P.3 Radicals & Rational Exponents

GOALS:

1. Perform multiplication of radicals.
2. Perform addition and subtraction of radicals.
3. Perform division of radicals by rationalizing the denominator.
4. Rationalize the denominator using conjugates.
5. Understand exponents and the rules of exponents including: the Product Rule, the Quotient Rule, and the Power Rule.
6. Understand the relationship between rational exponents and radical indices.

Study P.3 *CVC # 1-13;* #1,5,9,13,..... 113

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P.3 Radicals & Rational Exponents

$\sqrt{9}$ = ___ because ___^{2} = 9

$\sqrt{25}$ = ___ because ___^{2} = 25

$\sqrt{\frac{25}{9}}$

$\sqrt{a}$ The number, when multiplied by itself, results in a product = a

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P.3 Radicals & Rational Exponents

$\sqrt{9}$ = 3 because $3^2 = 9$

$\sqrt{25}$ = 5 because $5^2 = 25$

$\sqrt{\frac{25}{9}} = \frac{5}{3}$ $\left(\frac{5}{3}\right)^2 = \frac{25}{9}$

$\sqrt{a}$ The number, when multiplied by itself, results in a product = a

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P.3 Radicals & Rational Exponents

$\sqrt{2}$ = ___, 2 not a perfect square, leave as is

$\sqrt{-9}$ = ___ not real number,
because $3^2 = +9$ and $(-3)^2 = +9$

If $(-3)^2 = +9$, why can't $\sqrt{9}$ be -3 ?

$\sqrt{a}$ is the Principal Root which is non-negative

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P.3 Radicals & Rational Exponents

$\sqrt{2}$ = $\sqrt{2}$, 2 not a perfect square, leave as is

$\sqrt{-9}$ = $\emptyset$ not real number,

because $3^2 = +9$ and $(-3)^2 = +9$

DNE not real

If $(-3)^2 = +9$, why can't $\sqrt{9}$ be -3 ?

$\sqrt{a}$ is the Principal Root which is non-negative

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P.3 Radicals & Rational Exponents

$\sqrt{36} + \sqrt{64}$? $\sqrt{36 + 64}$

$\sqrt{a} + \sqrt{b}$? $\sqrt{a + b}$

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P.3 Radicals & Rational Exponents

$\sqrt{36} + \sqrt{64} \;?\; \sqrt{36+64}$

$6 + 8 \neq \sqrt{100}$

$14 > 10$

$\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$

?

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P.3 Radicals & Rational Exponents

Product Rule:

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

Quotient Rule:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

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P.3 Radicals & Rational Exponents

Product Rule:

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

$\sqrt{27}$

$\sqrt{6x}\ \sqrt{3x^2}$

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P.3 Radicals & Rational Exponents

Product Rule:

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

$\sqrt{27} = \sqrt{9 \cdot 3}$

$= \sqrt{9}\sqrt{3}$

$= 3\sqrt{3}$

$\sqrt{6x}\ \sqrt{3x^2}$

$\sqrt{6x}\ \sqrt{3}\ \sqrt{x^2}$

$\sqrt{18x}\ x = \sqrt{9}\sqrt{2x}\ x$

$= 3x\sqrt{2x}$

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P.3 Radicals & Rational Exponents

$$\sqrt{\frac{121}{9}}$$

Quotient Rule:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\frac{\sqrt{24x^4}}{\sqrt{3x}}$$

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P.3 Radicals & Rational Exponents

$$\sqrt{\frac{121}{9}} = \frac{11}{3}$$

Quotient Rule:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\frac{\sqrt{24x^4}}{\sqrt{3x}} = \sqrt{\frac{24}{3}x^3} \qquad \sqrt{\frac{24x^4}{3x}}$$

$$= \sqrt{8x^3} = \sqrt{4x^2}\sqrt{2x}$$

$$= 2x\sqrt{2x}$$

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P.3 Radicals & Rational Exponents

