1.5 Quadratic Equations

GOALS:
Find Solutions to quadratic equations by:

1. using Zero Product Principle
2. using the Square Root Property
3. Completing the Square Link to Complete Square
4. applying the Quadratic Formula Link to Quadratic Formula

Study 1.5 CVC # 1 - 7
1,5,9,13,... 105

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1.5 Quadratic Equations

General Form for Quadratic Equation in 1 Variable:

$$ax^2 + bx + c = 0$$

where a, b, c are real #s, $a \neq 0$

Find Solution to quadratic equation by:

1. factoring & Zero Product Principle
2. u^2 & Square Root Property
3. Completing the Square
4. Quadratic Formula

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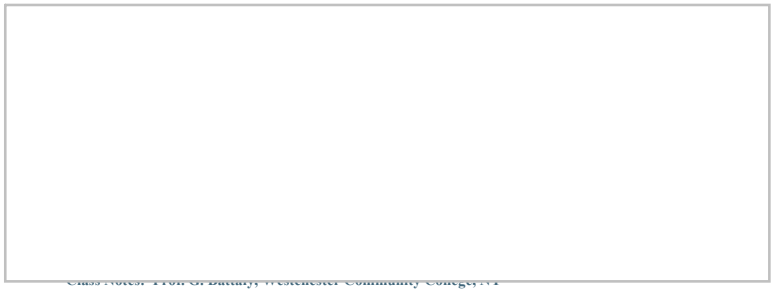
Skip to Quadratic Formula

1.5 Quadratic Equations

To solve a quadratic equation in factored form:

$u\,v = \mathbf{0}$	$u\,v = \mathbf{8}$	$\mathbf{u\,u} = 9$
$u = 0$	$u = ?$	$u^2 = 9$
$v = 0$	$v = ?$	$u = \pm 3$
can solve	can't solve: infinitely many	can solve



1.5 Quadratic Equations

To solve a quadratic equation in factored form:

$u\,v = \mathbf{0}$	$u\,v = \mathbf{8}$	$\mathbf{u\,u} = 9$
$u = 0$	$u = ?$	$u^2 = 9$
$v = 0$	$v = ?$	$u = \pm 3$
can solve	can't solve: infinitely many	can solve

(1)(8) = 8	(-1)(-8) = 8
(2)(4) = 8	(-2)(-4) = 8
(1/2)(16) = 8	(-1/2)(-16) = 8
(1/3)(24) = 8	(-1/3)(-24) = 8
...	...
...	...
...	...

1.5 Quadratic Equations

Solve: $x^2 - 16 = 0$ b=0

Can use different approaches to solving.

Factoring

$(x + 4)(x - 4) = 0$

$x + 4 = 0$ | $x - 4 = 0$

$x = -4$ | $x = 4$

Find Square Roots

$x^2 = 16$

$x = \pm\sqrt{16}$

$x = \pm 4$

$x = -4$ and $x = 4$ are both solutions.

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1.5 Quadratic Equations

Solve: $x^2 - 16 = 0$

Can use different approaches to solving.

Factoring

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1.5 Quadratic Equations

Solve: $x^2 - 19 = 0$

b=0

1.5 Quadratic Equations

Solve: $x^2 - 19 = 0$

$x^2 = 19$

$x = \pm\sqrt{19}$

$x = -\sqrt{19}$ and $x = \sqrt{19}$ are both solutions.

1.5 Quadratic Equations

Solve:

$$(x - 3)^2 = 36$$

b≠0, but left member is a perfect square

1.5 Quadratic Equations

Solve: $(x - 3)^2 = 36$

b≠0, but left member is a perfect square

$x - 3 = \pm\sqrt{36} = \pm 6$

$x^2 + 2x(-3) + 9 = 36$

$x^2 - 6x + 9 = 36$

$-36 = -36$

$x^2 - 6x - 27 = 0$

$(x - 9)(x + 3) = 0$

$x - 9 = 0$ | $x + 3 = 0$

$x = 9$ | $x = -3$

Expand, -> general form, factor, solve

$x - 3 = \pm 6$

$+3 \quad +3$

$x = 3 \pm 6 = \begin{cases} 3+6 \\ 3-6 \end{cases}$

$x = 9, -3$

$(3)(-9) = -27$

$(3) + (-9) = -6$

Square root property

1.5 Quadratic Equations

Solve:

$$(x-3)^2 = 36$$

$$x-3 = \pm 6$$
$$x = 3 \pm 6$$
$$x = -3,\ 9$$

1.5 Quadratic Equations

$b \neq 0$, but left member is almost a perfect square

Solve:

$$3(x + 4)^2 = 21$$

1.5 Quadratic Equations

Solve:

$$3(x + 4)^2 = 21$$

$$(x + 4)^2 = 7$$

$$x + 4 = \pm\sqrt{7}$$

$$x = -4 \pm \sqrt{7}$$

1.5 Quadratic Equations

Solve:

