P.6 Rational Expressions

GOALS:

1. Recognize Rational Expressions as the quotient of polynomials.
2. Understand restrictions on the Domain of Rational Expressions.
3. Simplify Rational Expressions by finding Common Factors in numerator and denominator and reducing to lowest terms.
4. Perform operations of addition, subtraction, multiplication and division of Rational Expressions.
5. Use the Multiplication Property of 1 to simplify complex fractions.

Study P.6 CVC 1-9, # 1-13;
15, 19, 23, ...77; 53

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P.6 Rational Expressions

Rational Expressions

1. Quotient of Two Polynomials

$\frac{1}{3x^2}$ $\frac{x+3}{x^2-9}$ $\frac{1-x}{2x^2-5x-3}$

2. Not defined when denominator = 0.

$x \neq 0$ $x \neq -3, +3$ $x \neq -1/2, 3$

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P.6 Rational Expressions

To Simplify Rational Expressions

$\frac{1}{3x^2}$ $\frac{x+3}{x^2-9}$ $\frac{1-x}{2x^2-5x-3}$

1. Look for common factors.

No common factors $x+3$ No common factors

Common factors include binomials as possibilities

2. Reduce to Lowest Terms (RLT)

$$\frac{x+3}{x^2-9}$$

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P.6 Rational Expressions

Simplify: $\frac{4x - 8}{x^2 - 4x + 4}$

Factor easiest first.
Look for the same factor(s) in other part.
If no common factor, then already simplified.

$\frac{4(x-2)}{(\ \ ?\ \ )(x-2)}$

4 is not a factor of the denominator.
Is (x-2) a factor of the denominator?
Substitute 2 into denominator:
$2^2 -4(2) + 4$? 0

Simplify: $\frac{x^2 - 8x + 16}{3x - 12}$

$= \frac{(\ \ ?\ \ )(x-4)}{3(x-4)}$

Factor easiest first.
Look for the same factor(s) in other part.
If no common factor, then already simplified.

3 is not a factor of the numerator.
Is (x-4) a factor of the numerator?
Substitute 4 into numerator:
$4^2 -8(4) + 16$? 0

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P.6 Rational Expressions

Simplify:

$$\frac{4x - 8}{x^2 - 4x + 4}$$

$$\frac{4(x-2)}{(x-2)(x-2)}$$

Factor easiest first.
Look for the same factor(s) in other part.
If no common factor, then already simplified.

4 is not a factor of the denominator.
Is (x-2) a factor of the denominator?
Substitute 2 into denominator: $2^2 - 4(2) + 4 ? 0$

RLT using multiplication property of 1

Simplify: $$\frac{x^2 - 8x + 16}{3x - 12}$$

$$= \frac{(x-4)(x-4)}{3(x-4)}$$

Factor easiest first.
Look for the same factor(s) in other part.
If no common factor, then already simplified.

RLT using multiplication property of 1

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P.6 Rational Expressions

Simplify:

$$\frac{4x - 8}{x^2 - 4x + 4}$$

$$\frac{4(x-2)}{(x-2)(x-2)}$$

$$\frac{4}{(x-2)} \bullet \frac{(x-2)}{(x-2)}$$

$$\frac{4}{(x-2)}$$

Factor easiest first.
Look for the same factor(s) in other part.
If no common factor, then already simplified.

RLT using multiplication property of 1

Simplify: $$\frac{x^2 - 8x + 16}{3x - 12}$$

$$= \frac{(x-4)^2}{3(x-4)} = \frac{(x-4)}{3}$$

Factor easiest first.
Look for the same factor(s) in other part.
If no common factor, then already simplified.

RLT using multiplication property of 1

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P.6 Rational Expressions

Simplify:

$$\frac{x^2 - 14x + 49}{x^2 - 49}$$

Factor easiest first.
Look for the same factor(s) in other part.
If no common factor, then already simplified.

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P.6 Rational Expressions

Simplify:

$$\frac{x^2 - 14x + 49}{x^2 - 49} = \frac{(\quad)(\quad)}{(x+7)(x-7)}$$

Factor easiest first.

$$= \frac{(x-7)(x-7)}{(x+7)(x-7)}$$

Look for the same factor(s) in other part.

$$= \frac{(x-7)\cancel{(x-7)}}{(x+7)\cancel{(x-7)}}$$

RLT: reduce to lowest terms.

$$= \frac{x-7}{x+7}$$

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P.6 Rational Expressions

Operations with Rational Expressions
Same as with Numerals

1. Multiply, Divide.

$$\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$$

2. Add, Subtract

$$\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$$

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$$

RLT; b,d ≠ 0

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P.6 Rational Expressions

Multiply: $\frac{6x + 9}{3x - 15} \bullet \frac{x - 5}{4x + 6}$

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P.6 Rational Expressions

Divide: $$\frac{x^2 - 4}{x^2 + 3x - 10} \div \frac{x^2 + 5x + 6}{x^2 + 8x + 15}$$