Like Radicals: terms with same radicands

$6\sqrt{3} - 4\sqrt{3}$

$6\sqrt{2} - 3\sqrt{32}$

P.3 Radicals & Rational Exponents

Like Radicals: terms with same radicands

$6\sqrt{3} - 4\sqrt{3} = (6-4)\sqrt{3} = 2\sqrt{3}$

$6\sqrt{2} - 3\sqrt{32}$

$6\sqrt{2} - 3\sqrt{16}\sqrt{2}$

$6\sqrt{2} - 3(4)\sqrt{2}$

$6\sqrt{2} - 12\sqrt{2}$

$= -6\sqrt{2}$

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P.3 Radicals & Rational Exponents

$a(b+c) = ab + ac$

$ab + ac = a(b+c)$

$2x + 3y - x + 2y$

$2x - x + 3y + 2y$

$x + 5y$

P.3 Radicals & Rational Exponents

Like Radicals: terms with same radicands

$\sqrt{20} + 6\sqrt{5}$

P.3 Radicals & Rational Exponents

Like Radicals: terms with same radicands

$$\sqrt{20} + 6\sqrt{5}$$

$$\sqrt{4}\sqrt{5} + 6\sqrt{5}$$

$$2\sqrt{5} + 6\sqrt{5} = 8\sqrt{5}$$

P.3 Radicals & Rational Exponents

Rationalize the Denominator

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot 1$$

Multiplication Property of 1

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{3}$$

P.3 Radicals & Rational Exponents

Rationalize the Denominator

$$\frac{1}{\sqrt{27}} = \frac{1}{\sqrt{27}} \cdot 1$$

Multiplication Property of 1

P.3 Radicals & Rational Exponents

Rationalize the Denominator

$$\frac{1}{\sqrt{27}} = \frac{1}{\sqrt{27}} \cdot 1$$

Multiplication Property of 1

$$\frac{1}{\sqrt{27}} = \frac{1}{\sqrt{27}} \cdot \frac{\sqrt{27}}{\sqrt{27}}$$

$$= \frac{\sqrt{27}}{27} = \frac{\sqrt{9}\sqrt{3}}{27}$$

$$= \frac{3\sqrt{3}}{27} = \frac{\sqrt{3}}{9}$$

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P.3 Radicals & Rational Exponents

Rationalize the Denominator using Conjugates

$$\frac{1}{2-\sqrt{5}} = \frac{1}{2-\sqrt{5}} \cdot 1$$

Multiplication Property of 1

Use the conjugate of the denominator as both the numerator and the denominator in the Multiplication Property of 1.

$$(2-\sqrt{5})(2+\sqrt{5}) = 4-5 = -1$$

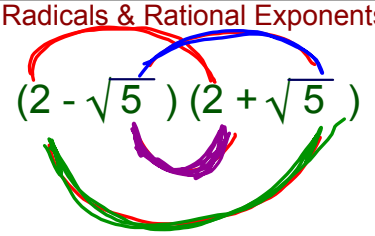
Rational!

$$\frac{1}{2-\sqrt{5}} = \frac{1}{2-\sqrt{5}} \cdot \frac{2+\sqrt{5}}{2+\sqrt{5}} = \frac{2+\sqrt{5}}{-1}$$

$$= -2-\sqrt{5}$$

P.3 Radicals & Rational Exponents

$$(2-\sqrt{5})(2+\sqrt{5}) = 4-5 = -1$$



FOIL

$$4 + 2\sqrt{5} - 2\sqrt{5} - \sqrt{5}\sqrt{5}$$

$$4 + 0 - 5 = -1$$

or apply: $(a+b)(a-b) = a^2 - b^2$

P.3 Radicals & Rational Exponents

G: $\frac{5}{\sqrt{3}-1}$ F: Rationalize the denominator.

P.3 Radicals & Rational Exponents

$\frac{5}{(\sqrt{3}-1)} \cdot \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$

$3 - \sqrt{3} + \sqrt{3} - 1$

$\frac{5(\sqrt{3}+1)}{3-1}$

$\frac{5\sqrt{3}+5}{2}$

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P.3 Radicals & Rational Exponents

G: $\frac{5}{\sqrt{3}+1}$ F: Rationalize the denominator.

