2.3 Linear Functions and Slope

GOALS:

1. Understand the concept of slope for linear equations.
2. Find the slope of a line through 2 given points.
3. Find the linear equation given:
   - the slope of the line and the y-intercept
   - the slope of the line and any point
   - two points on the line
4. Understand how the slope of a line relates to increasing, decreasing, vertical and horizontal lines.
5. Recognize different forms of a linear equation:
   - slope-intercept form: $y = mx + b$
   - point-slope form: $y - y_1 = m(x - x_1)$
   - vertical line: $x = c$ and horizontal line: $y = k$

Study 2.3 CVC # 1-11; # 1-11; 15, 19, 23, 27, 31; 35-45, 49-55, 59, 61, 67-71

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2.3 Linear Functions and Slope

$$f(x) = 4x + 5$$

What does the graph of this function look like?

How do we know this is a linear equation?

VIDEO of above geogebra
http://www.battaly.com/collegealgebra/geogebra/slopeIntercept/

What part of the linear equation deals with steepness?

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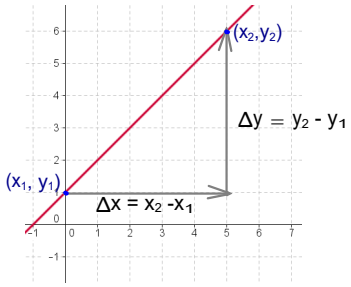
2.3 Linear Functions and Slope

What part of the linear equation deals with steepness?

Definition: Slope of a Non-vertical Line

Let (x_1, y_1) and (x_2, y_2) be two distinct points of a non-vertical line. Then,

$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$



VIDEO of above geogebra
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2.3 Linear Functions and Slope

Find the slope of the line through the points **(2, 3)** and **(4, 9)**.
Is the line increasing, decreasing, horizontal, or vertical?

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\square - \square}{\square - \square}$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\square - \square}{\square - \square}$$

$$m = \frac{\square}{\square} = \square$$

Is the line increasing, decreasing, horizontal, or vertical?

VIDEO of above geogebra
http://www.battaly.com/collegealgebra/geogebra/slopeIntercept/

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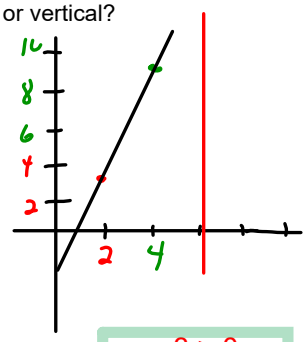
2.3 Linear Functions and Slope

(x_1, y_1) (x_2, y_2)

Find the slope of the line through the points (2, 3) and (4, 9).
Is the line increasing, decreasing, horizontal, or vertical?

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 3}{4 - 2}$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 3}{4 - 2}$$



$$m = \frac{6}{2} = 3 > 0$$

m = 3 > 0
so the line is **increasing**

Is the line increasing, decreasing, horizontal, or vertical?

VIDEO of above geogebra
http://www.battaly.com/collegealgebra/geogebra/slopeIntercept/

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2.3 Linear Functions and Slope

Find the slope of the line through the points **(-3,-2)** and **(3,8)**.
Is the line increasing, decreasing, horizontal, or vertical?

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\square - \square}{\square - \square}$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\square - \square}{\square - \square}$$

$$m = \frac{\square}{\square} = \square$$

Is the line increasing, decreasing, horizontal, or vertical?

VIDEO of above geogebra
http://www.battaly.com/collegealgebra/geogebra/slopeIntercept/

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2.3 Linear Functions and Slope

Find the slope of the line through the points (-3,-2) and (3,8).
Is the line increasing, decreasing, horizontal, or vertical?

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\square - \square}{\square - \square}$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - (-2)}{3 - (-3)}$$

If reverse points and start with (-3, -2)

$$\frac{-2 - (8)}{-3 - (3)} = \frac{-10}{-6} = \frac{5}{3}$$

$$m = \frac{10}{6} = \frac{5}{3} > 0$$

m = 5/3 > 0 so the line is **increasing**

Is the line increasing, decreasing, horizontal, or vertical?

