4.4 Exponential & Logarithmic Equations

GOALS:

1. Solve Exponential Equations by:
   a) Rewriting in exponential form
   b) Converting to logarithmic form
   c) Find the log of both members of the equation
2. Solve Logarithmic Equations by:
   a) Converting to exponential form
   b) Using properties of logarithms
   c) Checking solution to be sure it is in the domain of the function.

$$y = \log_b x \quad x > 0,\ b>0,\ b \neq 1$$

Study 4.4 CVC # 1-11 all # 1-5,9,13,17,21; 23-27,31,35,39,43; 49,53,57, ...89;103

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4.4 Exponential & Logarithmic Equations

To solve an exponential equation:

1. If possible, **rewrite** both members of the eq. as **a power of the same base**. Then set the **exponents equal** to each other, and solve.
   eg: $2^x = 32$ then $2^x = 2^5$ and $x = 5$.

2. If step 1 is not possible, **convert to logarithmic form**, or **find** either the ***log* or the *ln*** of both members of the equation.
   Use properties of logarithms to solve.
   eg: $10^x = 8.06$ then $\log 8.06 = x$
   or $\log 10^x = \log 8.06$
   $x \log 10 = \log 8.06$ or $x = \log 8.06$

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4.4 Exponential & Logarithmic Equations

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   eg: $2^x = 32$ then $2^x = 2^5$ and $x = 5$.

Solve for x: $5^x = 625$

4.4 Exponential & Logarithmic Equations

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   eg: $2^x = 32$ then $2^x = 2^5$ and $x = 5$.

Solve for x: $5^x = 625$

$5^x = 5^4$

$x = 4$

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4.4 Exponential & Logarithmic Equations

To solve an exponential equation:

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   eg: $2^x = 32$ then $2^x = 2^5$ and $x = 5$.

Solve for x: $9^x = \frac{1}{\sqrt[3]{3}}$

4.4 Exponential & Logarithmic Equations

To solve an exponential equation:

1. If possible, **rewrite** both members of the eq. as **a power of the same base**. Then set the **exponents equal** to each other, and solve.
   eg: $2^x = 32$ then $2^x = 2^5$ and $x = 5$.

Solve for x: $9^x = \frac{1}{\sqrt[3]{3}}$

$$(3^2)^x = 3^{(-1/3)}$$

$$\mathbf{3^{2x} = 3^{(-1/3)}}$$

$$\mathbf{2x = -1/3}$$

$$x = -1/6$$

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4.4 Exponential & Logarithmic Equations

3. If step 2 is not possible, **convert to logarithmic form**, or **find** either the ***log*** **or the** ***ln*** of both members of the equation. **Use properties of logarithms** to solve.

eg: $10^x = 8.06$ then $\log 8.06 = x$

or $\log 10^x = \log 8.06$

$x \log 10 = \log 8.06$ or $x = \log 8.06$

Solve for x: $10^x = 0.9$

4.4 Exponential & Logarithmic Equations

3. If step 2 is not possible, **convert to logarithmic form**, or **find** either the ***log*** **or the** ***ln*** of both members of the equation. **Use properties of logarithms** to solve.

eg: $10^x = 8.06$ then $\log 8.06 = x$

or $\log 10^x = \log 8.06$

$x \log 10 = \log 8.06$ or $x = \log 8.06$

Solve for x: $10^x = 0.9$

either: $\log 0.9 = x$

or: $\log 10^x = \log 0.9$

$x \log 10 = \log 0.9$

$x = \log 0.9$

4.4 Exponential & Logarithmic Equations

3. If step 2 is not possible, **convert to logarithmic form**, or **find** either the ***log*** **or the** ***ln*** of both members of the equation. **Use properties of logarithms** to solve.

eg: $10^x = 8.06$ then $\log 8.06 = x$

or $\log 10^x = \log 8.06$

$x \log 10 = \log 8.06$ or $x = \log 8.06$

Solve for x: $e^x = 0.83$

4.4 Exponential & Logarithmic Equations

3. If step 2 is not possible, **convert to logarithmic form**, or **find** either the ***log*** **or the** ***ln*** of both members of the equation. **Use properties of logarithms** to solve.

eg: $10^x = 8.06$ then $\log 8.06 = x$

or $\log 10^x = \log 8.06$

$x \log 10 = \log 8.06$ or $x = \log 8.06$

Solve for x: $e^x = 0.83$

either: $\ln 0.83 = x$

or: $\ln e^x = \ln 0.83$

$x \ln e = \ln 0.83$

$x = \ln 0.83$

4.4 Exponential & Logarithmic Equations

3. If step 2 is not possible, **convert to logarithmic form**, or **find** either the ***log*** **or the** ***ln*** of both members of the equation.
Use properties of logarithms to solve.
eg: $10^x = 8.06$ then $\log 8.06 = x$
or $\log 10^x = \log 8.06$
$x \log 10 = \log 8.06$ or $x = \log 8.06$

Solve for x: $e^{4x-5} - 7 = 243$

4.4 Exponential & Logarithmic Equations

3. If step 2 is not possible, **convert to logarithmic form**, or **find** either the ***log*** **or the** ***ln*** of both members of the equation.
Use properties of logarithms to solve.
eg: $10^x = 8.06$ then $\log 8.06 = x$
or $\log 10^x = \log 8.06$
$x \log 10 = \log 8.06$ or $x = \log 8.06$

Solve for x: $e^{4x-5} - 7 = 243$

$e^{4x-5} = 250$

either: $\ln 250 = 4x-5$

or: $\ln e^{4x-5} = \ln 250$

$(4x-5) \ln e = \ln 250$
$4x-5 = \ln 250$
$4x = \ln 250 + 5$
$x = (\ln 250 + 5) / 4$

4.4 Exponential & Logarithmic Equations

To solve a logarithmic equation:

1. Convert to exponential form and solve.
   eg: $\log_5 x = 3$ then $5^3 = x$ and $x = 125$

2. Use Properties of Logarithms to obtain the form $\log_b M = \log_b N$
   Then $M = N$
   eg: $3 \log x = \log 125$
   $\log x^3 = \log 125$
   then: $x^3 = 125$
   and: $x = 5$

3. Check that solution is in the domain.

$$y = \log_b x \quad x > 0,\ b>0,\ b \neq 1$$

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4.4 Exponential & Logarithmic Equations

To solve a logarithmic equation:

1. Convert to exponential form and solve.
   eg: $\log_5 x = 3$ then $5^3 = x$ and $x = 125$
3. Check that solution is in the domain.

