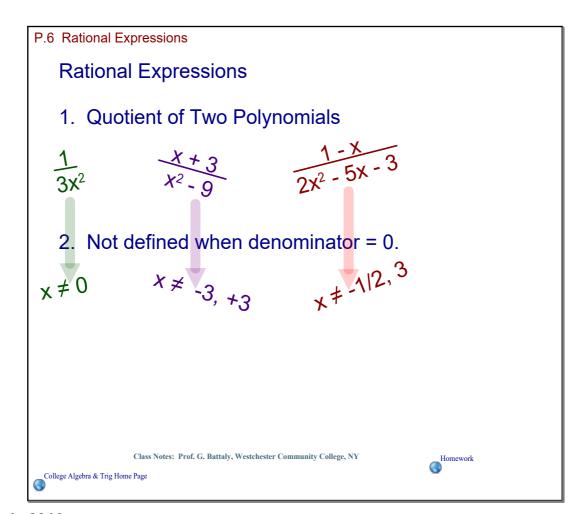
GOALS: P.6 Rational Expressions

- 1. Recognize Rational Expressions as the quotient of polynomials.
- 2. Understand restrictions on the Domain of Rational Expressions.
- 3. Simplify Rational Expressions by finding Common Factors in numerator and denominator and reducing to lowest terms.
- 4. Perform operations of addition, subtraction, multiplication and division of Rational Expressions.
- 5. Use the Multiplication Property of 1 to simplify complex fractions.

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Homework



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To Simplify Rational Expressions

$$\frac{1}{3x^2}$$

$$\frac{x+3}{x^2-9}$$

$$\frac{x+3}{x^2-9}$$
 $\frac{1-x}{2x^2-5x-3}$

1. Look for common factors.

No common factors





Common factors include binomials as possibilities

2. Reduce to Lowest Terms (RLT)

$$\frac{x+3}{x^2-9}$$

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P.6 Rational Expressions

Simplify:

Factor easiest first.

 $\frac{4x - 8}{x^2 - 4x + 4}$ Look for the same factor(s) in other part.

4 is not a factor of the denominator.

4(x-2) Is (x-2) a factor of the denominator??) (x-2) Substitute 2 into denominator: Substitute 2 into denominator:

$$2^2 - 4(2) + 4?0$$

Simplify:

 $\frac{x^2 - 8x + 16}{3x - 12}$

Look for the same factor(s) in other part.

= (?)(x-4)3(x-4)

If no common factor, then already simplified.

3 is not a factor of the numerator. Is (x-4) a factor of the numerator? Substitute 4 into numerator:

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Simplify:

$$\frac{4x - 8}{x^2 - 4x + 4}$$

4 is not a factor of the denominator. Is (x-2) a factor of the denominator? Substitute 2 into denominator: 22 -4(2) + 4 ? 0

RLT using multiplication property of 1

Simplify:
$$\frac{x^2 - 8x + 16}{3x - 12}$$

Factor easiest first.

Look for the same factor(s) in other part. If no common factor, then already simplified.

$$= \frac{(x-4)(x-4)}{3(x-4)}$$

RLT using multiplication property of 1

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P.6 Rational Expressions

Simplify:

$$\frac{4x - 8}{x^2 - 4x + 4}$$

$$\frac{4(x-2)}{(x-2)(x-2)}$$

Factor easiest first

Look for the same factor(s) in other part. If no common factor, then already simplified.

$$\frac{4}{(x-2)} \cdot \frac{(x-2)}{(x-2)}$$

$$\frac{4}{(x-2)}$$

RLT using multiplication property of 1

Simplify:

$$\frac{x^2 - 8x + 16}{3x - 12}$$

Factor easiest first.

Look for the same factor(s) in other part.

If no common factor, then already simplified.

$$= \frac{(x-4)^2}{3(x-4)} = \frac{(x-4)}{3}$$

RLT using multiplication property of 1

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3

Simplify:

$$\frac{x^2 - 14x + 49}{x^2 - 49}$$

Factor easiest first

Look for the same factor(s) in other part. If no common factor, then already simplified.

