\sum_{n=1}^{\infty} \frac{3n}{n+1} = \frac{3}{2} + 2 + \frac{9}{4} + \frac{12}{5} + \frac{15}{6} + \frac{18}{7} + ...$

Partial Sums: s_n is sum of first n terms, $n \to \infty$

$s_1 = 3/2$

$s_2 = 3/2 + 2 = 7/2$

$s_3 = 3/2 + 2 + 9/4 = 23/4$

$s_4 = 3/2 + 2 + 9/4 + 12/5 = 163/20$

$s_5 = 3/2 + 2 + 9/4 + 12/5 + 15/6 = 639/60$

Sequence: {1.5, 2, 2.25, 2.4, 2.5, ... }

Sequence of Partial Sums: {1.5, 3.5, 5.75, 8.15, 10.65, ...}

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11.2 Series

If the seq of partial sums converges, the **series converges**.
If the seq of partial sums diverges, the **series diverges**.

Series: $\sum_{n=1}^{\infty} \frac{3n}{n+1} = \frac{3}{2} + 2 + \frac{9}{4} + \frac{12}{5} + \frac{15}{6} + \frac{18}{7} + ...$

Sequence: {1.5, 2, 2.25, 2.4, 2.5, ..., a_{2000}=2.998501, ... }

Sequence converges to the limit 3 $\lim_{n\to\infty} \frac{3n}{n+1} = 3$

Sequence of Partial Sums: {1.5, 3.5, 5.75, 8.15, 10.65, ...,s_{2000}=5978.5, ...}

index	3n/(n+1)	sum
1	1.5	1.5
2	2	3.5
3	2.25	5.75
4	2.4	8.15
5	2.5	10.65
1997	2.998498	5969.468
1998	2.998499	5972.466
1999	2.9985	5975.465
2000	2.998501	5978.463
2001	2.998501	5981.462
2002	2.998502	5984.46

Sequence of partial sums **diverges**:
* near n=1, sums are increasing
* near n=2000, sums still increasing
* logic: if sequence is converging to 3, then for large n, adding 3 to each term

Series diverges

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11.2 Series

If the seq of partial sums converges, the **series converges**.
If the seq of partial sums diverges, the **series diverges**.

Series: $\sum_{n=1}^{\infty} \frac{1}{2^n}$ Converge? or diverge?

Sequence:

Sequence of Partial Sums:

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11.2 Series

If the seq of partial sums converges, the **series converges**.
If the seq of partial sums diverges, the **series diverges**.

Series: $\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots..$

Sequence: {0.5, 0.25, 0.125, 0.0625, 0.03125, ... }

Sequence converges to the limit 0 ← $\lim_{n\to\infty} \frac{1}{2^n} = 0$

Sequence of Partial Sums: {0.5, 0.75, 0.875, 0.9375, 0.96875, ...}

Series converge? or diverge?

Sequence of partial sums increasing and monotonic. Is it bounded?

* LOOK AT FRACTIONS for pattern

$\left\{ \frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots ?\ldots.. \right\}$ → Denominator = 2^n, numerator 1 less

$\left\{ \frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots \frac{2^n-1}{2^n} \right\}$

$\lim_{n\to\infty} \frac{2^n-1}{2^n} = 1$

Series converges to 1

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11.2 Series

For **telescoping series**, intermediate terms 'drop out' - opposites add to 0.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} = \frac{A(n+1) + Bn}{n(n+1)}$$

$$1 = A(n+1) + Bn = (A+B)n + A$$

$$0n + 1 = (A+B)n + A$$

$$A+B = 0, \quad A = 1, \quad B = -1$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$s_n = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) \dots + \dots \left(\frac{1}{n} - \frac{1}{n+1}\right) = 1 - \frac{1}{n+1}$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right) = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right) = 1$$

Series converges

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11.2 Series

For **telescoping series**, intermediate terms 'drop out' - opposites add to 0.

$$\sum_{n=1}^{\infty} \frac{4}{n(n+2)}$$

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11.2 Series

For **telescoping series**, intermediate terms 'drop out' - opposites add to 0.

