Rational Expressions

GOALS:
1. Recognize Rational Expressions as the quotient of polynomials.
2. Understand restrictions on the Domain of Rational Expressions.
3. Simplify Rational Expressions by finding Common Factors in numerator and denominator and reducing to lowest terms.
4. Perform operations of addition, subtraction, multiplication and division of Rational Expressions.
5. Use the Multiplication Property of 1 to simplify complex fractions.

Study P.6 CVC 1-9, # 1-13; 15, 19, 23, ...77; 53

Rational Expressions

1. Quotient of Two Polynomials
   \[
   \frac{1}{3x^2}, \quad \frac{x + 3}{x^2 - 9}, \quad \frac{1 - x}{2x^2 - 5x - 3}
   \]
   - \(x \neq 0\)
   - \(x \neq -3, +3\)
   - \(x \neq -1/2, 3\)
P.6 Rational Expressions

To Simplify Rational Expressions

\[
\frac{1}{3x^2} \quad \frac{x + 3}{x^2 - 9} \quad \frac{1 - x}{2x^2 - 5x - 3}
\]

1. Look for common factors.

No common factors \(x + 3\) No common factors

Common factors include binomials as possibilities

2. Reduce to Lowest Terms (RLT)

\[
\frac{x + 3}{x^2 - 9}
\]

Factor easiest first. Look for the same factor(s) in other part. If no common factor, then already simplified.

4 is not a factor of the denominator. Is (x-2) a factor of the denominator?

Substitute 2 into denominator:

\[
2^2 - 4(2) + 4 \neq 0
\]

Factor easiest first. Look for the same factor(s) in other part. If no common factor, then already simplified.

3 is not a factor of the numerator. Is (x-4) a factor of the numerator?

Substitute 4 into numerator:

\[
4^2 - 8(4) + 16 \neq 0
\]
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Simplify: \[
\frac{4x - 8}{x^2 - 4x + 4}
\]

Factor easiest first.
Look for the same factor(s) in other part.
If no common factor, then already simplified.

4 is not a factor of the denominator.
Is (x-2) a factor of the denominator?
Substitute 2 into denominator: \(2^2 - 4(2) + 4 \neq 0\)

\[
\frac{4(x-2)}{(x-2)(x-2)}
\]

RLT using multiplication property of 1

Simplify: \[
\frac{x^2 - 8x + 16}{3x - 12}
\]

Factor easiest first.
Look for the same factor(s) in other part.
If no common factor, then already simplified.

\[
= \frac{(x-4)(x-4)}{3(x-4)}
\]

RLT using multiplication property of 1

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Simplify: \[
\frac{4x - 8}{x^2 - 4x + 4}
\]

Factor easiest first.
Look for the same factor(s) in other part.
If no common factor, then already simplified.

\[
\frac{4(x-2)}{(x-2)(x-2)}
\]

RLT using multiplication property of 1

Simplify: \[
\frac{x^2 - 8x + 16}{3x - 12}
\]

Factor easiest first.
Look for the same factor(s) in other part.
If no common factor, then already simplified.

\[
= \frac{(x-4)^2}{3(x-4)} = \frac{(x-4)}{3}
\]

RLT using multiplication property of 1
Rational Expressions

Simplify:

\[
\frac{x^2 - 14x + 49}{x^2 - 49}
\]

Factor easiest first.
Look for the same factor(s) in other part.
If no common factor, then already simplified.

\[\frac{x^2 - 14x + 49}{x^2 - 49} = \frac{(x-7)(x-7)}{(x+7)(x-7)}\]

RLT: reduce to lowest terms.

\[= \frac{x-7}{x+7}\]
Operations with Rational Expressions
Same as with Numerals

1. Multiply, Divide.

\[
\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}
\]

\[
\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}
\]

2. Add, Subtract

\[
\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}
\]

\[
\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}
\]

(RLT; b,d ≠ 0)

Multiply:

\[
\frac{6x + 9}{3x - 15} \cdot \frac{x - 5}{4x + 6}
\]
Divide:
\[
\frac{x^2 - 4}{x^2 + 3x - 10} \div \frac{x^2 + 5x + 6}{x^2 + 8x + 15}
\]

\[
\frac{x^2 - 4}{x^2 + 3x - 10} \cdot \frac{x^2 + 8x + 15}{x^2 + 5x + 6}
\]

\[
\frac{(x+2)(x-2)}{(x+5)(x-2)} \cdot \frac{(x+5)(x+3)}{(x+2)(x+3)}
\]

\[
\frac{(x+2)(x-2)}{(x+2)(x+3)} = 1, \ x \neq -2, -5, -3
\]
Divide: \[
\frac{x^2 + x}{x^2 - 4} \div \frac{x^2 - 1}{x^2 + 5x + 6}
\]

\[
\frac{x(x+1)}{(x+2)(x-2)} \div \frac{(x+1)(x-1)}{(x+2)(x+3)}
\]

\[
= \frac{x(x+1)}{(x-2)(x+1)} \cdot \frac{(x+2)(x+3)}{(x+1)(x-1)}
\]

\[
\frac{x(x+3)}{(x-2)(x-1)} \quad x \neq -3, -2, -1, 1, 2
\]
Subtract:
\[
\frac{2x + 3}{3x - 6} - \frac{3 - x}{3x - 6}
\]

common denominators
perform subtraction

\[
= \frac{2x + 3 - (3 - x)}{3(x-2)} = \frac{2x + 3 - 3 + x}{3(x-2)}
\]

\[
= \frac{3x}{3(x-2)} = \frac{x}{x-2}
\]
Subtract: \( \frac{4}{x} - \frac{3}{x+3} \)

