1.5 Infinite Limits and Vertical Asymptotes

p.88 # 1- 9, 13-27, 33-47, 59, 61

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1.5 Infinite Limits

y values $\to +\infty$ or $-\infty$

p. 88 #34. $\lim_{x \to 1^+} \frac{2+x}{1-x} \to \frac{3}{0}$ unbounded

Is $y \to +\infty$ or $-\infty$?

as $x \Rightarrow 1$ from the right

$1 - x < 0$

$\therefore \frac{2+x}{1-x} \to -\infty$ as $x \to 1^+$

$$f(x) = \frac{1}{x^2-25}, \quad x \neq \pm 5$$

$$g(x) = \frac{x-5}{x^2-25} = \frac{x-5}{(x+5)(x-5)} = \frac{1}{x+5}, \quad x \neq -5$$

VA: num $\neq 0$, den $= 0$.

Find the vertical asymptotes.

f(x): 2 VAs: x = 5 and x = -5

g(x): 1 VA: x = - 5

for g(x), x = 5 is not VA, when substitute in numerator, get 0: 5 -5 = 0

$$\lim_{x \to -1} \frac{x^2-1}{x+1} \to \frac{1-1}{-1+1} \quad \frac{0}{0}$$

$$\frac{(x+1)(x-1)}{(x+1)} = x-1$$

$$\lim_{x \to -1} (x-1) = -1-1 = -2$$

Since x = -1 results in denominator=0,
and also numerator =0,
x = -1 is NOT a VA.

$\lim_{x \to 0} \frac{5}{x} \to \frac{5}{0}$ unbounded DNE

Continuity

i. $f(c)$ exist — f defined at $x = c$

ii $\lim_{x \to c} f(x)$ exists — lim of f as $x \to c$ exists.

iii $\lim_{x \to c} f(x) = f(c)$ — lim = f(c)

So, VA occurs where function is not continuous because the limits does not exist.

$$f(x) = \frac{x-1}{x^2+x-2}$$

num, den both polyn. .. cont everywhere except where den = 0

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

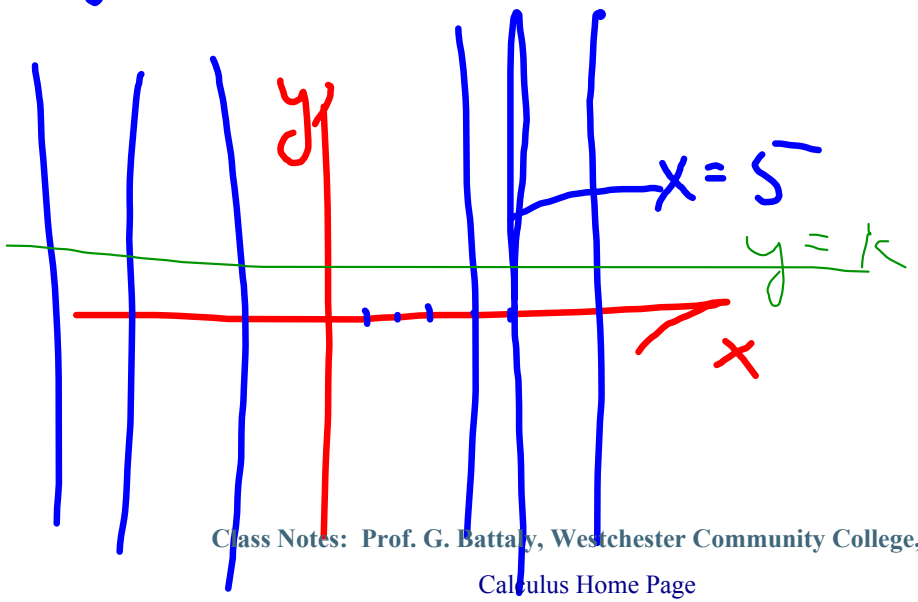
$x = -2$, $x = 1$ x values where f is discont.

For x = -2, numerator: x - 1≠ 0, so x = -2 is VA

For x = 1, numerator: x - 1= 0, so x = 1 is NOT a VA
(hole in graph)

VA : definition

If $f(x) \to \pm\infty$ as $x \to c$, fr. right or left, then the line $x = c$ is a VA of the graph of f.



$x = c$

To find VA:

1. den = 0 ... $x = c$
2. subst. c into num.

If num = 0, then $x = c$ Not VA

If num ≠ 0, then $x = c$ IS V.A.

want values of x that result in den = 0, num ≠ 0

p. 88 #10 $f(x) = \frac{4}{(x-2)^3}$ F: VA

$(x-2)^3 = 0 \therefore x = 2$, num $\neq 0$

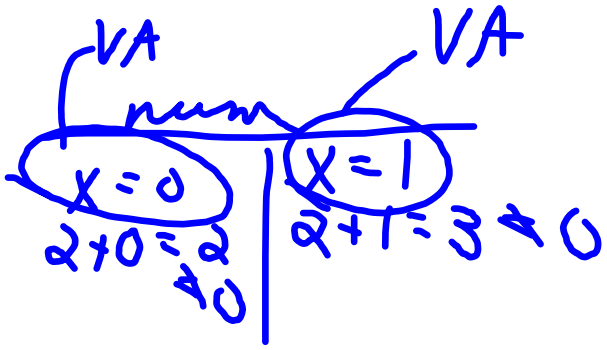
$\therefore x = 2$ VA

12. $g(x) = \frac{2+x}{x^2(1-x)}$ F: VA

$x^2(1-x) = 0$

$x = 0 \mid 1-x = 0$

$x = 1$



20. $g(x) = \dfrac{\frac{1}{2}x^3 - x^2 - 4x}{3x^2 - 6x - 24}$ F: VA

$3x^2 - 6x - 24 = 0$

$3(x^2 - 2x - 8) = 0$

$(x-4)(x+2) = 0$

$x = 4, -2$

num

$x = 4$	$x = -2$
$\frac{1}{2}\cdot 64 - 16 - 16$	$\frac{1}{2}(-8) - 4 - 4(-2)$
$32 - 16 - 16$	$-4 - 4 + 8$
0	0

$\therefore$ no VAs

$$\frac{(x-4)(x+1)(x)}{3(x-4)(x+2)}$$