P.3 Radicals & Rational Exponents

$\frac{5}{(\sqrt{3}+1)} \cdot \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$ F: rationalize the den.

$\frac{5(\sqrt{3}-1)}{3-1} = \frac{5(\sqrt{3}-1)}{2}$

$\frac{5}{2}(\sqrt{3}-1)$

$(a+b)(a-b) = a^2-b^2$

P.3 Radicals & Rational Exponents

G: $\dfrac{11}{\sqrt{7}-\sqrt{3}}$

F: Rationalize the denominator.

P.3 Radicals & Rational Exponents

F: Rationalize the denominator.

$$\frac{11}{(\sqrt{7}-\sqrt{3})}\cdot\frac{\sqrt{7}+\sqrt{3}}{(\sqrt{7}+\sqrt{3})}$$

$$\frac{11(\sqrt{7}+\sqrt{3})}{7+\sqrt{7}\sqrt{3}-\sqrt{7}\sqrt{3}-\sqrt{9}}$$

$$\frac{11(\sqrt{7}+\sqrt{3})}{7-3}=\frac{11}{4}(\sqrt{7}+\sqrt{3})$$

$$\frac{11\sqrt{7}+11\sqrt{3}}{4}$$

P.3 Radicals & Rational Exponents

Rational Exponents

$(3^2)^3 = (3 \cdot 3)(3 \cdot 3)(3 \cdot 3) = 3^6$

3 factors of 3^2

$(a^m)^n = a^{mn}$

$(3^{\square})^3 = 3^6$ $(3^{\square})^2 = 3^6$

$(3^{\square})^2 = 3^2$ $(3^{\square})^2 = 3^1$

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P.3 Radicals & Rational Exponents

Rational Exponents

$(3^2)^3 = (3 \cdot 3)(3 \cdot 3)(3 \cdot 3) = 3^6$

3 factors of 3^2

$(a^m)^n = a^{mn}$

$(3^{2})^3 = 3^6$ $(3^{3})^2 = 3^6$

$(3^{1})^2 = 3^2$ $(3^{\frac{1}{2}})^2 = 3^1$

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P.3 Radicals & Rational Exponents

Rational Exponents

$(a^m)^n = a^{mn}$

$(3\square)^2 = 3^1$

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P.3 Radicals & Rational Exponents

Rational Exponents

$(a^m)^n = a^{mn}$

$(3\square)^2 = 3^1$

Need to define $3^{1/2}$

Need to define $3^{1/n}$

$b^{1/n}$

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P.3 Radicals & Rational Exponents

Rational Exponents

$b^{1/n}$, n = 2, 3, 4, 5,...

Examples

$a^{1/2}$	$25^{1/2} = \sqrt[2]{25} = 5$
$b^{1/3}$	$27^{1/3} = \sqrt[3]{27} = 3$
$c^{1/4}$	$16^{1/4} = \sqrt[4]{16} = 2$

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P.3 Radicals & Rational Exponents

Rational Exponents

$b^{1/n}$, n = 2, 3, 4, 5,...

	n even	n odd
b ≥ 0	Principal n^{th} root of b $\sqrt[n]{b}$	n^{th} root of b $\sqrt[n]{b}$
b < 0	not a real #	n^{th} root of b $\sqrt[n]{b}$

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$\sqrt[n]{b}$

P.3 Radicals & Rational Exponents

$b^{1/n}$, n = 2, 3, 4, 5,...

	n even	n odd
b ≥ 0	Principal nth root of b $+\sqrt[n]{b}$	nth root of b $\sqrt[n]{b}$
b < 0	not a real #	nth root of b $\sqrt[n]{b}$

$4^{1/2} = 2$ (not -2)

