11.1 Sequences

Goals:

1. Understand that a sequence is an infinite list of real numbers.
2. If the limit of a sequence exists, then the sequence converges. If the limit does not exist, then the sequence diverges.
3. If $|r| < 1$, then, as n increases indefinitely, limit of r^n is 0.
4. A monotonic sequence is nondecreasing or nonincreasing.
5. Sequences can be bounded above, below, or both.
6. If a sequence is bounded and monotonic, it converges.

Study 11.1 # 1 - 43, 65

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11.1 Sequences

Sequence: $\{a_n\}=\{a_1, a_2, a_3, ... \}$

1. Infinite list of real numbers.
2. Function whose domain is the set of positive integers.

$$a_n = \frac{3n}{n+1}$$

n	1	2	3	4	5	6	....

$$\left\{ \frac{3}{2},\ 2,\ \frac{9}{4},\ \frac{12}{5},\ \frac{15}{6},\ \frac{18}{7},\ ..... \right\}$$

can start with a different number for n

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11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, ... \}$

List the first 5 terms:

$$a_n = \frac{n^2 - 1}{n^2 + 1}$$

n 1 2 3 4 5 ....

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Homework

11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, ... \}$

List the first 5 terms:

$$a_n = \frac{n^2 - 1}{n^2 + 1}$$

n 1 2 3 4 5 ....

$$\left\{ 0, \frac{3}{5}, \frac{8}{10}, \frac{15}{17}, \frac{24}{26}, \quad ..... \right\}$$

$$\left\{ 0, \frac{3}{5}, \frac{4}{5}, \frac{15}{17}, \frac{12}{13}, \quad ..... \right\}$$

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Homework

11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, ... \}$

Limits of Sequences

The sequence $\{a_n\}$ has a limit L

$$\lim_{n\to\infty} a_n = L$$

if the terms, a_n , approach L when $n \Rightarrow \infty$

Let L be a real number.
The **limit** of the sequence $\{ a_n \}$ is L if
for all $\varepsilon > 0$, there exists $M > 0$ such that
$|a_n - L| < \varepsilon$ whenever $n > M$.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2 - 1}{n^2 + 1} = 1$$

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11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, ... \}$

Limits of Sequences

The sequence $\{a_n\}$ has a limit L
if the terms, a_n , approach L when $n \Rightarrow \infty$

$$\lim_{n\to\infty} a_n = L$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2 - 1}{n^2 + 1} = 1$$

Is it legitimate to use previous techniques of finding limits to find these limits?

Theorem:
(connection to definition of limit for a function in Calc 1)

If $\lim_{n\to\infty} f(x) = L$ and $f(n) = a_n$ when n is an integer,

then $\lim_{n\to\infty} a_n = L$

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11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, ... \}$

Sequences: Convergent or Divergent?

If a sequence has a limit, it is convergent.

If a sequence does NOT have a limit, it is divergent.

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Homework

11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, ... \}$

Limits of Sequences

The sequence $\{a_n\}$ has a limit L if the terms, a_n , approach L when $n \Rightarrow \infty$

$$\lim_{n\to\infty} a_n = L$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2 - 1}{n^2 + 1} = 1$$

Convergent

$$\left\{ 0, \frac{3}{5}, \frac{4}{5}, \frac{15}{17}, \frac{12}{13}, \ ..... \right\}$$

0, 0.6, 0.8, 0.88235, 0.92308, ...

$$\lim_{n\to\infty} \frac{n^2-1}{n^2+1} = \lim_{n\to\infty} \frac{1-\frac{1}{n^2}}{1+\frac{1}{n^2}} = 1$$

Calculus Home Page

Homework

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11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, \ldots\}$

Limits of Sequences

The sequence $\{a_n\}$ has a limit L if the terms, a_n, approach L when $n \Rightarrow \infty$

$$\lim_{n\to\infty} a_n = L$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2-1}{3n^3} = 0$$

convergent

$$\frac{\frac{n^2}{n^3} - \frac{1}{n^3}}{\frac{3n^3}{n^3}} = \frac{\frac{1}{n} - \frac{1}{n^3}}{3}$$

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Homework

11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, \ldots\}$

Bounded and Monotonic Sequences

A sequence is **monotonic** if its terms are
nondecreasing $a_1 \le a_2 \le a_3 \le \ldots$
or nonincreasing $a_1 \ge a_2 \ge a_3 \ge \ldots$

A sequence is **bounded above** if there is a real number M such that $a_n \le M$ for all a_n

A sequence is **bounded below** if there is a real number N such that $a_n \ge N$ for all a_n

A sequence is **bounded** if it is both bounded above and bounded below

If a sequence is bounded and monotonic, then it **converges**.