$$\sum_{n=1}^{\infty} \frac{4}{n(n+2)}$$

$$\frac{4}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2} = \frac{A(n+2) + Bn}{n(n+2)}$$

$$4 = A(n+2) + Bn = (A+B)n + 2A$$

$$0n + 4 = (A+B)n + 2A$$

$$A+B = 0, \quad 2A = 4, \quad A=2 \therefore B = -2$$

$$\sum_{n=1}^{\infty} \left(\frac{2}{n} - \frac{2}{n+2}\right)$$

$$s_n = \left(\frac{2}{1} - \frac{2}{3}\right) + \left(\frac{2}{2} - \frac{2}{4}\right) + \left(\frac{2}{3} - \frac{2}{5}\right) + \left(\frac{2}{4} - \frac{2}{6}\right) + \ldots + \left(\frac{2}{n} - \frac{2}{n+2}\right)$$

$$s_n = \frac{2}{1} + \frac{2}{2} - \frac{2}{n+1} - \frac{2}{n+2}$$

1st term of n=1 and n=2
and last terms of n-1 and n

$$\sum_{n=1}^{\infty} \left(\frac{2}{n} - \frac{2}{n+2}\right) = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \left(3 + \frac{2}{n} - \frac{2}{n+2}\right) = 3$$

Series converges to 3

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11.2 Series

Geometric Series: fixed multiple for each subsequent term.

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$$

If $|r| < 1$, or $-1 < r < 1$

the Geometric Series Converges to:

$$\frac{a}{1 - r}$$

$$\sum_{n=1}^{\infty} \frac{3}{2^{(n-1)}}$$

Converge? or Diverge?

a = ___? r = ____?

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Calculus Home Page Homework $\sum_{n=0}^{\infty}$ $\sum_{n=1}^{\infty}$

11.2 Series

Geometric Series: fixed multiple for each subsequent term.

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$$

If $|r| < 1$, or $-1 < r < 1$

the Geometric Series Converges to: $\dfrac{a}{1 - r}$

$$\sum_{n=1}^{\infty} \frac{3}{2^{(n-1)}} = \sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{(n-1)}$$ a = 3 r = 1/2

$$\sum_{n=1}^{\infty} \frac{3}{2^{(n-1)}} = \sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{(n-1)} = \frac{3}{1-1/2} = \frac{3}{1/2} = 6$$

Series converges to 6

$\sum_{n=0}^{\infty}$ $\sum_{n=1}^{\infty}$

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11.2 Series

Geometric Series: fixed multiple for each subsequent term.

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$$

$$\sum_{n=0}^{\infty} \left(\frac{-1}{3}\right)^{n} =$$

$$\sum_{n=1}^{\infty} \left(\frac{-1}{3}\right)^{n} =$$

Which of the following should be used?

$$\sum_{n=1}^{\infty} \left(\frac{-1}{3}\right)\left(\frac{-1}{3}\right)^{n-1} \quad \text{or} \quad \sum_{n=1}^{\infty} \left(\frac{-1}{3}\right)^{n-1}$$

Hint: Check 1st term.

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11.2 Series

Geometric Series: fixed multiple for each subsequent term.

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$$

$$\sum_{n=0}^{\infty}\left(\frac{-1}{3}\right)^n = \sum_{n=1}^{\infty}\left(\frac{-1}{3}\right)^{n-1}$$

a = 1, r = -1/3

| r | = |-1/3| < 1

Series converges, to what?

Sequence converges to 0

$$\frac{a}{1-r} = \frac{1}{1-(-1/3)} = \frac{1}{1+1/3} = \frac{3}{3+1} = \frac{3}{4}$$

Series converges to 3/4

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11.2 Series

Geometric Series: fixed multiple for each subsequent term.

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$$

$$\sum_{n=1}^{\infty}\left(\frac{-1}{3}\right)^n = \sum_{n=1}^{\infty}\left(\frac{-1}{3}\right)\left(\frac{-1}{3}\right)^{n-1}$$

a = -1/3, r = -1/3

| r | = |-1/3| < 1

Series converges, to what?