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P.6 Rational Expressions

$$\frac{x^2 - 14x + 49}{x^2 - 49} = \frac{()()}{(x+7)(x-7)}$$

$$= \frac{(x-7)(x-7)}{(x+7)(x-7)}$$
 Look for the same factor(s) in other part.

$$= \frac{(x-7)(x-7)}{(x+7)(x-7)}$$

$$=\frac{x-7}{x+7}$$

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Operations with Rational Expressions Same as with Numerals

1. Multiply, Divide.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

2. Add, Subtract

$$\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$$

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$$
RLT; b,d \neq 0

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P.6 Rational Expressions

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Divide:

$$\frac{x^2 - 4}{x^2 + 3x - 10} \div \frac{x^2 + 5x + 6}{x^2 + 8x + 15}$$

Z,ε-,Σ-,δ- ≠ x , Γ

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P.6 Rational Expressions

Divide:
$$\frac{x^2 - 4}{x^2 + 3x - 10} \div \frac{x^2 + 5x + 6}{x^2 + 8x + 15}$$

$$\frac{x^2 - 4}{x^2 + 3x - 10} \cdot \frac{x^2 + 8x + 15}{x^2 + 5x + 6}$$

$$\frac{(x+3)(x-2)}{(x+5)(x+3)} \cdot \frac{(x+5)(x+3)}{(x+2)(x+3)}$$

$$\frac{(x+2)(x-2)}{(x+2)(x-2)}\frac{(x+5)(x+3)}{(x+5)(x+3)}$$

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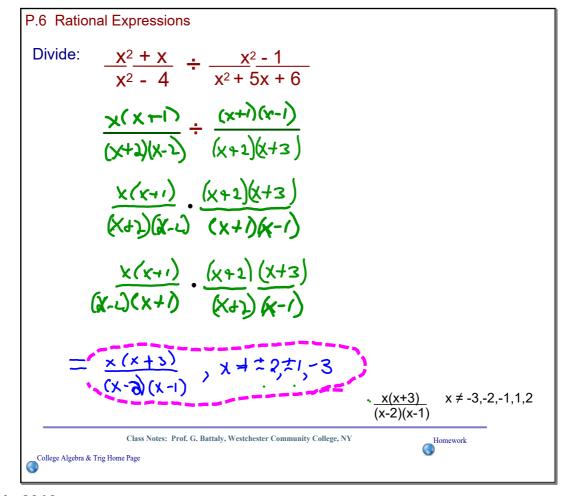
P.6 Rational Expressions

Divide:
$$\frac{x^2 + x}{x^2 - 4} \div \frac{x^2 - 1}{x^2 + 5x + 6}$$

$$\frac{x^2 + x}{x^2 - 4} \div \frac{x^2 - 1}{x^2 + 5x + 6}$$

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Subtract:

$$\frac{2x+3}{3x-6} - \frac{3-x}{3x-6}$$

common denominators perform subtraction

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P.6 Rational Expressions

Subtract: 2x + 3 = 3 - x common denominators 3x - 6

$$\frac{2x+3-(3-x)}{3(x-2)} = \frac{2x+3-3+x}{3(x-2)}$$

$$-\frac{3x}{3(x-2)} - \frac{x}{x-2}$$

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Subtract:

$$\frac{4}{x} - \frac{3}{x+3}$$

Find common denominator:

Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

Perform subtraction.

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P.6 Rational Expressions

Subtract:

$$\frac{4}{x} - \frac{3}{x+3}$$

Find common denominator: X (X + 3

 $\frac{4}{x} \cdot \frac{x+3}{x+3} - \frac{x}{x} \cdot \frac{3}{x+3}$

Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

Perform subtraction.

$$\frac{4x+12-3x}{x(x+3)}=\frac{x+12}{x(x+3)}$$

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 $\frac{3x}{(x-3)} \frac{(?)}{(x+2)} - \frac{(?)}{(x-3)} \frac{(x+4)}{(x+2)}$

$$\frac{1}{a} - \frac{2}{3} = \frac{3}{6} - \frac{4}{6} = \frac{-1}{6}$$

Find common denominator: (x-3)(x+2)

Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

Perform subtraction.