$b \neq 0$, but left member is a perfect square

$$(x + 2)^2 = -7$$

1.5 Quadratic Equations

Solve:

$$(x + 2)^2 = -7$$

$$x + 2 = \pm\sqrt{-7}$$
$$x = -2 \pm i\sqrt{7}$$

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1.5 Quadratic Equations

b=0

$$x^2 - 9 = 8$$

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1.5 Quadratic Equations

b=0

$x^2 - 9 = 8$

$x^2 = 17$

$x = \pm\sqrt{17}$

$x^2 - 17 = 0$

$(x+\sqrt{17})(x-\sqrt{17}) = 0$

$x+\sqrt{17} = 0 \quad | \quad x-\sqrt{17} = 0$

$x = -\sqrt{17} \quad | \quad x = \sqrt{17}$

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1.5 Quadratic Equations

Solve: $x^2 + 5x + 6 = 0$ b≠0

Not a perfect square trinomial.

? $(\ )(\ ) = 6$?
$(\ ) + (\ ) = 5$

$(2)(3) = 6$
$(2) + (3) = 5$

Can factor and use Zero Product Principal (ZPP)

$(x + 2)(x + 3) = 0$

$x + 2 = 0 \quad | \quad x + 3 = 0$

$x = -2 \quad | \quad x = -3$

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1.5 Quadratic Equations

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

$b \neq 0$

Solve:

$$x^2 + 6x + 9 = 0$$

x 2 $\qquad$ 3 2

$$6x = 2\,(3)\,x$$

$\therefore$ Perfect square trinomial. Use square root property.

$$(x + 3)^2 = 0$$

$$x + 3 = \pm 0 = 0$$

$$x = -3$$

1.5 Quadratic Equations

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

$b \neq 0$

Solve:

$$x^2 + 8x + 16 = 0$$

1.5 Quadratic Equations

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

$b \neq 0$

Solve:

$$x^2 + 8x + 16 = 0$$

$$(x + 4)^2 = 0$$

$$x + 4 = 0$$

$$x = -4$$

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1.5 Quadratic Equations

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

$b \neq 0$

Solve: $x^2 + 6x = -9$

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1.5 Quadratic Equations

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

Solve: $x^2 + 6x = -9$ $b \neq 0$

Write in standard form.

$x^2 + 6x + 9 = 0$

Same as previous.

$(x + 3)^2 = 0$

$x + 3 = 0$

$x = -3$

1.5 Quadratic Equations

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

$b \neq 0$

Solve: $x^2 + 6x + 7 = 0$

Not a Perfect square trinomial. Not factorable w. integers.

Can rewrite as a perfect square trinomial?

$x^2 + 6x = -7$ APE

$x^2 + 6x + \_\_\_ = -7 + \_\_\_$

What can we add to make the left member a perfect square trinomial?

$x^2 + 6x + 9 = -7 + 9 = 2$ APE

1.5 Quadratic Equations

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

Solve: $x^2 + 6x + 7 = 0$ $b \neq 0$

$x^2 + 6x + \underline{9} = -7 + \underline{9} = 2$

$(x + 3)^2 = 2$

$x + 3 = \pm\sqrt{2}$

$x = -3 \pm \sqrt{2}$

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1.5 Quadratic Equations

Solve: $x^2 + 6x + 5 = 0$

Can we solve using the square root property?

Need a perfect square. > Create one by APE.

$x^2 + 6x = -5$ APE

$x^2 + 6x + \_\_\_ = -5 + \_\_\_$

What can we add to make the left member a perfect square trinomial?

Add $\left(\frac{b}{2}\right)^2$ to get: $\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2$

$= 3^2 = 9$

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

Note: coefficient on x^2 term = 1

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1.5 Quadratic Equations

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

Solve: $x^2 + 6x + 5 = 0$ $b = 2c \neq 0$

$x^2 + 6x + \underline{9} = -5 + \underline{9} = 4$

$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2$

$= 3^2 = 9$

$(x + 3)^2 = 4$

$x + 3 = \pm 2$

$x = -3 \pm 2$

$x = -5, -1$

Was factorable, but this might be easier for problems that are more difficult to factor.

1.5 Quadratic Equations

Solve: $x^2 + 10x - 11 = 0$

Can we solve using completing the square?

Need a perfect square. > Create one by APE.

$x^2 + 10x = 11$

$x^2 + 10x + \_\_\_ = 11 + \_\_\_$

What can we add to make the left member a perfect square trinomial?

