1.4 Continuity

Homework for Section 1.4, Continuity and One-sided Limits

Study 1.4, p. 78 # 1 - 21, 29, 31, 37, 38,
45 - 53, 61, 69, 87, 91, 93

Homework on the Web *Part 1* *Part 2*


How would you describe the graph of a curve that is continuous on an interval?

unbroken

- asymptotes $y = \tan x$

- openings $y = \frac{x-3}{x^2-9}$ at $x = 3$

sketch: $y = \frac{x^2-9}{x-3}$ Is it continuous? at $x=3$

No! y not def at $x=3$

sketch: $y = \frac{1}{x}$

Is it continuous? at $x = 0$

sketch: $y = \begin{cases} x, & x < 1 \\ 2 & x = 1 \\ 2x-1, & x > 1 \end{cases}$

Is it continuous at $x = 1$?

NO.

At x = 1, y = 2, but as x -> 1 from both sides, the curve -> 1. So, there is a hole in the graph because the lim as x -> 1 does not equal the value of y when x = 1.

$\lim_{x \to 1} f(x) = 1$

$\lim_{x \to 1^-} x = 1 \overset{?}{=} \lim_{x \to 1^+} 2x-1 = 1$

What is needed for continuity at a point?

f(x)	c	$\lim_{x \to c} f(x)$	f(c)	Continuous at x = c?
$\frac{1}{x}$	0	DNE	DNE	NO
$\frac{x^2-9}{x-3}$	3	6	DNE	NO
$\begin{cases} x, & x<1 \\ 2 & x=1 \\ 2x-1, & x>1 \end{cases}$	1	1	2	NO
$x+3$	3	6	6	YES

What is needed for continuity at a point?

Sep 10-4:52 PM

Continuity at a Pt.

A function f cont. at c if:

1. $f(c)$ is defined
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

Required to show continuity.

What about continuity on an interval?

Consider $f(x) = \sqrt{(9-x^2)}$ $\qquad g(x) = \dfrac{1}{\sqrt{(9-x^2)}}$

What about continuity on an interval?

Consider $f(x) = \sqrt{(9-x^2)}$ $\quad g(x) = \dfrac{1}{\sqrt{(9-x^2)}}$

I. A function is continuous on an **open** interval (a,b) if it is continuous at each point in (a,b).

II. A function is continuous on an **closed** interval [a,b] if it is continuous at each point in (a,b), and

$$\lim_{x \to a^+} f(x) = f(a) \quad \textit{and} \quad \lim_{x \to b^-} f(x) = f(b)$$

III. Functions continuous on entire real number line are everywhere continuous.

#39 $G: g(x) = \frac{1}{x^2-4}$ F: cont. on $[-1, 2]$?

Sep 10-5:04 PM

IVT

If ① f is cont. on closed interval $[a,b]$

② $f(a) \neq f(b)$

③ and k is any # betw. $f(a)$ and $f(b)$

then $\exists$ at least one $c \in [a,b]$

$\ni f(c) = k$

IVT

If ① f is cont. on closed interval $[a,b]$

② $f(a) \neq f(b)$

③ and k is any # betw. $f(a)$ and $f(b)$

then $\exists$ at least one $c \in [a,b]$

$\ni f(c)=k$