1, x ≠ -5,-2,-3,2

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P.6 Rational Expressions

Divide: $$\frac{x^2 - 4}{x^2 + 3x - 10} \div \frac{x^2 + 5x + 6}{x^2 + 8x + 15}$$

$$\frac{x^2 - 4}{x^2 + 3x - 10} \cdot \frac{x^2 + 8x + 15}{x^2 + 5x + 6}$$

$$\frac{(x+2)(x-2)}{(x+5)(x-2)} \cdot \frac{(x+5)(x+3)}{(x+2)(x+3)}$$

$$\frac{(x+2)}{(x+2)} \frac{(x-2)}{(x-2)} \cdot \frac{(x+5)}{(x+5)} \frac{(x+3)}{(x+3)}$$

$$= 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

1, x ≠ -2,2,-5,-3

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P.6 Rational Expressions

Divide: $$\frac{x^2 + x}{x^2 - 4} \div \frac{x^2 - 1}{x^2 + 5x + 6}$$

$$\frac{x(x+3)}{(x-2)(x-1)} \quad x \neq -3,-2,-1,1,2$$

P.6 Rational Expressions

Divide: $$\frac{x^2 + x}{x^2 - 4} \div \frac{x^2 - 1}{x^2 + 5x + 6}$$

$$\frac{x(x+1)}{(x+2)(x-2)} \div \frac{(x+1)(x-1)}{(x+2)(x+3)}$$

$$\frac{x(x+1)}{(x+2)(x-2)} \cdot \frac{(x+2)(x+3)}{(x+1)(x-1)}$$

$$\frac{x(x+1)}{(x-2)(x+1)} \cdot \frac{(x+2)(x+3)}{(x+2)(x-1)}$$

$$= \frac{x(x+3)}{(x-2)(x-1)}, \quad x \neq \pm 2, \pm 1, -3$$

$$\frac{x(x+3)}{(x-2)(x-1)} \quad x \neq -3,-2,-1,1,2$$

P.6 Rational Expressions

Subtract:

$$\frac{2x+3}{3x-6} - \frac{3-x}{3x-6}$$

common denominators

perform subtraction

P.6 Rational Expressions

Subtract: $\frac{2x+3}{3x-6} - \frac{3-x}{3x-6}$

common denominators

perform subtraction

$$\frac{2x+3-(3-x)}{3(x-2)} = \frac{2x+3-3+x}{3(x-2)}$$

$$= \frac{3x}{3(x-2)} = \frac{x}{x-2}$$

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P.6 Rational Expressions

Subtract: $\dfrac{4}{x} - \dfrac{3}{x+3}$

Find common denominator: ______________

Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

Perform subtraction.

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P.6 Rational Expressions

Subtract: $\dfrac{4}{x} - \dfrac{3}{x+3}$

Find common denominator: $x(x+3)$

Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

Perform subtraction.

$$\frac{4}{x}\cdot\frac{x+3}{x+3} - \frac{x}{x}\,\frac{3}{x+3}$$

$$\frac{4x+12-3x}{x(x+3)} = \frac{x+12}{x(x+3)}$$

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P.6 Rational Expressions

Subtract: $\dfrac{3x}{x-3} - \dfrac{x+4}{x+2}$

$\dfrac{1}{2} - \dfrac{2}{3} = \dfrac{3}{6} - \dfrac{4}{6} = \dfrac{-1}{6}$

Find common denominator: (x-3)(x+2)

Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

$\dfrac{3x}{(x-3)}\,\dfrac{(\,?\,)}{(x+2)} - \dfrac{(\,?\,)}{(x-3)}\,\dfrac{(x+4)}{(x+2)}$

Perform subtraction.

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P.6 Rational Expressions

Subtract: $\dfrac{3x}{x-3} - \dfrac{x+4}{x+2}$

Find common denominator: (x-3)(x+2)

$\dfrac{3x\ (x+2)}{(x-3)(x+2)} - \dfrac{(x-3)(x+4)}{(x-3)(x+2)}$

Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

Perform subtraction.

$\dfrac{3x\,(x+2)}{(x-3)(x+2)} - \dfrac{(x-3)(x+4)}{(x-3)(x+2)}$

$= \dfrac{3x^2+6x-[x^2+x-12]}{(x-3)(x+2)}$

$= \dfrac{3x^2+6x-x^2-x+12}{(x-3)(x+2)}$

$\dfrac{2x^2+5x+12}{(x-3)(x+2)}$

$(\quad)(\quad) = 24$

$(\quad) + (\quad) = 5$

1·24

2·12

3·8   5

4·6

5 from diff; ∴ opp. signs, but then get -24 ≠ 24. So PRIME

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P.6 Rational Expressions

Subtract: $\dfrac{x}{x^2 - 2x - 24} - \dfrac{x}{x^2 - 7x + 6}$

Find common denominator: ______________

Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

Perform subtraction and simplify.