Add $\left(\frac{b}{2}\right)^2$ to get:

$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2$

$(x + b/2)^2 = x^2 + bx + (b/2)^2$ $= 5^2 = 25$

Note: coefficient on x^2 term = 1

1.5 Quadratic Equations

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

Solve: $x^2 + 10x - 11 = 0$ $b \neq 0$

$x^2 + 10x + \underline{25} = 11 + \underline{25}$

$(x + 5)^2 = 36$

$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2$

$= 5^2 = 25$

$x + 5 = \pm 6$

$x = -5 \pm 6$

$x = -11, 1$

Was factorable, but this might be easier.

1.5 Quadratic Equations

Solve: $2x^2 + 20x - 12 = 0$ $\frac{2(x^2 + 10x - 6)}{2} = \frac{0}{2}$

Can we solve using completing the square?
1st need a=1.

$x^2 + 10x - 6 = 0$

Need a perfect square. > Create one by APE.

$x^2 + 10x = 6$

$x^2 + 10x + \underline{\quad} = 6 + \underline{\quad}$

What can we add to make the left member a perfect square trinomial?

Add $\left(\frac{b}{2}\right)^2$ to get:

$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2$

$= 5^2 = 25$

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

Note: coefficient on x^2 term = 1

1.5 Quadratic Equations

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

Solve: $2x^2 + 20x - 12 = 0$ $\quad b \neq 0$

$x^2 + 10x + \underline{25} = 6 + \underline{25}$

$(x + 5)^2 = 31$

$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2$

$= 5^2 = 25$

$x + 5 = \pm\sqrt{31}$

$x = -5 \pm \sqrt{31}$

1.5 Quadratic Equations

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

$2x^2 + 5x - 3 = 0$

$x = -3,\ 1/2$

1.5 Quadratic Equations

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

$$2x^2 + 5x - 3 = 0$$

$$\frac{2x^2}{2} + \frac{5}{2}x - \frac{3}{2} = \frac{0}{2}$$ MPE: to get $(1)x^2$

$$x^2 + \frac{5}{2}x - \frac{3}{2} = 0$$

$$x^2 + \frac{5}{2}x = \frac{3}{2}$$

$b = 5/2$
$b/2 = 5/4$
$(b/2)^2 = (5/4)^2 = 25/16$

$$x^2 + \frac{5}{2}x + \boxed{\frac{25}{16}} = \frac{3}{2} + \boxed{\frac{25}{16}}$$

$$\left(x + \frac{5}{4}\right)^2 = \frac{24}{16} + \frac{25}{16} = \frac{49}{16}$$

$$x + \frac{5}{4} = \pm\sqrt{\frac{49}{16}} = \pm\frac{7}{4}$$

$$x = -\frac{5}{4} \pm \frac{7}{4} = \frac{2}{4}, \frac{-12}{4} = \frac{1}{2}, -3$$

or:

$$x = \begin{cases} (-5+7)/4 = 2/4 = 1/2 \\ (-5-7)/4 = -12/4 = -3 \end{cases}$$

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1.5 Quadratic Equations

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

$2x^2 - 4x - 1 = 0$

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1.5 Quadratic Equations

$(x + b/2)^2 = x^2 + bx + (b/2)^2$

$2x^2 - 4x - 1 = 0$

$x^2 - 2x - \frac{1}{2} = 0$

$\frac{-2}{2} = -1$

$(-1)^2 = 1$

$x^2 - 2x + 1 = \frac{1}{2} + 1$

$(x-1)^2 = \frac{1}{2} + \frac{2}{2} = \frac{3}{2}$

$x - 1 = \pm\sqrt{\frac{3}{2}}$

$x = 1 \pm \sqrt{3/2}$

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1.5 Quadratic Equations

Solve:

$$x^2 + 11x - 20 = 0$$

Use APE with $\left(\frac{b}{2}\right)^2$

$u^2 = n$

$u = \pm\sqrt{n}$

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1.5 Quadratic Equations

Solve: $x^2 + 11x = 20$

$u^2 = n$

$u = \pm\sqrt{n}$

$$x^2 + 11x + \left[\frac{11}{2}\right]^2 = 20 + \left[\frac{11}{2}\right]^2$$

$$\left(x + \frac{11}{2}\right)^2 = 20 + \left[\frac{11}{2}\right]^2$$

$$x + \frac{11}{2} = \pm\sqrt{20 + \left[\frac{11}{2}\right]^2}$$

$$x = -\frac{11}{2} \pm \sqrt{(80/4) + (121/4)}$$

$$x = -\frac{11}{2} \pm \sqrt{(201/4)} = \frac{-11 \pm \sqrt{201}}{2}$$

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1.5 Quadratic Equations

Solve: $ax^2 + bx + c = 0$

ALWAYS:
1. Write formula.
2. Write values for a, b, and c.
3. Be careful with substitution, esp. negatives.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$x^2 + 11x - 20 = 0$