Solve for x: $\log_5 (x-7) = 2$

$$y = \log_b x \quad x > 0,\ b>0,\ b \neq 1$$

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4.4 Exponential & Logarithmic Equations

To solve a logarithmic equation:

1. Convert to exponential form and solve.
   eg: $\log_5 x = 3$ then $5^3 = x$ and $x = 125$
3. Check that solution is in the domain.

Solve for x: $\log_5 (x-7) = 2$

then $5^2 = x - 7$

and $x = 25 + 7 = 32$

check: $x - 7 = 32-7 > 0$

$y = \log_b x \quad x > 0, b>0, b \neq 1$

4.4 Exponential & Logarithmic Equations

To solve a logarithmic equation

2. Use Properties of Logarithms to obtain the form $\log_b M = \log_b N$
   Then $M = N$
   eg: $3 \log x = \log 125$ $\log x^3 = \log 125$
   then: $x^3 = 125$ and: $x = 5$
3. Check that solution is in the domain.

Solve for x: $\log (5x+1) = \log (2x+3) + \log 2$

$\log (5x+1) = \log [2(2x+3)]$

$\log (5x+1) = \log [4x+6]$

$5x+1 = 4x+6$

$x = 5$

$y = \log_b x \quad x > 0, b>0, b \neq 1$

4.3 Properties of Logarithms

Properties of Logarithms

1. $\log_b (1) = 0$
2. $\log_b (b) = 1$
3. $\log_b (MN) = \log_b(M) + \log_b(N)$
4. $\log_b \frac{M}{N} = \log_b(M) - \log_b(N)$
5. $\log_b M^n = n \log_b(M)$

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4.4 Exponential & Logarithmic Equations

Solve for x: $\log(x-2) + \log 5 = \log 100$

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4.4 Exponential & Logarithmic Equations

Solve for x: $\log(x-2) + \log 5 = \log 100$

$$\log[5(x-2)] = \log 100$$

$$5(x-2) = 100$$

$$x - 2 = 20$$

$$x = 22$$

$y = \log_b x \quad x > 0, b>0, b \neq 1$

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4.4 Exponential & Logarithmic Equations

Solve for x: $\log_4(x+2) - \log_4 (x-1) = 1$

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4.4 Exponential & Logarithmic Equations

Solve for x: $\log_4(x+2) - \log_4(x-1) = 1$

$x \neq 1$

$$\log_4\left[\frac{(x+2)}{(x-1)}\right] = 1$$

$$4^1 = \frac{(x+2)}{(x-1)}$$

$$4(x-1) = (x+2)$$

$$4x - 4 = x + 2$$

$$3x = 6$$

$$x = 2$$

$\log_4(2+2) - \log_4(2-1)$

$\log_4(4) - \log_4(1)$ OK x=2 is in the domain

$y = \log_b x \quad x > 0, b>0, b \neq 1$

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4.4 Exponential & Logarithmic Equations

$6 \ln(2x) = 30$

$7 + 3 \ln x = 6$

$3^{x/7} = 0.2$

$7^{(2x+1)} = 3^{(x+2)}$

$2^{2x} + 2^x - 12 = 0$

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4.4 Exponential & Logarithmic Equations

$6 \ln(2x) = 30$

$\ln(2x) = 5$

$e^5 = 2x$

$x = \frac{e^5}{2}$

$7 + 3 \ln x = 6$

$3 \ln x = -1$

$\ln x = -1/3$

$e^{-1/3} = x$

$x = \frac{1}{e^{1/3}}$

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4.4 Exponential & Logarithmic Equations

$3^{x/7} = 0.2$

$\log 3^{x/7} = \log 0.2$

$\frac{x}{7} \log 3 = \log 0.2$

$x = 7 \frac{\log 0.2}{\log 3}$

or

$\log_3 0.2 = x/7$

$x = 7 \log_3 0.2$

$7^{(2x+1)} = 3^{(x+2)}$

$\log 7^{(2x+1)} = \log 3^{(x+2)}$

$(2x+1)\log 7 = (x+2)\log 3$

$2x\log7+1\log7 = x\log3+2\log3$

$-x\log3 -1\log7 = -x\log3 -1\log7$

$x(2\log7 - \log3) = 2\log3 - \log7$

$x = \frac{2\log3 - \log7}{2\log7 - \log3}$

$x = \frac{\log(9/7)}{\log(49/3)} = 0.08997$

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4.4 Exponential & Logarithmic Equations

$$2^{2x} + 2^x - 12 = 0$$

$$u^2 + u - 12 = 0 \qquad u = 2^x$$

$$(u + 4)(u - 3) = 0 \qquad u^2 = (2^x)^2 = 2^{2x}$$

$u + 4 = 0$	$u - 3 = 0$
$u = -4$	$u = 3$
$2^x = -4$	$2^x = 3$
$+ \neq -$	$x \log 2 = \log 3$
Φ	$x = \frac{\log 3}{\log 2}$

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