$\sqrt[2]{4} = 2$

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P.3 Radicals & Rational Exponents

m, n rational numbers qualifiers

$$\frac{a^m}{a^n} = a^{m-n} \qquad a \neq 0$$

$$(ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \qquad b \neq 0$$

$$(a^m)^n = a^{mn}$$

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P.3 Radicals & Rational Exponents

$$(3\,x^{3/2})\,(4\,x^{1/2})$$

P.3 Radicals & Rational Exponents

$$(3x^{\frac{3}{2}})(4x^{\frac{1}{2}})$$

$$12x^{\left(\frac{3}{2}+\frac{1}{2}\right)} = 12x^{\frac{4}{2}} = 12x^{2}$$

[similar to $(3x^{2})(4x^{3}) = 12x^{5}$]

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P.3 Radicals & Rational Exponents

$$(3\,x^{2/3})\,(4\,x^{3/4})$$

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P.3 Radicals & Rational Exponents

$$(3x^{\frac{2}{3}})(4x^{\frac{3}{4}}) = 12x^{\left(\frac{2}{3}+\frac{3}{4}\right)}$$

$$= 12x^{\frac{17}{12}}$$

$$\frac{4}{4}\cdot\frac{2}{3} + \frac{3}{4}\cdot\frac{3}{3}$$

$$\frac{8}{12} + \frac{9}{12} = \frac{17}{12}$$

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P.3 Radicals & Rational Exponents

$$(9\,x^{1/2})\,(-3\,x^{5/2})$$

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P.3 Radicals & Rational Exponents

$$(9\,x^{1/2})\,(-3\,x^{5/2})$$

$$-27\,x^{\left(\frac{1}{2}+\frac{5}{2}\right)}$$

$$= -27\,x^{\frac{6}{2}} = -27x^{3}$$

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P.3 Radicals & Rational Exponents

$$(9\,x^{1/2})\,(2\,x^{-3/2})$$

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P.3 Radicals & Rational Exponents

$$(9\,x^{1/2})\,(2\,x^{-3/2})$$

$= 18x^{\frac{1}{2}-\frac{3}{2}} = 18x^{-\frac{2}{2}}$

$= 18x^{-1}$

$= \frac{18}{x}$

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P.3 Radicals & Rational Exponents

$$(125\,x^9\,y^6)^{1/3}$$

P.3 Radicals & Rational Exponents

$$(125\,x^9\,y^6)^{1/3}$$

$$= 125^{-\frac{1}{3}}\left(x^9\right)^{\frac{1}{3}}\left(y^6\right)^{\frac{1}{3}}$$

$$= \sqrt[3]{125\,x^3\,y^2}$$

$$5x^3y^2$$

$$(a^m)^n = a^{mn}$$

$$(x^9)^{\frac{1}{3}} = x^{(9\cdot\frac{1}{3})}$$

P.3 Radicals & Rational Exponents

$$\frac{(2y^{\frac{1}{5}})^4}{y^{\frac{3}{10}}}$$

P.3 Radicals & Rational Exponents

$$\frac{(2y^{\frac{1}{5}})^4}{y^{\frac{3}{10}}} = \frac{2^4 (y^{1/5})^4}{y^{3/10}}$$

$$= \frac{16\, y^{4/5}}{y^{3/10}} = 16\, y^{\left(\frac{4}{5}-\frac{3}{10}\right)} = 16\, y^{1/2}$$

$$\frac{2}{2}\cdot\frac{4}{5} - \frac{3}{10} = \frac{8}{10} - \frac{3}{10} = \frac{5}{10} = \frac{1}{2}$$

P.3 Radicals & Rational Exponents

Consider:

Start with -10 and square it: $(-10)^2$

Then raise to 1/2 power: $[(-10)^2]^{(1/2)}$

What value do you have?

$$[(-10)^2]^{(1/2)} = [100]^{(1/2)}$$
$$= 10$$

How does this compare to the -10 at start?

$$= 10 = |-10|$$

If n is even and b<0,

$$[(b)^n]^{(1/n)} = |b| \quad \text{Or} \quad \sqrt[n]{(b)^n} = |b|$$

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