$a = 1,\ b = 11,\ c = -20$

$$x = \frac{-11 \pm \sqrt{11^2-4(1)(-20)}}{2(1)} = \frac{-11 \pm \sqrt{121+80}}{2}$$

$$x = \frac{-11 \pm \sqrt{201}}{2}$$

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1.5 Quadratic Equations

Solve: $ax^2 + bx + c = 0$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

ALWAYS:
1. Write formula.
2. Write values for a, b, and c.
3. Be careful with substitution, esp. negatives.

1. $x^2 + 10x - 11 = 0$
2. $2x^2 + 5x - 3 = 0$
3. $2x^2 - 4x - 1 = 0$

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1.5 Quadratic Equations

Solve: $ax^2 + bx + c = 0$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$x^2 + 10x - 11 = 0$

ALWAYS:
1. Write formula. ✓
2. Write values for a, b, and c. ✓
3. Be careful with substitution, esp. negatives.

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1.5 Quadratic Equations

Solve: $ax^2 + bx + c = 0$

$$x^2 + 10x - 11 = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a = 1$
$b = 10$
$c = -11$

$$x = \frac{-10 \pm \sqrt{10^2-4(1)(-11)}}{2(1)}$$

$$= \frac{-10 \pm \sqrt{100+44}}{2} = \frac{-10 \pm \sqrt{144}}{2}$$

$$= \frac{-10 \pm 12}{2} = -\frac{10}{2} \pm \frac{12}{2} = -5 \pm 6 = \begin{cases} -5+6 = 1 \\ -5-6 = -11 \end{cases}$$

ALWAYS:
1. Write formula.
2. Write values for a, b, and c.
3. Be careful with substitution, esp. negatives.

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1.5 Quadratic Equations

Solve: $ax^2 + bx + c = 0$

2. $2x^2 + 5x - 3 = 0$

ALWAYS:
1. Write formula.
2. Write values for a, b, and c.
3. Be careful with substitution, esp. negatives.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

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1.5 Quadratic Equations

$ax^2 + bx + c = 0$

Solve: 2. $2x^2 + 5x - 3 = 0$

ALWAYS:
1. Write formula. ✓
2. Write values for a, b, and c. ✓
3. Be careful with substitution, esp. negatives.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a = 2$
$b = 5$
$c = -3$

$$x = \frac{-5 \pm \sqrt{5^2-4(2)(-3)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{25+24}}{4}$$

$$= \frac{-5 \pm \sqrt{49}}{4} = \frac{-5 \pm 7}{4} = \begin{cases} \frac{-5+7}{4} = \frac{2}{4} = \frac{1}{2} \\ \frac{-5-7}{4} = \frac{-12}{4} = -3 \end{cases}$$

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1.5 Quadratic Equations

$ax^2 + bx + c = 0$

Solve: 3. $2x^2 - 4x - 1 = 0$

ALWAYS:
1. Write formula.
2. Write values for a, b, and c.
3. Be careful with substitution, esp. negatives.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

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1.5 Quadratic Equations $ax^2 + bx + c = 0$

Solve: 3. $2x^2 - 4x - 1 = 0$

ALWAYS:
1. Write formula.
2. Write values for a, b, and c.
3. Be careful with substitution, esp. negatives.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a = 2$
$b = -4$
$c = -1$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2-4(2)(-1)}}{2(2)}$$

$\sqrt{24} = \sqrt{4}\sqrt{6}$

$$= \frac{4 \pm \sqrt{16+8}}{4} = \frac{4 \pm \sqrt{24}}{4} = \frac{4 \pm 2\sqrt{6}}{4}$$

$$= 1 \pm \frac{1}{2}\sqrt{6} \qquad \frac{2 \pm \sqrt{6}}{2}$$

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1.5 Quadratic Equations $ax^2 + bx + c = 0$

Derivation of the Quadratic Formula: not required

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\frac{\frac{b}{a}}{2} = \frac{b}{2a} \qquad \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a}\cdot\frac{4a}{4a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}} = \pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Quadratic Formula

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1.5 Quadratic Equations

Practice

1. $5x^2 - 20x = 0$
2. $2x^2 - 7x + 3 = 0$
3. $2x^2 = 250$

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1. $x = 0, 4$
2. $x = 1/2, 3$
3. $x = \pm 5\sqrt{5}$