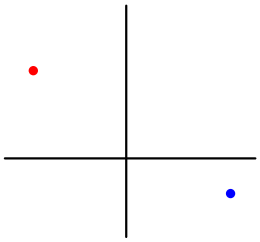
2.3 Linear Functions and Slope

Find the slope of the line through the points (-3,3) and (3,-1).
Is the line increasing, decreasing, horizontal, or vertical?

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\square - \square}{\square - \square}$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\square - \square}{\square - \square}$$

$$m = \frac{\square}{\square} = \square$$



Is the line increasing, decreasing, horizontal, or vertical?

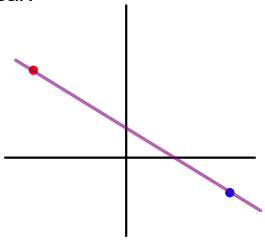
2.3 Linear Functions and Slope

Find the slope of the line through the points **(-3,3)** and **(3,-1)**.
Is the line increasing, decreasing, horizontal, or vertical?

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{3 - (-3)}$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{3 - (-3)}$$

$$m = \frac{-4}{6} = -2/3 < 0$$



m = -2/3 < 0
so the line is
decreasing

Is the line increasing, decreasing, horizontal, or vertical?

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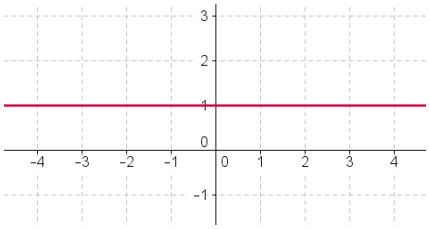
2.3 Linear Functions and Slope

Describe a line with slope, m = 0

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0}{a}, \quad a \neq 0$$

A line with slope = 0:

1. has the same y value for every x:
   $y = mx + b = 0 + b$
2. has the form $y = b$
3. is a horizontal line



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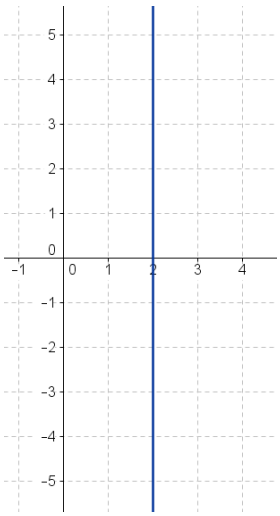
2.3 Linear Functions and Slope

Describe a line with an undefined slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{a}{0}$$

Division by 0 is not defined.



A line with an undefined slope:

1. has the same x value for every y.
2. has the form $x = c$
3. is a vertical line

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2.3 Linear Functions and Slope

If slope **m > 0**, then as x increases, y **increases**

If slope **m < 0**, then as x increases, y **decreases**

If slope **m = 0**, then as x increases, y **horizontal**

$m = \frac{\Delta y}{\Delta x} = 0 \quad \Delta y = 0$

If slope **m DNE**, then as x increases, y **vertical**

$m = \frac{\Delta y}{\Delta x}$ DNE $\Delta x = 0$

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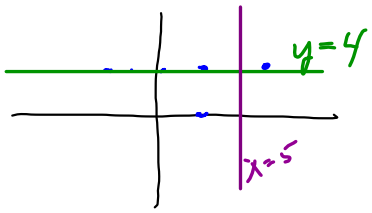
2.3 Linear Functions and Slope

$y=k$ $m=0$



$m=\frac{\Delta y}{\Delta x}$

vertical $\Delta x=0$

m undef. for. vertical line.

$x=k$

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2.3 Linear Functions and Slope

line eq

$(2,3)$ $(2,-4)$

$(-3,4)$ $(-3,2)$

$(\frac{1}{2},0)$ $(\frac{1}{2},-\frac{3}{4})$

$(\frac{1}{2},0)$ $(\frac{1}{3},0)$

$(2,-1)$ $(2,\frac{1}{10})$

$(2,-1)$ $(3,-1)$

$(2,-1)$ $(0,3)$

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2.3 Linear Functions and Slope

line

(2, 3) (2, −4) vertical $x = 2$

(−3, 4) (−3, 2) vertical $x = -3$

$(\frac{1}{2}, 0)$ $(\frac{1}{2}, -\frac{3}{4})$ vertical $x = \frac{1}{2}$

$(\frac{1}{2}, 0)$ $(\frac{1}{3}, 0)$ horiz $y = 0$

$(2, -1)$ $(2, \frac{1}{10})$ vertical $x = 2$

$(2, -1)$ $(3, -1)$ horiz. $y = -1$

$(2, -1)$ $(0, 3)$ decr.