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P.6 Rational Expressions

Subtract:

Find common denominator: $(x+4)(x-6)(x-1)$

$$\underset{(x+4)(x-6)}{\dfrac{x}{x^2 - 2x - 24}} - \underset{(x-1)(x-6)}{\dfrac{x}{x^2 - 7x + 6}}$$

Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

Perform subtraction and simplify.

$$\frac{x}{(x+4)(x-6)} \cdot \frac{x-1}{x-1} - \frac{x}{(x-1)(x-6)} \cdot \frac{x+4}{x+4}$$

$$\frac{x(x-1) - x(x+4)}{(x+4)(x-6)(x-1)} = \frac{x^2 - x - x^2 - 4x}{(x+4)(x-6)(x-1)}$$

$$= \frac{-5x}{(x+4)(x-6)(x-1)}$$

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P.6 Rational Expressions

video: complex fractions
http://www.battaly.com/algebra_trig/videos/complexFractions_small.mp4
http://www.battaly.com/algebra_trig/videos/complexFractions/

Simplify Complex Fractions:
use the Multiplication Property of One

$$\frac{\frac{x}{4} - 1}{x - 4}$$

$$= \frac{1}{4}, \ x \neq 4$$

RLT

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P.6 Rational Expressions

Simplify Complex Fractions:
use the Multiplication Property of One

$$\frac{\frac{x}{4} - 1}{x - 4} \cdot \frac{4}{4} = \frac{\frac{x}{4} \cdot 4 - 1 \cdot 4}{(x - 4) \cdot 4}$$

MP1, DP

$$\frac{x - 4}{4(x-4)} = \frac{1}{4}, \ x \neq 4$$

RLT

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P.6 Rational Expressions

Simplify Complex Fractions: use the Multiplication Property of One

$$\frac{\frac{x}{x-2}+1}{\frac{3}{x^2-4}+1}$$

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P.6 Rational Expressions

Simplify Complex Fractions: use the Multiplication Property of One

$$\frac{\frac{x}{x-2}+1}{\frac{3}{x^2-4}+1} \cdot \frac{(x+2)(x-2)}{(x+2)(x-2)}$$

$$\left[\frac{\frac{x}{\cancel{x-2}}\cdot\frac{(x+2)\cancel{(x-2)}}{1}+1(x+2)(x-2)}{\frac{3}{x^2-4}\cdot(x+2)(x-2)+1(x+2)(x-2)}\right]$$

$$\frac{x(x+2)+1(x+2)(x-2)}{3+(x+2)(x-2)}$$

$$\frac{x^2+2x+x^2-4}{3+x^2-4}=\frac{2x^2+2x-4}{x^2-1}$$

$$=\frac{2(x^2+x-2)}{(x+1)(x-1)}$$

$(2)(-1)=-2$

$(2)+(-1)=1$

$$\frac{2(x+2)(x-1)}{(x+1)(x-1)}=\frac{2(x+2)}{x+1}$$

$x \neq \pm 1, \pm 2$

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Another approach:

$$\frac{\frac{x}{x-2}+1}{\frac{3}{x^2-4}+1} = \frac{\frac{x}{x-2}+\frac{x-2}{x-2}}{\frac{3}{x^2-4}+\frac{x^2-4}{x^2-4}} = \frac{\frac{x+x-2}{x-2}}{\frac{3+x^2-4}{x^2-4}}$$

$$= \frac{\frac{2x-2}{x-2}}{\frac{x^2-1}{x^2-4}}$$

then simplify (divide)

$$= \frac{2x-2}{x-2} \div \frac{x^2-1}{x^2-4}$$

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P.6 Rational Expressions

Simplify:

$$\frac{\frac{6}{\cancel{x^2+2x-15}\ (x+5)(x-3)} - \frac{1}{x-3}}{\frac{1}{x+5}+1}$$

LCD = ___________

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P.6 Rational Expressions

Simplify:

$$\frac{\dfrac{6}{\underset{(x+5)(x-3)}{x^2+2x-15}} - \dfrac{1}{x-3}}{\dfrac{1}{x+5} + 1} \cdot \frac{(x+5)(x-3)}{(x+5)(x-3)}$$

$$\frac{6-(x+5)}{x-3+(x+5)(x-3)} = \frac{1-x}{x-3+x^2+2x-15}$$

$$= \frac{1-x}{x^2+3x-18} =$$

$1-x$ is not a factor of denominator

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P.6 Rational Expressions

Divide:

$$\frac{1}{x^2-2x-8} \div \left(\frac{1}{x-4} - \frac{1}{x+2}\right)$$

Can rewrite as complex fraction:

$$\frac{\dfrac{1}{x^2-2x-8}}{\dfrac{1}{x-4} - \dfrac{1}{x+2}} \cdot \frac{(x-4)(x+2)}{(x-4)(x+2)} = \frac{1}{(x+2)-(x-4)}$$

$$= \frac{1}{6}$$

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