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P.6 Rational Expressions

 $\frac{3x}{x-3}$ - $\frac{x+4}{x+2}$ Find common denominator: (x-3)(x+2)

 $\frac{3x}{(x-3)(x+2)} - \frac{(x-3)(x+4)}{(x-3)(x+2)}$

Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

 $\frac{3x(X+2)}{(x-3)(X+2)} - \frac{(x-3)(x+4)}{(x-3)(X+2)}$

Perform subtraction.

$$= \frac{3\chi^{2}+6\chi-[\chi^{2}+\chi-12]}{(\chi\cdot3)(\chi+\lambda)}$$

$$=\frac{3\chi^{1}+\zeta\chi-x^{2}-\chi+12}{(\chi\cdot3)(\chi+\lambda)}$$

2x75x+12 (x-3)(x+2)

(1()= 24 ()+()=5

5 from diff; ∴ opp. signs, but then get $-24 \neq 24$. So PRIME

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Subtract:
$$\frac{x}{x^2 - 2x - 24} - \frac{x}{x^2 - 7x + 6}$$

Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

Perform subtraction and simplify.

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P.6 Rational Expressions

Subtract:

$$\frac{x}{x^2 - 2x - 24} - \frac{x}{x^2 - 7x + 6}$$

$$(x+4)(x-6) \qquad (x-1)(x-6)$$

$$\frac{x}{(x+4)(x-6)} \cdot \frac{x-1}{x-1} - \frac{x}{(x-1)(x-6)} \cdot \frac{x+4}{x+4}$$

$$\frac{x(x-1)-x(x+4)}{(x+4)(x-6)(x-1)} = \frac{x^2-x^2-x^2-4x}{(x+4)(x-6)(x-1)}$$

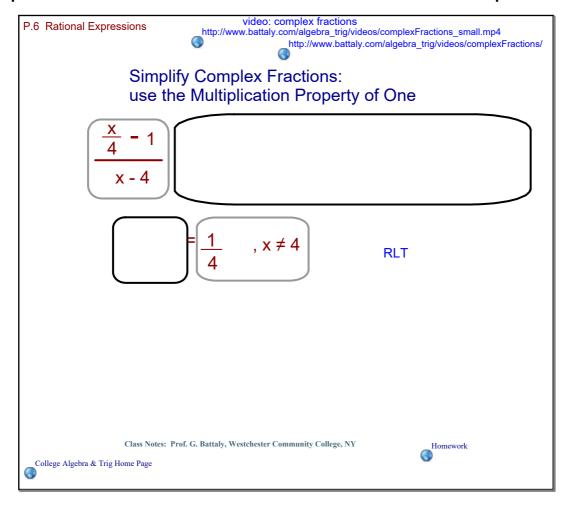
$$= \frac{-5x}{(x+4)(x-6)(x-1)}$$

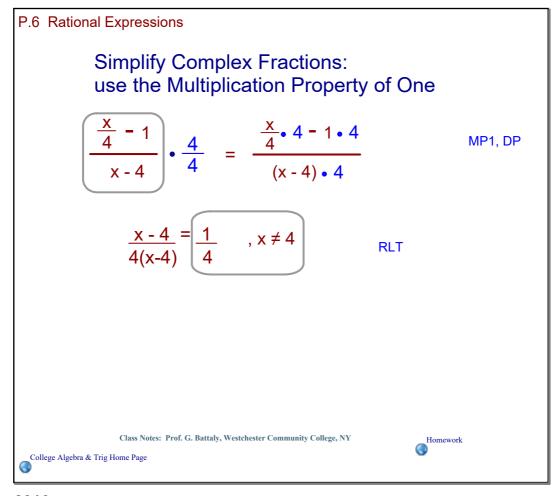
Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

Perform subtraction and simplify.