Find common denominator: \( \frac{4(x+3)}{x(x+3)} - \frac{3x}{x(x+3)} \)

Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

Perform subtraction.

\[
\frac{4(x+3) - 3x}{x(x+3)} = \frac{x+12}{x(x+3)}
\]
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Subtract: \[ \frac{3x}{x - 3} - \frac{x + 4}{x + 2} \]

Find common denominator: \((x-3)(x+2)\)

Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

Perform subtraction.

\[ \frac{3x}{(x-3)} \left( \frac{x+2}{x+2} \right) - \frac{?}{(x-3)} \left( \frac{x+4}{x+2} \right) \]

\[ \frac{3x(x+2)}{(x-3)(x+2)} - \frac{(x-3)(x+4)}{(x-3)(x+2)} \]

\[ = \frac{3x^2 + 6x - 12}{(x-3)(x+2)} \]

\[ = \frac{3x^2 + 6x - x^2 - x + 12}{(x-3)(x+2)} \]

\[ \frac{2x^2 + 5x + 12}{(x-3)(x+2)} \]

\[ 5 \text{ from diff; } \because \text{ opp. signs, but then get } -24 \neq 24. \]

So PRIME
P.6 Rational Expressions

Subtract: \( \frac{x}{x^2 - 2x - 24} - \frac{x}{x^2 - 7x + 6} \)

Find common denominator: ____________

Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

Perform subtraction and simplify.

\[
\frac{x}{(x+4)(x-6)} - \frac{x}{(x-1)(x-6)}
\]

Find common denominator: \((x+4)(x-6)(x-1)\)

Rewrite each as an equivalent expression with the common denominator, using the multiplication property of 1.

Perform subtraction and simplify.

\[
\frac{x}{(x+4)(x-6)} \cdot \frac{x-1}{x-1} - \frac{x}{(x-1)(x-6)} \cdot \frac{x+4}{x+4}
\]

\[
\frac{x(x-1) - x(x+4)}{(x+4)(x-6)(x-1)} = \frac{5x}{(x+4)(x-6)(x-1)}
\]
Simplify Complex Fractions: use the Multiplication Property of One

\[
\frac{\frac{x}{4} - 1}{x - 4} = \frac{1}{4}, \quad x \neq 4
\]
Simplify Complex Fractions:
use the Multiplication Property of One

\[
\frac{\frac{x}{x - 2} + 1}{\frac{3}{x^2 - 4} + 1} \cdot \frac{(x + 2)(x - 2)}{(x + 2)(x - 2)}
\]

\[
= \frac{x(x + 2)}{3} \cdot \frac{x - 2}{x + 2}(x - 2)
\]

\[
= \frac{x^2 + 2x + x^2 - 4}{3 + x^2 - 4}
\]

\[
= \frac{2x^2 + 2x - y}{x^2 - 1}
\]

\[
= \frac{2(x^2 + x - 2)}{(x + 1)(x - 1)}
\]

\[
= \frac{2(x + 2)(x - 1)}{(x + 1)(x - 1)} \cdot \frac{2}{x + 1}
\]

\[
= \frac{2(x + 2)}{x + 1}
\]

\[
= \frac{2}{x + 1}
\]
Another approach:

\[
\frac{\frac{x}{x^2 - 4} + \frac{1}{x^2 - 4}}{\frac{3}{x^2 - 4} + \frac{x - 1}{x^2 - 4}} = \frac{\frac{x - 2}{x - 2}}{\frac{3 + x^2 - y}{x^2 - 4}}
\]

then simplify (divide)

\[
= \frac{2x - 2}{x - 2} \div \frac{x^2 - 1}{x^2 - 4}
\]

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Simplify:

\[
\frac{6}{x^2 + 2x - 15} - \frac{1}{x - 3}
\]

\[
\frac{1}{x + 5} + 1
\]

LCD = ___________
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Simplify:
\[ \frac{6}{x^2 + 2x - 15} - \frac{1}{x - 3} \frac{(x+5)}{(x-3)} \cdot \frac{1}{x + 5} + 1 \]

\[ = \frac{6 - (x+5)}{x-3 + (x+5)(x-3)} = \frac{1-x}{x-3 + x^2 + 2x - 15} \]

\[ = \frac{1-x}{x^2 + 3x - 18} \]

\[ \text{is not a factor of denominator} \]

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Divide:
\[ \frac{1}{x^2 - 2x - 8} \div \left( \frac{1}{x - 4} - \frac{1}{x + 2} \right) \]

Can rewrite as complex fraction:
\[ \frac{1}{x^2 - 2x - 8} \cdot \frac{(x-4)(x+2)}{(x-4)(x+2)} = \frac{1}{(x+2) - (x-4)} \]

\[ = \frac{1}{6} \]