$y = mx + b$

$y = -2x + 3$

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2.3 Linear Functions and Slope

$y = mx + b$

m slope

b y-intercept

(0, 1)

Consider the equation: $y = 4x + 1$

$$m = 4 = \frac{\Delta y}{\Delta x} = \frac{+4}{+1}$$

If x increases 1 unit,

then y _____

VIDEO of above geogebra
http://www.battaly.com/collegealgebra/geogebra/slopeIntercept/

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2.3 Linear Functions and Slope

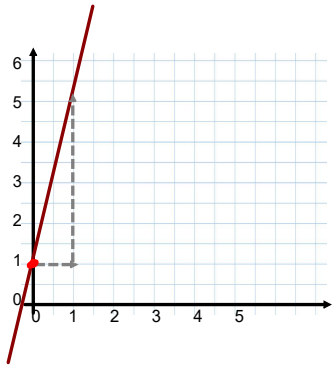
$x=0,\ y=1$

Consider the equation: $y = 4x + 1$

$$m = 4 = \frac{\Delta y}{\Delta x} = \frac{+4}{+1}$$

If x increases 1 unit,

then y incr. 4 units



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2.3 Linear Functions and Slope

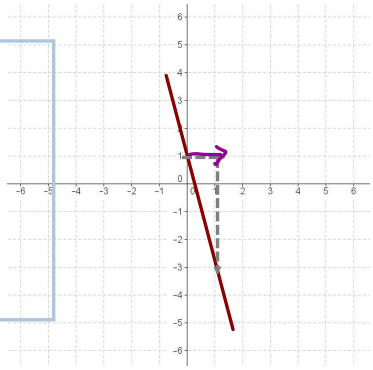
$-4 = \frac{-4}{1} = \frac{4}{-1}$

Consider the equation: $y = -4x + 1$

$$m = -4 = \frac{\Delta y}{\Delta x} = \frac{-4}{+1}$$

If x increases 1 unit,

then y _____



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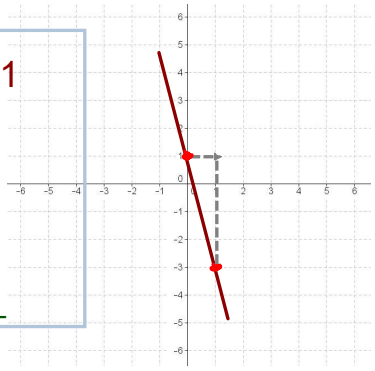
2.3 Linear Functions and Slope

Consider the equation: $y = -4x + 1$

$$m = -4 = \frac{\Delta y}{\Delta x} = \frac{-4}{+1}$$

If x increases 1 unit,

then y decr. 4 units



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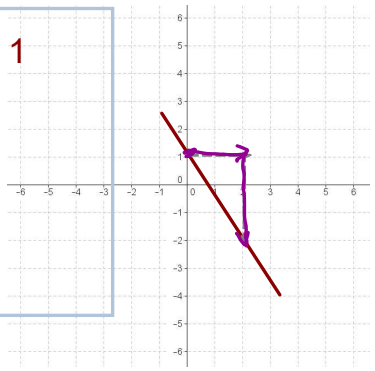
2.3 Linear Functions and Slope

Consider the equation: $y = \frac{-3}{2}x + 1$

$$m = \frac{-3}{2} = \frac{\Delta y}{\Delta x} = \frac{-3}{+2}$$

If x increases 2 units,

then y _____



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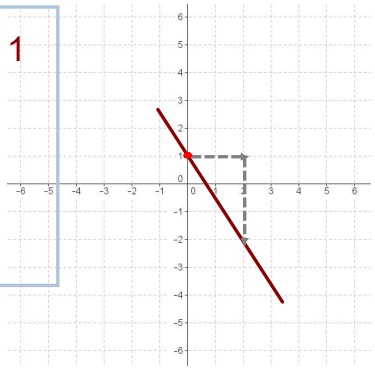
2.3 Linear Functions and Slope

Consider the equation: $y = \frac{-3}{2}x + 1$

$m = \frac{-3}{2} = \frac{\Delta y}{\Delta x} = \frac{-3}{+2}$

If x increases 2 units,

then y deer. 3 units



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2.3 Linear Functions and Slope

Slope-intercept Form
of a Linear Equation

$y = m x + b$

where m = slope
and b represents y-intercept (0,b)

What is the coefficient on y?