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11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, ... \}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2-1}{n^2+1} = 1$$

$$\left\{ 0, \frac{3}{5}, \frac{4}{5}, \frac{15}{17}, \frac{12}{13}, ... \right\}$$

0 , 0.6, 0.8, 0.88235, 0.92308 , ...

monotonic? and bounded?

YES YES ∴ Convergent

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11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, ... \}$

$$\lim_{n\to\infty}(3+(-1)^n)$$

$$\lim_{n\to\infty} \sin\frac{1}{n}$$

$$\lim_{n\to\infty}\left(5-\frac{1}{n^2}\right)$$

$$\lim_{n\to\infty} n\sin\frac{1}{n}$$

Example 2: Limits of Sequences

a. $\lim_{n\to\infty}\left(3+(-1)^n\right)$ does not exist.

b. $\lim_{n\to\infty}\left(5-\frac{1}{n^2}\right)=5$.

c. $\lim_{n\to\infty}\sin\frac{1}{n}=0$.

d. $\lim_{n\to\infty} n\sin\frac{1}{n}=\lim_{n\to\infty}\frac{\sin 1/n}{1/n}=\lim_{x\to 0}$ ...

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11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, ... \}$

$$\lim_{n\to\infty}(3+(-1)^n)$$

Diverges

$\{2, 4, 2, 4, ...\}$

$$\lim_{n\to\infty}\sin\frac{1}{n} = 0$$

Converges

$$\lim_{n\to\infty}\left(5-\frac{1}{n^2}\right) = 5$$

Converges

$$\lim_{n\to\infty} n\sin\frac{1}{n} = 1$$

Converges

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Homework

11.1 Sequences

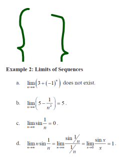
$\{a_n\}=\{a_1, a_2, a_3, ... \}$

$$\lim_{n\to\infty}(3+(-1)^n)$$

$\{2, 4, 2, 4, 2, 4, ...\}$

cycles DNE

Divergent



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11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, \ldots\}$

$$\lim_{n\to\infty} \sin\frac{1}{n} = 0$$

as $n \to \infty$, $\frac{1}{n} \to 0$

convergent

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11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, \ldots\}$

$$\lim_{n\to\infty} n \sin\frac{1}{n} = 1 \to \infty \cdot 0$$

$$\lim_{x\to 0} \frac{\sin x}{x}$$

$$\lim_{n\to\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}} \to \frac{0}{0}$$

$$\overset{LH}{=} \lim_{n\to\infty} \frac{\frac{-1\cos\frac{1}{n}}{n^2}}{-\frac{1}{n^2}} \cdot \frac{n^2}{n^2} = \frac{-\cos\frac{1}{n}}{-1}$$

$$= \cos\frac{1}{n} \to 1$$

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11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, ... \}$

F: a_n

$$\left\{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \quad ..... \right\}$$

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11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, ... \}$

F: a_n

$$\left\{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \quad ..... \right\}$$

$$a_n = \frac{n}{n+1}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{n+1} = 1$$

{ 0.5, 0.6667, 0.75, 0.80, ...}

monotonic, bounded

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11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, ... \}$

Fibonacci sequence:
1, 1, 2, 3, 5. 8, ...
the next term = sum of previous 2 terms

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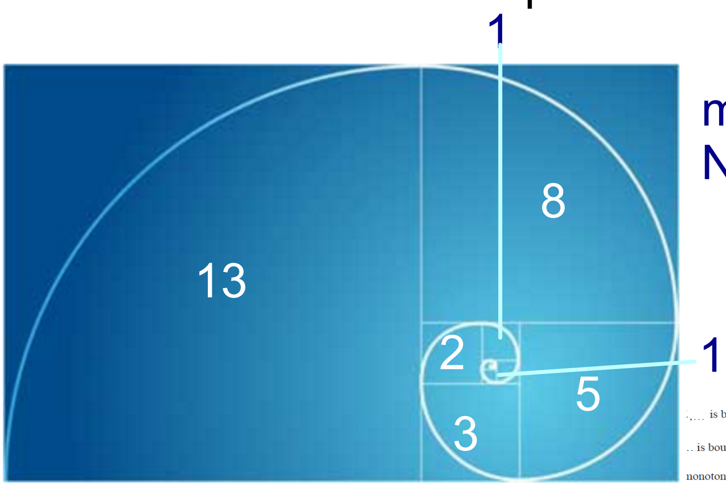
11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, ... \}$

Fibonacci sequence:
1, 1, 2, 3, 5. 8, ...
the next term = sum of previous 2 terms



monotonic,
NOT bounded

..., is bounded and monotonic.
.. is bounded but not monotonic.
nonotonic but not bounded.

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11.1 Sequences

$\{a_n\}=\{a_1, a_2, a_3, ... \}$

$\{ 2, 4, 2, 4, 2, 4, ... \}$

NOT monotonic, bounded

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