(**Sequence** converges to 0)

$$\frac{a}{1-r} = \frac{-1/3}{1-(-1/3)} = \frac{-1}{3+1} = \frac{-1}{4}$$

Series converges to 1/4

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11.2 Series

Repeating decimals as a geometric series.

Write the decimal number 0.48484848...as a fraction.

$$0.48484848... = \frac{48}{10^2} + \frac{48}{10^4} + \frac{48}{10^6} + \frac{48}{10^8} + ...$$

$$\sum_{n=1}^{\infty} \frac{48}{10^2}\left(\frac{1}{10^2}\right)^{n-1}$$

$a = 48/10^2$ $r = 1/10^2$

$$0.\overline{48} = \frac{a}{1 - r} = \frac{48/\,10^2}{1 - 1/10^2} = \frac{48}{10^2-1} = \frac{48}{99}$$

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11.2 Series

Repeating decimals as a geometric series.

Write the decimal number 0.123123123...as a fraction.

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11.2 Series

Repeating decimals as a geometric series.

Write the decimal number 0.123123123...as a fraction.

$$0.123123123... = \frac{123}{10^3} + \frac{123}{10^6} + \frac{123}{10^9} + \frac{123}{10^{12}} + ...$$

$$\sum_{n=1}^{\infty} \frac{123}{10^3}\left(\frac{1}{10^3}\right)^{n-1}$$

$a = 123/10^3$ $r = 1/10^3$

$$0.\overline{123} = \frac{a}{1 - r} = \frac{123/10^3}{1 - 1/10^3} = \frac{123}{10^3-1} = \frac{123}{999}$$

11.2 Series

Convergent or Divergent?.

Theorum: If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

If an infinite series is convergent, then the terms approach zero.

Test for Divergence: If the $\lim_{n\to\infty} a_n$ does not exist, or $\lim_{n\to\infty} a_n \neq 0$ then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

nth Term Test for Divergence If the terms of an infinite series **do not approach zero, then the series diverges.**

Note: If the terms approach zero, series may or may not converge.

Logic: If A, then B is true then also true: If not B, then not A

but it is **not** necessarily **true**: If B, then A

eg: If baby, then either boy or girl. Does not mean that if boy or girl, then baby.

$\lim_{n\to\infty}$ $\sum_{n=1}^{\infty}$

11.2 Series

Convergent or Divergent?.

Series that diverge:

$\sum_{n=1}^{\infty} (-1)^{n-1}$

$\sum_{n=1}^{\infty} \left(\frac{7}{6}\right)^{n-1}$

$\sum_{n=1}^{\infty} \frac{n}{n+1}$

$\lim_{n\to\infty}$

$\lim_{n\to\infty}$

$\sum_{n=1}^{\infty}$

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11.2 Series

Convergent or Divergent?.

Series that diverge:

$\sum_{n=1}^{\infty} (-1)^{n-1}$ $r = -1$, $|r| \not< 1$ ∴ diverges

$\sum_{n=1}^{\infty} \left(\frac{7}{6}\right)^{n-1}$ $r = \frac{7}{6} > 1$ ∴ diverges

$\sum_{n=1}^{\infty} \frac{n}{n+1}$ $\lim_{n\to\infty} a_n = 1 \not< 1$ ∴ diverges

$\sum_{n=1}^{\infty}$

$\lim_{n\to\infty}$

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11.2 Series

Series that diverge:

Note: If the terms approach zero, series may or may not converge.

Logic: If A, then B is true then also true: If not B, then not A
but it is **not** necessarily **true**: If B, then A

Harmonic Series

$$\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots \qquad \lim_{n\to\infty} a_n = 0$$

Does series Converge or Diverge?

text: shows $s_{2^n} > 1 + n/2 \longrightarrow \infty$ as $n \longrightarrow \infty$

$\therefore$ Series Diverges

$\sum_{n=1}^{\infty}$

$\sum_{n=1}^{\infty}$ $\sum_{n=1}^{\infty}$

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$\lim_{n\to\infty}$