$\square y = m x + b$

$1 y = m x + b$

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2.3 Linear Functions and Slope

Given: m = 2, (0,1) Find: equation of line

1. Start with
Slope-intercept Form: _____

2. Substitute m = 2: _____

3. Substitute b = 1 _____

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2.3 Linear Functions and Slope

Given: m = 2, (0,1) Find: equation of line

1. Start with
Slope-intercept Form: y = mx + b

2. Substitute m = 2: y = 2x + b

3. (0,1) is the y-intercept since x=0
Substitute b = 1: y = 2x + 1

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2.3 Linear Functions and Slope

(0, b)

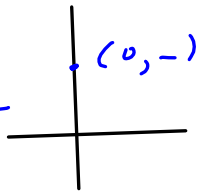
Given: m = 2, **(1, 0)** Find: equation of line

1. Start with

Slope-intercept Form: y = mx + b

2. Substitute m = 2: y = 2x + b

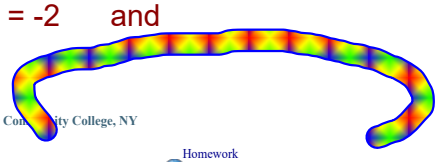
3. Substitute b = ?: y = 2x + ?



(1, 0) is NOT a y-intercept need (0, ___)

But, (1, 0) is a point on the line, so it is a solution of the equation. Therefore, to find b, we can **substitute x=1 and y=0**:

y = 2x + b looking for b
0 = 2(1) + b substitute x=1 and y=0
0 = 2 + b then b = -2 and
y = 2x - 2



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2.3 Linear Functions and Slope

Given: m = 3, **(-1, 0)** Find: equation of line

1. Start with

Slope-intercept Form: y = mx + b

2. Substitute m = ___: _____

3. Substitute x = ___, y = ___: _____

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2.3 Linear Functions and Slope

(0, b)

Given: m = 3, **(-1, 0)** Find: equation of line

1. Start with
Slope-intercept Form: $y = mx + b$

2. Substitute m = _3_: $y = 3x + b$

3. Substitute x = _-1_, y = _0_: $0 = 3(-1) + b$

Solve:
$0 = -3 + b$
$b = 3$
$y = 3x + 3$

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2.3 Linear Functions and Slope

Given: **m = 3**, **(-1, 0)** Find: equation of line

Alternative: $(y - y_1) = m(x - x_1)$ Point-slope Form

Substitute: $(y - \mathbf{0}) = \mathbf{3}(x - (\mathbf{-1}))$

Simplify:
$y = \mathbf{3}(x + 1)$
$y = 3x + 3$

Direct Substitution, but requires remembering

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2.3 Linear Functions and Slope

Derivation of Pt-Slope Form

not required -> can skip

Start: $m = \frac{y_2 - y_1}{x_2 - x_1}$

(x,y) represents any point on the line, each of which is a solution to the equation

Replace (x_2,y_2) with (x,y):

$$m = \frac{y - y_1}{x - x_1}$$

Multiply both members of eq. by $(x - x_1)$:

$$(x - x_1)\, m = \frac{y - y_1}{x - x_1}\,(x - x_1)$$

Simplify: $(x - x_1)\, m = (y - y_1)$

Rewrite: $(y - y_1) = m(x - x_1)$

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2.3 Linear Functions and Slope

Given: m = 2, (1, 0) Find: equation of line

$(y - y_1) = m(x - x_1)$ | $y = mx + b$

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2.3 Linear Functions and Slope

Given: m = 2, (1, 0) Find: equation of line

$(y - y_1) = m(x - x_1)$

$(y - 0) = 2\,(x - 1)$

$y = 2x - 2$

OR

$y = mx + b$

$y = 2x + b$
$0 = 2(1) + b$
$0 = 2 + b$
$b = -2$
$y = 2x - 2$

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2.3 Linear Functions and Slope

G: $m = \frac{1}{2}$, $(0, 0)$ F: sg.