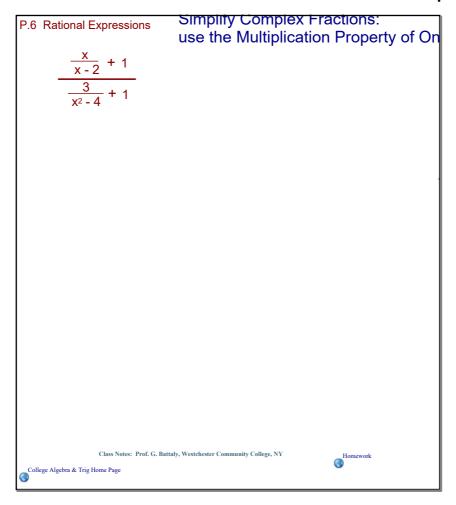
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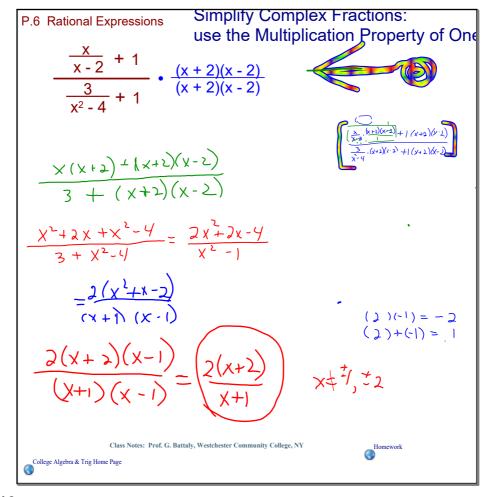






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$$\frac{\frac{x}{x-2} + 1}{\frac{3}{x^2-4} + 1} = \frac{\frac{x}{x-2} + \frac{x-2}{x-2}}{\frac{3}{x^2-4} + \frac{x^2-4}{x^2-4}} = \frac{\frac{x+x-2}{x-2}}{\frac{3+x^2-4}{x^2-4}} = \frac{\frac{x+x-2}{x-2}}{\frac{3+x^2-4}{x^2-4}}$$

$$= \frac{2x-2}{x-2}$$
then simplify (divide)

$$=\frac{2x-2}{x-2} \cdot \frac{x^2-1}{x^2-4}$$

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P.6 Rational Expressions

Simplify:

$$\frac{\frac{6}{x^2 + 2x - 15} - \frac{1}{x - 3}}{\frac{1}{x + 5}(x - 3)} + 1$$

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Homework

P.6 Rational Expressions

Simplify:
$$\frac{6}{x^2 + 2x - 15} - \frac{1}{x - 3} \quad (x + 5)(x - 3)$$

$$\frac{1}{x + 5} + 1 \quad (x + 5)(x - 3)$$

$$\frac{6}{(x + 5)(x - 3)} - \frac{1}{x + 5} + 1$$

$$\frac{6}{(x + 5)(x - 3)} - \frac{1}{x + 5} + 1$$

$$\frac{1 - x}{x - 3 + x + 2x - 15}$$

$$\frac{1 - x}{x - 3 + x + 2x - 15}$$

$$\frac{1 - x}{x - 3 + x + 2x - 15}$$

$$\frac{1 - x}{x - 3 + x + 2x - 15}$$

$$\frac{1 - x}{x - 3 + x + 2x - 15}$$

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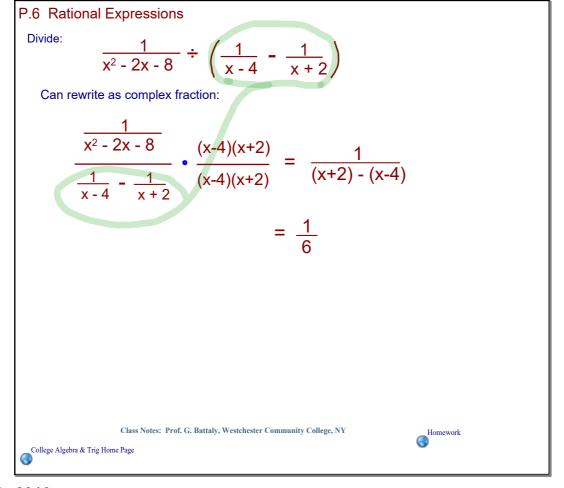
$$\frac{1 - x}{x - 3 + x + 2x - 15}$$

$$\frac{1 - x}{x - 3 + x + 2x - 15}$$

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$$\frac{1 - x}{x - 3 + x + 2x - 15}$$
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