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2.3 Linear Functions and Slope

G: $m = \frac{1}{2}$, $(0,0)$ F: eq.

$y = mx + b$

$b = 0$

$y = \frac{1}{2}x + 0$

$y = \frac{1}{2}x$

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2.3 Linear Functions and Slope

G: $m = -2$, $(0, -3)$ F: eq.

b

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2.3 Linear Functions and Slope

G: $m = -2$, $(0, -3)$ F: eq.

$y = mx + b$

$y = -2x - 3$

b = -3

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2.3 Linear Functions and Slope

G: $m = 8$, $(4, -1)$ F: eq.

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2.3 Linear Functions and Slope

G: $m = 8$, $(4, -1)$ F: eq.

$y - y_1 = m(x - x_1)$

$y - (-1) = 8(x - 4)$

$y + 1 = 8x - 32$

$y = 8x - 33$

$y = mx + b$

$y = 8x + b$

$-1 = 8(4) + b$

$-1 = 32 + b$

$b = -33$

$y = 8x - 33$

$x = 4$: $y = 8(4) - 33 = -1$ ✓

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2.3 Linear Functions and Slope

How do find equation when **2 points are given and no slope**?

Given: (___, ___) , (___, ___) Find: equation of line

Start by finding the slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Then use either the Slope-Intercept or
the Point-Slope Form of a Linear Equation

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2.3 Linear Functions and Slope

G: pts. (-2, -4) and (1, -1) F: linear eq. thru pts

2.3 Linear Functions and Slope

(x_1, y_1) (x_2, y_2)

G: pts. (-2, -4) and (1, -1) F: linear eq. thru pts

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-4)}{1-(-2)}$$

$$m = \frac{-1+4}{1+2} = \frac{3}{3} = 1$$

$y = mx + b$

$y - y_1 = m(x - x_1)$

$y - y_1 = m(x - x_1)$

$y - (-4) = 1(x - (-2))$

$y + 4 = x + 2$

$y = x - 2$

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2.3 Linear Functions and Slope

G: pts. (-2, -5) and (6, -5) F: linear eq. thru pts

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2.3 Linear Functions and Slope

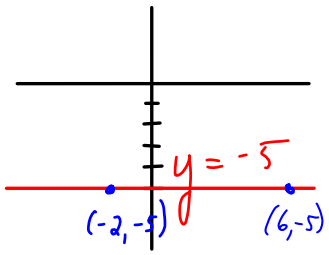
G: pts. (-2, -5) and (6, -5) F: linear eq. thru pts

$y = mx + b$

$m = \frac{\Delta y}{\Delta x} = \frac{-5-(-5)}{6-(-2)} = \frac{-5+5}{8}$

$m = 0$ $\therefore$ horiz. line $y = c$

$y = -5$



$y = 0x + b$

$y = 0x + b$

$(6, -5)$: $-5 = 0(6) + b = 0 + b$

$b = -5$

$y = 0x - 5$

$y = -5$

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2.3 Linear Functions and Slope

G: pts. (-3, 2) and (-3, -4) F: linear eq. thru pts

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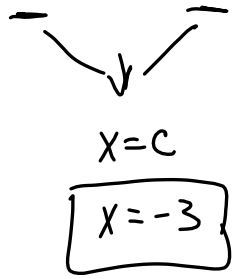

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2.3 Linear Functions and Slope

G: pts. (-2, 0) and (0, 2) F: linear eq. thru pts

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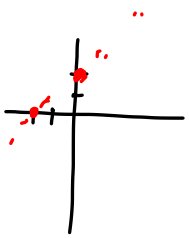
2.3 Linear Functions and Slope

y intercept

G: pts. (-2, 0) and (0, 2) F: linear eq. thru pts

$y - y_1 = m(x - x_1)$

$m = \frac{\Delta y}{\Delta x} = \frac{2-0}{0-(-2)} = \frac{2}{0+2} = 1$



$y - y_1 = m(x - x_1)$

$y - 2 = 1(x - 0)$

$y - 2 = x$

$y = x + 2$

$y = mx + b$

$= 1x + 2$

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Attachments

slope_intercept.mp4